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Incentive Effects of Bonus Taxes in a Principal-Agent Model*

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Abstract

Several countries have implemented bonus taxes for corporate executives in response to the financial crisis of 2007-2010. Using a principal-agent model, this paper investigates the incentive effects of bonus taxes by analyzing the agent’s and principal’s behavior. Specifically, we show how bonus taxes affect the agent’s incentives to exert effort and the principal’s decision regarding the composition of the compensation package (fixed salary and bonus rate). We find that, surprisingly, a bonus tax can increase the bonus rate and decrease the fixed salary. In addition, a bonus tax can induce the principal to pay higher bonuses even though the agent’s effort always decreases.

Keywords: Principal-agent model, bonus tax, executive compensation, incentive, pay regulation

JEL Classification: H24, J30, M52

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1 Introduction

In response to the recent financial crisis that occurred from 2007 to 2010, several countries have implemented bonus taxes for corporate executives in firms that received large amounts of federal bailout funds. For example, the US House of Representatives approved a 90% tax on bonuses in such firms. Similarly, Ireland introduced in January 2011 a 90% tax on executives’ bonuses in banks that received government support. Moreover, in the UK, a bonus tax of 50% was imposed on bankers’ bonuses for a period of several months in 2010. In Switzerland, the Council of States approved a tax on executive bonuses above CHF 3 million in December 2010.

Despite their political relevance, the economic effects of bonus taxes have received little attention in academic research. This paper tries to fill part of this gap by developing basic insights into the functioning and consequences of bonus taxes on executive pay based on the principal-agent model of Holmstrom and Milgrom (1987). We introduce a tax that is levied on the agent’s bonus to analyze how it affects the agent’s incentive to exert effort, the composition of executives’ compensation packages (fixed salary and bonus rate) and bonus payments. The objective of our paper, however, is not to provide a normative analysis about the desirability of bonus taxes but rather, given their existence in the real world, to analyze the incentive effects of such taxes.\(^1\)

In our model, the principal chooses the fixed salary and the bonus rate by satisfying the agent’s participation constraint and anticipating the agent’s optimal effort level. For a general effort cost function, the agent reacts to a higher bonus tax with lower effort, while the behavior of the principal depends on the risk parameter (product of the agent’s risk aversion and variance in the firm value). Basic intuition might suggest that by taxing the agent’s bonus, the compensation package is shifted to the fixed salary. However, the opposite can occur. Moreover, a bonus tax can lead to the counterintuitive result that both the fixed salary and the bonus rate increase. We can show that a substitution effect between these components exists for a quadratic effort cost function. In this case, an increase (decrease) in the bonus rate always leads to a decrease (increase) in the fixed salary. For quadratic effort costs, we also find that the agent’s tax-induced effort reduction decreases with a higher risk parameter. Surprisingly, a higher bonus tax induces the principal to pay higher bonuses if the risk parameter is sufficiently high.

In what follows, we provide a short literature review. In the early 1980s, research on executive compensation paralleled the emergence and acceptance of agency theory. Jensen and Meckling (1976), Mirrlees (1976), Holmstrom (1979, 1982), Lazear and Rosen (1981), and Grossman and Hart (1983) provide the theoretical framework for the literature on executive compensation. Diverging interests between shareholders and executives pose a

\(^1\)Moreover, we do not aim to investigate how a bonus tax influences the risk-taking behavior of corporate executives.
prototypical agency problem, which may be solved by appropriate compensation. The theory of optimal contracts suggests that compensation has to be set such that executives maximize shareholder value (Murphy, 1999 and Core et al., 2003). Bebchuk et al. (2002) and Bebchuk and Fried (2003) distinguish this view from the perspective that executives control their own compensation. According to their analysis, executives may be paid more than necessary to work for a certain firm. Both market forces as well as an executive’s power exert influence on the level of compensation.

In addition to the categorization of the literature in terms of approaches between the perspective of optimal contracting and the managerial power approach, the literature has also been categorized in articles on the relationship between pay and performance and on the relationship between pay and behavior. Research on executive pay analyzes not only the character and the strength of the link between pay and performance indicators but also the consequences of pay on executives’ behavior and the alignment of executive and firm objectives. For comprehensive surveys of research on executive compensation, see Gomez-Mejia et al. (1985), Murphy (1999), Core et al. (2003), and Devers et al. (2007).

Despite the large body of literature and numerous theoretical and empirical studies on executive compensation, only a few papers have addressed the consequences of executive compensation regulation in general and the effects of bonus taxes in particular. For example, Dew-Becker (2009) reviewed the history of government rules and regulations in the US that affect executive compensation. By discussing disclosure rules, advancements in corporate governance, and say-on-pay, Dew-Becker analyzed the evolution of pay regulation and concluded that mandatory say-on-pay could be the most effective and least harmful measure of controlling executive compensation. Knutt (2005) examined diverse regulatory issues from a legal point of view. He claimed that the various attempts to regulate executive compensation, such as the disclosure and tax regulations, have not yet been effective.

Hall and Liebman (2000) analyzed the extent to which tax policy influences the composition of executive compensation and discussed the consequences of rising stock-based pay. First, they found that tax rate changes have not played a major role in the dramatic explosion in executive stock-option pay since 1980. Second, they found evidence that the million-dollar rule, which limits the corporate deductibility of non-performance-related executive compensation to US$ 1 million, leads firms to adjust the composition of their payments towards performance-related pay. Third, they examined whether there is evidence that firms significantly shift the timing of option exercises in response to changes in tax rates. Unlike Hall and Liebman (2000), who concentrated on a tax on stock-based

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2Gaston (1997) shows that the separation of ownership and control may increase the value of the firm’s implicit contracts with workers.

pay, we studied a tax that is levied on the agent’s bonus.

Katuščák (2004) focused on the incentives of compensation packages and the consequences of different tax rate implementations by separating the compensation package in a salary into an option gain and a stock gain component. Based on this framework, a tax on the entire compensation package decreases the equilibrium effort and the after-tax pay-performance sensitivity (PPS), and the effect on the pre-tax PPS is ambiguous. In addition, he empirically investigated variations in personal income tax rates and combined federal and state tax rates in the US during the period 1992-1996. Contrary to Katuščák (2004), this paper focuses on the consequences of a bonus tax levied on the variable salary. Moreover, our analysis focuses on the tax-induced effects on the principal and agent’s behaviors regarding the composition of the compensation package and effort level.

Radulescu (2010) analyzed the effects of bonus taxes on manager compensation and welfare. She based her results on a principal-agent model with quadratic effort costs in a two-country framework and considered two different scenarios for the firm’s relocation possibilities. She showed that the pay-performance sensitivity unambiguously increases in the country with a bonus tax. Furthermore, the country that does not implement a bonus tax suffers from lower welfare. If, however, relocation is not possible, welfare in the tax-free country is higher than it is in the country that has implemented a bonus tax. Radulescu (2010) focused her model on a macroeconomic two-country perspective including relocation possibilities and concentrated on tax incidence. In contrast, we focus on the incentive effects of bonus taxes in a microeconomic one-country perspective and show how that a bonus tax can have an ambiguous effect on the agent’s compensation package (fixed salary and bonus rate).

The remainder of the paper is structured as follows. Section 2 introduces our principal-agent model with its main assumptions and notation. It also presents the computations for the optimality conditions and derives the effects of bonus taxes for a general cost function. In Section 3, we specify the agent’s cost function and consider quadratic effort costs. Section 4 extends the analysis for a general polynomial cost function. Finally, Section 4 discusses the main insights and presents our conclusions.

2 Model

2.1 Notation and Assumptions

Our model is based on the principal-agent model of Holmstrom and Milgrom (1987) and introduces a tax denoted by $\tau \in (0, 1)$ that is levied on the agent’s variable salary (bonus). We consider a single-period employment relationship in a firm between a risk-neutral principal (e.g., a firm’s owner) and a risk-averse agent (e.g., CEO). The agent chooses
the unobservable action (effort) $a \in \mathbb{R}_0^+$ to produce a firm value given by $x = a + \varepsilon$, where $\varepsilon$ is a normally distributed error term with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ representing potential effects on the firm value beyond the agent’s control.\textsuperscript{4} A high variance in the error term $\sigma_\varepsilon^2$ can be interpreted as a more uncertain economic environment that creates a high variance in the firm value or a situation in which the agent’s performance cannot be measured precisely.

Because the principal can only observe the firm value $x$, the agent’s effort $a$ cannot be specified in a legally enforceable contract. The agent’s effort generates costs according to a strictly convex cost function $c(a) \in C^3$ that satisfies the following assumptions:\textsuperscript{5}

A1. $c'(a) > 0, c''(a) > 0$ for $a > 0, c'(0) = 0, c''(0) = 0$ and $\lim_{a\to\infty} c'(a) = \infty$.

In line with the agency literature, we assume that the principal offers the agent a linear employment contract that generates a payoff to the agent according to\textsuperscript{6}

$$p(x) \equiv \delta + (1 - \tau)\gamma x = \delta + (1 - \tau)\gamma (a + \varepsilon),$$

where $\delta \in \mathbb{R}_0^+$ is the fixed salary component and $\gamma x$ is the variable salary or bonus paid by the principal. We follow Gibbons (1998, p. 116) and refer to the incentive-based component $\gamma \in (0, 1)$ as the bonus rate.\textsuperscript{7} It should be noted that the gross salary paid by the principal is given by $s(x) \equiv \delta + \gamma x$, and that $p(x) = \delta + (1 - \tau)\gamma x$ is the net-of-tax salary received by the agent. We further assume that the agent has an exogenously-given outside option, represented by his reservation utility $\hat{u} \in \mathbb{R}_0^+$.\textsuperscript{8} The reservation utility can be interpreted as the utility the agent would receive in another firm or in a country without a bonus tax and therefore is assumed to be exogenous. The state receives the difference between gross and net salary as tax revenue $\tau\gamma x$. Because our paper focuses on bonus taxes, we do not consider other taxes such as income taxes on the agent’s salary.

From the properties of the normal distribution, we derive that the agent’s (net-of-tax)\textsuperscript{4}

Because we want focus on the incentive effects of bonus taxes in a microeconomic environment of a firm and its employee, we neglect possible negative externalities of the agent’s behavior on the economy as a whole. Hence, in our model, we do not analyze how a bonus tax might correct potential inefficiencies of the employment contract between the principal and the agent.

\textsuperscript{5}Note that we will require a weaker condition than $\lim_{a\to\infty} c'(a) = \infty$ in our proof for the existence and uniqueness of the equilibrium.

\textsuperscript{6}Linear contracts are widely used in the literature because of their analytical convenience (see, e.g., Feltham and Xie, 1994; Baker, 2002; Hughes et al., 2005). Holmstrom and Milgrom (1987) found the optimal dynamic compensation scheme to be linear. Hellwig and Schmidt (2002) show that the result of Holmstrom and Milgrom does not only apply to continuous-time but also to discrete-time settings. Bhattacharyya and Lafontaine (1995) and Kim and Wang (1998) further show that linear contracts can be an optimal means to resolve double moral hazard problems; Laffont and Tirole (1986) find that linear contracts can constitute optimal contracts in adverse selection settings.

\textsuperscript{7}Note that in the literature on executive compensation $\gamma$ is also referred to as the pay-performance sensitivity (PPS) or piece rate (see, e.g., Jensen and Murphy, 1990; Murphy, 1999; Conyon and Sadler, 2001; and Faulkender et al., 2010).

\textsuperscript{8}In Section 2.2, we analyze the consequences of an endogenous outside option.
salary $p(x)$ is also normally distributed with

$$p(x) \sim N \left( \delta + (1 - \tau)\gamma a; (1 - \tau)^2\gamma^2\sigma^2_x \right).$$

Thus, the expected salary of the agent is given by $E[p] = \delta + (1 - \tau)\gamma a \equiv \bar{p}$, and the variance of the salary is $V[p] = (1 - \tau)^2\gamma^2\sigma^2_x \equiv \sigma^2_{p}$. We assume that the agent is risk-averse with a constant absolute risk-averse (CARA) utility function that is given by the negative exponential function

$$U(p, a) = -e^{-r(p - c(a))},$$

where $r \in \mathbb{R}^+$ is the Arrow-Pratt measure of the agent’s level of absolute risk aversion. The expected value of this utility $E[U]$ yields $E[U] = \bar{p} - r\sigma^2_x/(1 - \tau)^2\gamma^2 - c(a)$. Using a monotonic transformation, which preserves the ordering, we conclude that the agent’s expected utility $E[U_A]$ (i.e., the certainty equivalent minus costs) yields

$$E[U_A] = \delta + (1 - \tau)\gamma a - \frac{r\sigma^2_x}{2}(1 - \tau)^2\gamma^2 - c(a).$$

The term $RP \equiv (1/2)r\sigma^2_x/(1 - \tau)^2\gamma^2$ is the agent’s risk premium required to compensate him for the uncertainty in his expected salary.

The principal is assumed to be risk neutral because she is well diversified. Her profit $\pi_P$ is the difference between firm value and the agent’s gross salary: $\pi_P \equiv x - s(x) = (1 - \gamma)x - \delta$. Hence, the principal’s expected profit is given by

$$E[\pi_P] \equiv (1 - \gamma)a - \delta.$$

The timing is as follows. In $t = 0$, the state sets a certain level for the bonus tax $\tau \in (0, 1)$ that is levied on the agent’s bonus payment. In $t = 1$, the principal offers the agent an employment contract with a fixed salary of $\delta$ and a bonus rate of $\gamma$. The agent accepts this contract if it guarantees him at least his reservation utility, which is given by $\hat{u}$. In $t = 2$, after accepting the contract, the agent exerts effort $a$. In $t = 3$, the firm value $x$ is realized, and all the payments are made in $t = 4$.

### 2.2 Optimality Conditions and General Results

The agent maximizes his expected net utility $E[U_A]$ with respect to the effort level $a$ such that the maximization problem is given by

$$\max_{a \geq 0} \left\{ E[U_A] = \delta + (1 - \tau)\gamma a - \frac{r\sigma^2_x}{2}(1 - \tau)^2\gamma^2 - c(a) \right\}.$$ 

\[9\text{See Appendix A.1 for a detailed derivation of the agent’s expected net utility.}\]
The principal maximizes her expected profit $E[\pi_P]$ and solves the following maximization problem:

$$\max_{(\delta, \gamma) \geq 0} \{ E[\pi_P] = (1 - \gamma)a - \delta \},$$

subject to

$$E[U_A(a^*)] = \delta + (1 - \tau)\gamma a^* - \frac{r\sigma^2}{2}\gamma^2(1 - \tau)^2 - c(a^*) \geq \hat{u}, \quad (1)$$

$$a^* \in \arg\max_{a \geq 0} E[U_A]. \quad (2)$$

The first constraint is the participation constraint, which guarantees that the agent receives at least his reservation utility $\hat{u}$. The second constraint represents the incentive compatibility constraint derived from the agent’s maximization problem.

The following proposition establishes the existence and uniqueness of the equilibrium and presents the optimality conditions deduced from the maximization problems of the agent and the principal.

**Proposition 1** For a cost function $c(a)$ that satisfies A1, the following are true:

(i) The equilibrium $(\gamma^*, \delta^*, a^*)$ exists and is unique.

(ii) The principal sets the optimal compensation package $(\gamma^*, \delta^*)$ as

$$\gamma^* = \frac{(1 - \tau) - a^*c''(a^*)\tau}{(1 - \tau)[1 + (1 - \tau)c''(a^*)r\sigma^2]^{1/2}} \quad \text{and} \quad \delta^* = \hat{u} - c'(a^*)a^* + \frac{c''(a^*)^2}{2}r\sigma^2 + c(a^*). \quad (3)$$

(iii) The agent exerts optimal effort $a^*$ according to

$$c'(a^*) = (1 - \tau)\gamma^*. \quad (4)$$

**Proof.** See Appendix A.2. ■

According to Proposition 1, a unique equilibrium exists for a general cost function. The optimal bonus rate $\gamma^*$, the optimal fixed salary $\delta^*$, and the optimal effort level $a^*$ in equilibrium are defined implicitly by equations (3) and (4). The proposition further shows that the bonus rate $\gamma^*$ in equilibrium depends on the following factors: the agent’s absolute risk aversion $r$, the variance in the firm value $\sigma^2$, the curvature of the agent’s effort cost $c''(a^*)$, the bonus tax $\tau$ and the agent’s equilibrium effort $a^*$.

For notational simplicity, the parameter $\rho$ stands for the product $r\sigma^2$ of the agent’s level of risk aversion $r$ and the variance in the firm value $\sigma^2$, i.e., $\rho \equiv r\sigma^2$. We refer to $\rho$ as the “risk parameter”. If the risk parameter $\rho$ is zero, that is, if there is no uncertainty in the economic environment such that the variance in the firm value is zero ($\sigma^2 = 0$) and/or the agent is risk neutral ($r = 0$), then the optimal bonus rate is $\gamma^* = 1 - \frac{a^*c''(a^*)\tau}{1 - \tau}$. In this scenario, the introduction of a bonus tax always induces the principal to set a lower bonus rate than in a scenario without a bonus tax (i.e., $\gamma^*(a^*, \tau) < \gamma^*(a^*, 0)$.
∀τ ∈ (0, 1)). A converse result can occur in a scenario with a positive risk parameter (i.e., the introduction of a bonus tax can induce the principal to set a higher bonus rate). We derive this counterintuitive result in Proposition 2, in which we analyze how a bonus tax τ affects the bonus rate γ∗, the fixed salary δ∗ and the agent’s effort level a∗.

**Proposition 2** For a cost function c(a) that satisfies A1, a higher bonus tax τ has the following effects in equilibrium:

(i) The principal increases or decreases the bonus rate γ∗ depending on the following condition:

\[
\frac{dγ^*}{dτ} \gtrless 0 \Leftrightarrow ρ \gtrless ρ_γ ≡ \frac{c''(a^*) (1 - (1 + τ)γ^* + c''(a^*)a^*) - τγ^* c'''(a^*)a^*}{c'(a^*) [2c''(a^*)^2 + c'(a^*)c'''(a^*)]}
\]

(ii) The principal decreases or increases the fixed salary δ∗ depending on the following condition:

\[
\frac{dδ^*}{dτ} \lesssim 0 \Leftrightarrow ρ \gtrless ρ_δ \equiv \frac{a^* c'(a^*)}{c''(a^*)}.
\]

(iii) The agent reduces equilibrium efforts, i.e., \( \frac{da^*}{dτ} < 0 \).

**Proof.** See Appendix A.3. □

The proposition shows that the reaction of the principal to a higher bonus tax depends on the risk parameter ρ, and that the agent always reduces his effort in equilibrium.\(^{10}\) Note that γ∗ and a∗ depend on ρ such that the threshold values ργ and ρδ themselves depend on ρ. To clarify the intuition behind the principal’s behavior in part (i) of Proposition 2, we proceed in two steps: (ia) First, we analyze the individual terms of the principal’s first-order condition to understand her incentives for determining the fixed salary component and the bonus rate. (ib) Second, we explain the effects of a bonus tax on the individual terms of the principal’s first-order condition.

ad (ia): Recall that the principal maximizes her profit by forming an optimal compensation package based on the conditions that the agent reaches at least his reservation utility \( \hat{u} \) and the compensation package is incentive compatible. Using the participation constraint and the incentive compatibility constraint, we rearrange the principal’s first-order condition and obtain

\[
\frac{1}{MR \text{ effect}} - \left( \frac{c''(a^*)a^*}{1 - τ} \right) + c''(a^*)a^* + (1 - τ)γ^* - ρ(1 - τ)γ^* c''(a^*) - (1 - τ)γ^* = 0.
\]

\(^{10}\) Based on a principal-agent model, Graziano and Parigi (1998) show that more competition represented by a higher number of firms induces the agent to decrease efforts in the case of a low product differentiation. Based on a panel of US executives, Cuijat and Guadalupe (2009) find that an increase in the financial sector through deregulations in the 1990s induced an increase in the variable and a decrease in the fixed components of executive pay. Using data from Germany, Kraft and Niederprüm (1999) show that a higher variance in firm profits decreases the variable salary component.
We derive that a one-unit increase in the agent’s effort (induced by a higher bonus rate) has the following effects for the principal:  

- **Marginal revenue (MR) effect**: One-to-one higher expected revenue, which yields a marginal revenue of one.

- **Direct marginal cost (MC) effect**: Higher effort generates higher costs for the principal, given by \( \gamma^* + c''(a^*)a^*/(1 - \tau) \), as she must pay the agent a higher bonus. The direct marginal cost effect consists of two terms. The first term represents the higher bonus paid by the principal, induced by a one-unit increase in the agent’s effort. The second term reflects the effort-induced effect on the bonus rate. It should be noted that a change of the bonus rate also affects the bonus.

- **Income effect**: Higher effort also implies that there is an income effect for the agent such that the participation constraint is relaxed. Hence, the income effect enters the principal’s first-order condition with a positive sign. The income effect consists of two terms. The first term reflects the effort effect on the net-of-tax bonus rate for the agent. The second term represents the higher net-of-tax revenue from a one-unit increase in the agent’s effort. Both effects relax the principal’s participation constraint.

- **Risk effect**: The fourth term indicates that higher effort implies higher uncertainty for the agent regarding his expected salary because the salary variance increases such that the participation constraint is tightened. It follows that the principal has to compensate the agent with a larger risk premium to accept the higher risk, which yields a negative sign for the risk effect in the principal’s first-order condition. It is important to mention that the risk parameter \( \rho \) influences the magnitude of the risk effect. If the risk parameter is zero, the risk effect will disappear. In this case, we have already shown that the bonus rate decreases through the introduction of a bonus tax. Hence, the risk effect is crucial to obtaining the positive effect of a bonus tax on the bonus rate as derived in Proposition 2.

- **Indirect marginal cost (MC) effect**: The last term shows that the principal has to compensate the agent for his higher effort costs, such that the participation constraint is tightened. Because the agent’s costs affect the principal’s incentives, we refer to it as the ”indirect marginal cost effect”. It should be noted that this effect enters the principal’s first-order condition with a negative sign.

We derive that the second term of the income effect compensates for the indirect marginal cost effect because the agent’s marginal effort costs equal the marginal revenue.

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11In equilibrium, the sum of the below-mentioned effects must equal zero.

12The difference between the direct marginal cost effect and income effect stems from the fact that the state receives fraction \( \tau \) of the bonus paid by the principal.
of effort in the participation constraint. Furthermore, the second term of the direct marginal cost effect sets off the first term of the income effect. It results in the negative term $-\tau a^* c''(a^*)/(1 - \tau)$. The intuition behind this result is that the effect of effort on the bonus rate is stronger in the direct marginal cost effect than it is in the income effect. This result occurs because the principal pays the full amount of the bonus, but the agent only receives fraction $(1 - \tau)$ of it.

After rearranging equation (5), we obtain

$$1 - \gamma^* = c''(a^*) \left[ \rho(1 - \tau)\gamma^* + a^* \frac{\tau}{1 - \tau} \right].$$

(6)

(ib) In a next step, we analyze the effects of a bonus tax on the principal’s first-order condition using equation (6). It is easy to see that a higher bonus tax does not influence the principal’s marginal revenue; the marginal revenue of effort is constant and equals one. The effect of a higher bonus tax on the direct marginal cost effect is ambiguous, however. Because a higher tax affects the agent’s optimal effort and the optimal bonus rate, it is ambiguous whether a higher tax increases or decreases the direct marginal cost effect. A higher bonus tax decreases the income effect because the agent’s expected net-of-tax variable salary decreases. At the same time, the risk effect diminishes due to a lower variance in the agent’s salary because the state shares part of the risk. Similarly, a higher bonus tax decreases the indirect marginal cost effect because the agent reduces his effort in equilibrium.

If $\rho > \rho_\gamma$, then the risk effect dominates the sum of the other effects. It follows that a higher bonus tax induces the principal to increase the bonus rate $\gamma^*$ to satisfy her first-order condition. Formally, it must be the case that the rhs of (6) decreases in $\tau$ because the principal lowers $\gamma^*$ such that the lhs decreases. If $\rho < \rho_\gamma$, then the principal decreases the bonus rate $\gamma^*$ through a higher bonus tax because the risk effect is dominated by the sum of the other effects. Because the principal sets a lower $\gamma^*$, it must be the case that the rhs of (6) increases in $\tau$. Note that if $\rho = \rho_\gamma$, then the above mentioned effects balance each other out such that $\gamma^*$ is not affected by a change in the bonus tax.

(ii) We provide the intuition for the principal’s behavior with respect to the fixed salary $\delta^*$ as follows. The partial derivative of $\delta^*$ with respect to $\tau$ is given by

$$\frac{d\delta^*}{d\tau} = -\frac{c''(a^*)a^* + (1 - \tau)\gamma^* - \rho(1 - \tau)\gamma^* c''(a^*) - (1 - \tau)\gamma^*}{\text{income effect}} \cdot \frac{da^*}{d\tau} \cdot \text{ind. MC effect} < 0.$$

If $\rho > \rho_\delta$, then the combined risk effect and the indirect marginal cost effect dominate the income effect such that the principal has an incentive to decrease the fixed salary in equilibrium: that is, $\frac{d\delta^*}{d\tau} < 0$. The opposite holds true if $\rho < \rho_\delta$. In this case, the principal has an incentive to increase the fixed salary: that is, $\frac{d\delta^*}{d\tau} > 0$. Note that if $\rho = \rho_\delta$, then
\(\delta^*\) is not affected by a change in \(\tau\).

From parts (i) and (ii) of Proposition 2, we can derive the following corollary:

**Corollary 1**

*For a general cost function \(c(a)\) that satisfies A1, the introduction of a bonus tax does not necessarily imply a substitution effect between the fixed salary \(\delta^*\) and the bonus rate \(\gamma^*\).*

Basic intuition might suggest that a bonus tax induces a substitution between the fixed salary and the bonus rate. However, such as a substitution effect is not guaranteed because the threshold values \(\rho_\delta\) and \(\rho_\gamma\) are not necessarily equal. For example, in Section 4, we show for a cubic cost function that a higher bonus tax can induce a simultaneous increase in the fixed salary and the bonus rate. However, for a quadratic cost function the threshold values \(\rho_\delta\) and \(\rho_\gamma\) are equal such that an increase (decrease) in \(\gamma^*\) always induces a decrease (increase) in \(\delta^*\). In this case, a substitution effect is present between \(\delta^*\) and \(\gamma^*\) (see Section 3).

Regarding part (iii) of Proposition 2, the intuition for the agent’s behavior in equilibrium is as follows. According to the agent’s incentive compatibility constraint (4), the marginal effort costs equal the marginal revenue of effort in equilibrium:

\[
\underbrace{c'(a^*)}_{\text{marginal cost}} = (1 - \tau)\gamma^* = \underbrace{(1 - \tau)\gamma^*}_{\text{marginal revenue}}.
\]

The direct tax effect \((1 - \tau)\) has a negative effect on the marginal revenue of effort (rhs). Based on part (i) of Proposition 2, we know that \(\gamma^*\) decreases in \(\tau\) if \(\rho < \rho_\gamma\), which further reduces the marginal revenue. It follows that marginal revenue unambiguously decreases if the bonus tax increases. In this case, it is clear that the agent reduces his effort in equilibrium. If \(\rho > \rho_\gamma\), then the bonus rate \(\gamma^*\) increases through a higher bonus tax, which results in a positive effect on the marginal revenue of effort. However, the direct tax effect always overcompensates for a higher \(\gamma^*\) such that the marginal revenue still decreases. Also in this case, the agent will reduce his effort in equilibrium.

Finally, we deduce that in the case of an endogenously-determined outside option \(\hat{u}(\tau)\), which negatively depends on \(\tau\), the results of Proposition 2 remain unchanged except for the effect of a bonus tax on the fixed salary \(\delta^*\). In this case, only the condition in part (ii) of the proposition, whether the principal decreases or increases the fixed salary, changes to

\[
\frac{d\delta^*}{d\tau} = \frac{d\hat{u}(\tau)}{d\tau} + \frac{da^*}{d\tau}c''(a^*)\left[\rho c'(a^*) - a^*\right] \geq 0.
\]

In the next section, to derive further insights, we simplify the model and analyze a quadratic cost function. In Section 4, we analyze the model with a general polynomial cost function and simulate it for a cubic cost function.
3 Quadratic Effort Costs

In this section, we analyze a quadratic cost function given by \( c(a) = \frac{b}{2}a^2 \) with \( b > 0 \). This function, which satisfies A1, has well-defined properties that allow us to calculate the equilibrium values \((a^*, \gamma^*, \delta^*)\) in closed form.\(^{13}\) The following lemma provides additional insights regarding the incentive effects of bonus taxes for a quadratic cost function.

**Lemma 1**

(i) A higher bonus tax induces the principal to increase (decrease) the bonus rate \( \gamma^* \) with an increasing (decreasing) rate if the risk parameter \( \rho \) is sufficiently large (small). Formally, \((\frac{d\gamma^*}{d\tau}) \gtrless 0 \leftrightarrow \rho \gtrless \frac{1}{b}\) and \(\frac{d^2\gamma^*}{d\tau^2} > 0\).

(ii) A change in the bonus tax always yield opposite movements in the bonus rate and the fixed salary, i.e., we observe a substitution effect between \( \gamma^* \) and \( \delta^* \).

(iii) A higher bonus tax induces the agent to reduce equilibrium effort \( a^* \) with an increasing (decreasing) rate if the risk parameter \( \rho \) is sufficiently large (small). Formally, \(\frac{da^*}{d\tau} < 0 \) and \(\frac{d^2a^*}{d\tau^2} \gtrless 0 \leftrightarrow \rho \gtrless \frac{1}{b}\).

**Proof.** See Appendix A.4. ■

Part (i) analyzes the effects of bonus taxes on the principal’s behavior. Based on the participation constraint, the incentive compatibility constraint and the quadratic cost function, we can rearrange the principal’s first-order condition and obtain

\[
\frac{1 - 2\gamma^*}{\gamma^*} = b(1 - \tau) \left(\rho - \frac{1}{b}\right). \tag{7}
\]

This equation corresponds to equation (6) in the general model. If \( \rho > 1/b \), then the risk effect dominates the sum of the other effects. It follows that the rhs of (7) is positive, and therefore it decreases linearly with a higher bonus tax. Hence, the principal increases the bonus rate \( \gamma^* \) on the lhs with a decreasing rate to satisfy her first-order condition.\(^{14}\)

If \( \rho < 1/b \), then the rhs of (7) is negative. Hence, it increases with a higher bonus tax. It follows that the principal lowers \( \gamma^* \) with an increasing rate to satisfy her first-order condition.

Part (ii) posits that, for a quadratic cost function, there is a substitution effect between \( \gamma^* \) and \( \delta^* \) because the threshold values \( \rho_\gamma \) and \( \rho_\delta \) from Proposition 2 are equal: \( \rho_\gamma = \rho_\delta = 1/b \). That is, whenever a tax change induces the principal to increase the bonus rate, she will lower the fixed salary and vice versa. In Section 4, however, we show that this substitution effect is not necessarily present for a polynomial cost function of higher degree.

\(^{13}\)See the proof of Lemma 1.

\(^{14}\)Notice that \( \gamma^* \in (1/2, 1) \) for \( \rho < 1/b \) and \( \gamma^* \in (0, 1/2) \) for \( \rho > 1/b \) \( \forall \tau \in (0, 1) \). Hence, \((1 - 2\gamma^*)/\gamma^* \) on the lhs of (7) is positive for \( \rho > 1/b \) and it decreases in \( \gamma^* \) with a decreasing rate.
Part (iii) of Lemma 1 shows that the rate of change in the agent’s effort reduction decreases with a higher bonus tax if the risk parameter is smaller than $1/b$ and that the opposite holds true for $\rho > 1/b$. It should be noted that the agent’s optimality condition (4) for a quadratic cost function is given by $a^* = (1 - \tau)\gamma^*/b$, such that

$$
\frac{d^2a^*}{d\tau^2} = \frac{1}{b} \left( -2\frac{d\gamma^*}{d\tau} + (1 - \tau)\frac{d^2\gamma^*}{d\tau^2} \right).
$$

If $\rho < 1/b$, then the agent reduces his effort with a decreasing rate because the bonus rate decreases with a higher bonus tax; that is, $d\gamma^*/d\tau < 0$. If, on the other hand, $\rho > 1/b$, then $2(d\gamma^*/d\tau) > (1 - \tau)(d^2\gamma^*/d\tau^2)$ such that the rate of change in the agent’s effort reduction increases with a higher bonus tax.

In the next proposition, we analyze the effect of a higher bonus tax on the bonus paid by the principal.

**Proposition 3 (Bonus Payment)** Suppose that $\rho \in \left( \frac{3}{b}, \frac{5 - \tau}{b(1 - \tau)} \right)$.

(i) A higher bonus tax increases the expected bonus $E[BP^*] = \gamma^* a^*$ paid by the principal until the maximum is reached for a bonus tax given by $\tau_{BP} \equiv \frac{bp-3}{bp-1}$.

(ii) The agent always receives a lower expected bonus $(1 - \tau)E[BP^*]$ after the introduction of a bonus tax.

**Proof.** See Appendix A.5. ■

Part (i) of the proposition posits that the bonus paid by the principal can increase if these payments are taxed. This surprising result emerges if the risk parameter is sufficiently large. Recall that the agent always reduces effort $a^*$ with a higher bonus tax and that the principal’s reaction depends on the risk parameter $\rho$. It is clear that the bonus payment cannot increase if $\rho < 1/b$ because, then, the principal also decreases $\gamma^*$. Hence, a necessary condition to increase the bonus payment with a higher tax rate is $\rho > 1/b$. According to the proposition, the threshold value of the risk parameter above which the bonus payment increases with the introduction of a bonus tax, is given by $\rho = 3/b$. That is, if $\rho > 3/b$, then the increase in the bonus rate $\gamma^*$ compensates for the decrease in the agent’s effort level $a^*$. Thus, the bonus payment increases with a higher tax until the maximum is reached for $\tau = \tau_{BP}$. Raising the bonus tax above $\tau_{BP}$ decreases the bonus such that it can be even lower than in the benchmark case without a tax.

For a given tax rate, a higher risk parameter always decreases the bonus payment (i.e., $\partial E[BP^*]/\partial \rho < 0$) because both the agent’s effort and the bonus rate decrease with a higher $\rho$. However, the bonus-maximizing tax $\tau_{BP}$ increases with a higher risk parameter because the increase in the bonus rate offsets the decrease in effort more easily if $\rho$ is higher.
Part (ii) of the proposition shows that a bonus tax always reduces the expected bonus received by the agent. As the state keeps $\tau E[BP^*]$, the agent only receives $(1 - \tau) E[BP^*]$ of the bonus paid by the principal, which is always lower than without a tax. That is, the tax-induced decrease in $(1 - \tau)$ always compensates for a potential increase in the bonus paid by the principal.

4 Extension

In this section, we extend our model to verify whether our results on the incentive effects of bonus taxes carry over to a polynomial cost function with a degree larger than two. We assume that effort costs are given by a polynomial cost function

$$c(a) = b \phi a^\phi$$

with $b \in \mathbb{R}^+$ and $\phi \in (2, \infty)$. This function fulfills the properties for the cost function required in Section 2.1. According to Proposition 1, the existence and uniqueness of the equilibrium is guaranteed, and we derive the optimality conditions as

$$1 - \phi \gamma^* = \gamma^*(1 - \tau)\left(\phi - 1\right)\left(b(a^*)^{\phi - 2} - 1\right),$$

$$b(a^*)^{\phi - 1} = (1 - \tau)\gamma^*.$$ (8)

The first equation is the principal’s optimality condition, and the second equation is the agent’s optimality condition. It should be noted that, in contrast to the quadratic effort costs with $\phi = 2$, the risk effect in the principal’s optimality condition given by $\rho(1 - \tau)\gamma^* b(\phi - 1) (a^*)^{\phi - 2}$ now depends on the agent’s effort $a^*$. We derive that a higher effort level $a^*$, which also depends on the tax rate $\tau$, strengthens the risk effect. Rearranging the principal’s optimality condition yields

$$1 - \phi \gamma^* = \gamma^*(1 - \tau)\left(\phi - 1\right)\left(b(a^*)^{\phi - 2} - 1\right).$$

As it is not possible to calculate the equilibrium values $(a^*, \gamma^*)$ in closed-form, we simulate the system of equations (8) simultaneously. For the simulation, we set $b = 1$ and $\phi = 3$ to obtain the cubic cost function $c(a) = \frac{1}{3}a^3$. Moreover, we specify the agent’s reservation utility as $\hat{u} = 5$ and calculate the fixed salary $\delta^*$ from the agent’s participation constraint. Our simulation results are depicted in Figure 1.15

Similar to a quadratic cost function, Panel (c) shows that the agent’s optimal effort $a^*$ decreases with a higher bonus tax. Furthermore, the negative relationship between effort and bonus tax is weaker with a higher risk parameter $\rho$. That is, for a given $\tau$, a higher $\rho$ induces the agent to reduce his effort $a^*$ less strongly with a higher bonus tax.

In contrast to the quadratic cost function, Panel (a) shows that the relationship between the bonus rate $\gamma^*$ and $\tau$ is no longer monotonic, but can follow an inverted

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15The results remain qualitatively the same for different parameter values $\phi > 2$ and $b > 0$. 

14
Figure 1: Bonus Tax Effects for Polynomial Cost Functions

(a) Effect on Bonus Rate

(b) Effect on Fixed Salary

(c) Effect on Effort

U-shaped pattern. Furthermore, Panel (b) illustrates that the relationship between the fixed salary $\delta^*$ and $\tau$ can follow a U-shaped pattern. That is, there exists a tax rate for which the bonus rate is maximal and another tax rate for which the fixed salary is minimal. For example, compare Figure 2, which is a rescaled version of Figure 1, in which we focus on $\rho = 2$.

The bonus rate $\gamma^*$ increases in $\tau$ until $\tau \approx 0.7$ and subsequently it decreases for a higher bonus tax. The fixed salary $\delta^*$, on the other hand, decreases in $\tau$ until $\tau \approx 0.25$ and subsequently it increases for a higher bonus tax. From these results, we derive the following lemma:

Lemma 2

For a polynomial cost function with a degree larger than two, a higher bonus tax can induce a simultaneous increase in the fixed salary and the bonus rate.

The lemma shows that we no longer observe a substitution effect between $\gamma^*$ and $\delta^*$. That is, we find parameter constellations in which both the fixed salary and the bonus rate increase with a higher bonus tax. The intuition behind these results is as follows. Based on Proposition 2, we derive that for a cubic cost function, the bonus rate
\( \gamma^* \) increases with a higher bonus tax for
\[
\rho > \rho_\gamma = \frac{(1 - \gamma^*) + 2 \left[ (a^*)^2 - \tau \gamma^* \right]}{5 (a^*)^3}.
\]

For a low bonus tax, \( \rho_\gamma \) is small and therefore \( \gamma^* \) increases with a higher bonus tax (see Panel (a) in Figure 2). In this case, the risk effect dominates the sum of the effects mentioned in Section 2.2. However, if the bonus tax increases, the agent’s effort decreases, and therefore, \( \rho_\gamma \) endogenously increases. If the bonus tax is larger than \( \tau \approx 0.7 \), then \( \rho_\gamma > 2 \). Therefore, the bonus rate decreases with a higher bonus tax. In this case, the risk effect is dominated by the sum of the other effects.

On the other hand, the fixed salary \( \delta^* \) increases with a higher bonus tax for \( \rho < \rho_\delta = 1/a^* \). For a low bonus tax, effort \( a^* \) is relatively high and yields a relatively low \( \rho_\delta \). Thus, the fixed salary \( \delta^* \) decreases with a higher bonus tax (see Panel (b) in Figure 2). In this case, the combined risk effect and the indirect marginal cost effect dominate the income effect such that the principal has an incentive to decrease the fixed salary \( \delta^* \). If, however, the bonus tax increases, then the agent’s effort decreases, and \( \rho_\delta \) endogenously increases. If the bonus tax is larger than \( \tau \approx 0.25 \), then \( \rho_\delta > 2 \) such that the fixed salary \( \delta^* \) increases with a higher bonus tax. In this case, the income effect dominates the combined risk effect and the indirect marginal cost effect; thus the principal has an incentive to increase the fixed salary \( \delta^* \). Because \( \rho_\gamma \) and \( \rho_\delta \) are not equal for a cubic cost function, there is a range of bonus taxes in which the bonus rate \( \gamma^* \) and the fixed salary \( \delta^* \) increase with a higher bonus tax. This behavior is not observable for a quadratic cost function.
5 Conclusion and Discussion

This paper has investigated the consequences of taxing the agent’s bonus in a principal-agent model. In particular, we have analyzed the effects of a bonus tax on the agent’s incentives to exert effort, on the composition of the agent’s compensation package and on the bonuses paid by the principal. Our model shows that a bonus tax does not necessarily induce the principal to substitute variable salary with fixed salary for a general effort cost function. In particular, it is possible that a higher bonus tax simultaneously increases the fixed salary component and the bonus rate.\textsuperscript{16}

A bonus tax has different effects on the fixed salary and the bonus rate because the principal sets an optimal compensation package on the conditions that the agent reaches at least his reservation utility and the package is incentive compatible. To determine the composition of the package, she anticipates that a higher bonus tax will decrease the agent’s expected net-of-tax variable salary (income effect), decrease the agent’s risk premium due to a lower salary variance (risk effect) and lower the agent’s effort costs (indirect marginal cost effect). The product of the agent’s risk aversion and the variance of the firm value (risk parameter) plays a crucial role in determining the effect of a bonus tax. We provided separate threshold values for the risk parameter for which a bonus tax positively or negatively influences the fixed salary and the bonus rate, respectively.

Moreover, our model shows that the agent always reacts to a higher bonus tax with lower effort. For quadratic effort costs, despite of a tax-induced effort reduction, a higher bonus tax induces the principal to pay higher bonuses if the risk parameter is sufficiently large. In this case, the increase in the bonus rate overcompensates for the decrease in the agent’s effort. Therefore, it is not guaranteed that firms will have to pay lower bonuses after implementing a bonus tax.

Our model yields potentially testable comparative statics results. In particular, our model might help to predict the sectors and firms in which bonuses would decrease or increase. In uncertain economic environments and/or in firms, in which the monitoring and evaluation of the manager’s performance is comparatively hard, we expect that bonus taxes will induce a firm to pay higher bonuses. Additionally, we do not expect a bonus tax to have the effect of shifting the bonus rate towards a fixed salary. That is, we anticipate low fixed salaries and high bonus rates. We can derive the following examples of firms for these predictions:

(i) New-economy firms: These firms tend to operate in more uncertain economic environments than old-economy firms. In addition, it might be more difficult to observe the manager’s marginal contribution in new-economy firms because these firms grow faster, are more R&D intensive and have larger market-to-book ratios than old-economy firms.\textsuperscript{16} For quadratic effort costs, however, we show that a substitution effect between the fixed salary component and the bonus rate is always present. Particularly, a bonus tax can induce a decrease in the fixed salary component and an increase in the bonus rate.
firms (see Ittner et al. 2003).

(ii) Large firms: According to Schaefer (1998), large firms have more noisy measures of individual performance than do small firms. Moreover, in large firms, one manager’s action has less influence on the firm value than it might in small firms.

(iii) Privately held firms: Marino and Zabojnik (2008) suggest that it is harder for privately held firms to evaluate their managers because a public firm’s stock price provides an informative measure of performance which is less available in privately held firms.

Our simple model may serve as a basic framework to further analyze bonus taxes in a principal-agent model. There is a broad range of further applications and model extensions. For instance, an interesting avenue for further research could be the extension of our model to more than one period. An agent’s effort decisions are often connected over time, and working contracts extend over several periods. The implementation of these dynamics in the model could shed more light on the impact of bonus taxes on executive pay. Furthermore, an interesting extension would be to analyze the effects of bonus taxes on executives’ risk-taking behaviors or welfare implications of bonus taxes.
A Appendix

A.1 Derivation of the Agent’s Expected Net Utility

Given the agent’s negative exponential utility function \( U(p) = -e^{-r(p-c(a))} \), we derive

\[
E[U] = \int_{-\infty}^{\infty} U(p) f(p) dp = \int_{-\infty}^{\infty} -e^{-r(p-c(a))} f(p) dp = -\int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_p} e^{-\frac{(p-\bar{p})^2}{2\sigma_p^2}} \frac{1}{2} e^{-r(p-c(a))} dp.
\]

Because the agent’s salary is normally distributed with \( N(\delta + (1-\tau)\gamma a; (1-\tau)^2\gamma^2\sigma_p^2) \), it follows

\[
E[U] = -\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_p} e^{\frac{-p^2+2(p-p)\bar{p}-2r\sigma_p^2}{2\sigma_p^2}+r c(a)} dp
\]

The second part of the above equation equals one because it corresponds to the integral of a normal distribution function with \( N(\bar{p} - r\sigma_p^2, \sigma_p^2) \) over the entire interval. Therefore, \( E[U] = -e^{\frac{-p^2+2(p-p)\bar{p}-2r\sigma_p^2}{2\sigma_p^2}-r(p-c(a))} \). By transforming this utility function with the positive monotonic natural logarithm function and by dividing it with \( r \), we obtain the agent’s utility \( \bar{p} - r\sigma_p^2/2 - c(a) \) with \( \bar{p} = \delta + (1-\tau)\gamma a \) and \( \sigma_p^2 = \gamma^2(1-\tau)^2\sigma_e^2 \). It follows that the agent’s expected utility (certainty equivalent) is given by

\[
E[U_A] = \bar{p} - \frac{r\sigma_p^2}{2} - c(a) = \delta + (1-\tau)\gamma a - \frac{r\sigma^2}{2}\gamma^2(1-\tau)^2 - c(a).
\]

A.2 Proof of Proposition 1

The maximization problem of the agent is given by

\[
\max_{a \geq 0} \left\{ E[U_A] = \delta + (1-\tau)\gamma a - \frac{r\sigma^2}{2}\gamma^2(1-\tau)^2 - c(a) \right\}.
\]

The corresponding first-order condition is computed as \( \partial E[U_A]/\partial a = (1-\tau)\gamma - c'(a) = 0 \), yielding the agent’s optimality condition

\[
c'(a^*) = (1-\tau)\gamma.
\]

We now turn our attention to the maximization problem of the principal, which is

\[17\]Note that given the properties of exponential functions, the ordering will be preserved by using \( \bar{p} - r\sigma_p^2/2 \) as an equivalent utility measure.
given by

\[
\max_{(\delta, \gamma) \geq 0} \{ E[\pi_P] = (1 - \gamma)a - \delta \} \text{ subject to } E[U_A(a)] = \delta + (1 - \tau)\gamma a - \frac{r\sigma^2}{2} \gamma^2 (1 - \tau)^2 - c(a) \geq \hat{u} \text{ and } c'(a) = (1 - \tau)\gamma.
\]

Note that the principal is able to control the agent’s effort \(a\) by choosing an appropriate bonus rate \(\gamma\). Therefore, instead of replacing \(a\), we use the last condition (incentive compatibility constraint) and replace \(\gamma\) with \(c'(a)/(1 - \tau)\) to set up the associated Lagrangian \(\mathcal{L}_P\). Then, the Lagrangian with multiplier \(\lambda\) has the following form:

\[
\mathcal{L}_P(a, \delta, \lambda) \equiv a - \frac{ac'(a)}{1 - \tau} - \delta + \lambda \left[ \delta + c'(a)a - \frac{r\sigma^2}{2} c'(a)^2 - c(a) - \hat{u} \right].
\]

The corresponding first-order conditions are computed as

\[
\begin{align*}
\frac{\partial \mathcal{L}_P}{\partial a} &= 1 - \frac{c'(a)}{1 - \tau} - \frac{c''(a)}{1 - \tau} a^* + \lambda \left[ c''(a^*) a^* - c'(a^*) c''(a^*) r \sigma^2 \right] = 0, \\
\frac{\partial \mathcal{L}_P}{\partial \delta} &= -1 + \lambda = 0, \\
\frac{\partial \mathcal{L}_P}{\partial \lambda} &= \delta + c'(a^*) a^* - \frac{r\sigma^2}{2} c'(a)^2 - c(a) - \hat{u} = 0.
\end{align*}
\]

Note that the principal has an incentive to provide a compensation package such that the participation constraint is binding, i.e., the agent receives exactly his reservation utility. Therefore, the derivative of \(\mathcal{L}_P\) with respect to \(\lambda\) equals zero. With the above system, the agent’s first-order condition \(c'(a^*) = (1 - \tau)\gamma^*\) and \(\lambda = 1\), we derive the first-order condition of the principal as

\[
1 - \gamma^* - \frac{c''(a^*)}{1 - \tau} a^* + c''(a^*) a^* - (1 - \tau)\gamma^* c''(a^*) r \sigma^2 = 0. \tag{9}
\]

To guarantee that the second-order condition for a maximum is satisfied, a lower bound of the third derivative of the cost function is required. Formally,

\[
c'''(a^*) > \Psi \equiv -c''(a^*) \frac{1 + \tau + (1 - \tau)c''(a^*) r \sigma^2}{a^* \tau + (1 - \tau)c'(a^*) r \sigma^2}. \tag{10}
\]

Note that \(\Psi < 0\), i.e., \(c''(a^*)\) can be negative to satisfy the second-order condition.

By solving equation (9), we obtain the optimal \(\gamma^*\) as

\[
\gamma^* = \frac{(1 - \tau) - a^* c''(a^*) \tau}{(1 - \tau) [1 + (1 - \tau)c''(a^*) r \sigma^2]}.
\]
With the binding participation constraint, we derive the optimal fixed salary $\delta^*$ as

$$\delta^* = \hat{u} - c'(a^*)a^* + \frac{c''(a^*)^2}{2}r\sigma_z^2 + c(a^*). \quad (12)$$

In a next step, we show that the equilibrium in $(\gamma^*, \delta^*, a^*)$ exists and is unique. Combining the two optimality conditions

$$c'(a) = (1 - \tau)\gamma \quad \text{and} \quad \gamma = \frac{(1 - \tau) - ac''(a)\tau}{(1 - \tau)[1 + (1 - \tau)c''(a)r\sigma_z^2]},$$

we obtain

$$1 - \tau = \underbrace{c'(a) + c''(a)(a\tau + (1 - \tau) r\sigma_z^2 c'(a))}_{\equiv \kappa_{rhs}(a)} \equiv \kappa_{lhs}(a). \quad (13)$$

Note that the left-hand side $\kappa_{lhs}$ of equation (13) is independent of $a$. Using the assumptions regarding the cost function, the right-hand side $\kappa_{rhs}(a)$ of equation (13) is zero for $a = 0$ and we obtain $\kappa_{rhs}(0) = 0 < (1 - \tau) = \kappa_{lhs}$. We can show that $\kappa_{rhs}(a)$ is a monotonically increasing function in $a$ if the requirement for $c'''(a)$ regarding the second-order condition holds, i.e., $c'''(a) > \Psi$ (see (10)). Formally,

$$\frac{\partial \kappa_{rhs}(a)}{\partial a} > 0 \iff c'''(a) > c''(a)\frac{1 + \tau + (1 - \tau)c''(a)r\sigma_z^2}{a\tau + (1 - \tau)c''(a)r\sigma_z^2}.$$}

Using the assumption $\lim_{a \to \infty} c'(a) = \infty$, it is guaranteed that $\kappa_{rhs}(a)$ passes the constant $\kappa_{lhs}$ for a certain effort $a = a^*$. Hence, there exist exactly one intersection, which defines the unique equilibrium $a^*$. Plugging $a^*$ into (11) and (12) yields the other equilibrium values $(\gamma^*, \delta^*)$. Note that the weaker assumption $\lim_{a \to \infty} c'(a) > (1 - \tau)$ would be sufficient to guarantee the existence and uniqueness of the equilibrium.

### A.3 Proof of Proposition 2

**Parts (i) and (iii).** We use the optimality conditions

$$c'(a^*) = (1 - \tau)\gamma^* \quad \text{and} \quad \gamma^* = \frac{(1 - \tau) - a^*c''(a^*)\tau}{(1 - \tau)[1 + (1 - \tau)c''(a^*)\rho]},$$

rearrange them and obtain

$$g_1(a^*, \gamma^*, \tau) := c'(a^*) - (1 - \tau)\gamma^* = 0,$$

$$g_2(a^*, \gamma^*, \tau) := (1 - \tau) - a^*c''(a^*)\tau - \gamma^*(1 - \tau)[1 + (1 - \tau)c''(a^*)\rho] = 0.$$
Next, we derive the total differential of \( g_1(a^*, \gamma^*, \tau) = 0 \) and \( g_2(a^*, \gamma^*, \tau) = 0 \):

\[
\begin{align*}
\frac{\partial g_1}{\partial a^*} da^* + \frac{\partial g_1}{\partial \gamma^*} d\gamma^* + \frac{\partial g_1}{\partial \tau} d\tau &= 0, \\
\frac{\partial g_2}{\partial a^*} da^* + \frac{\partial g_2}{\partial \gamma^*} d\gamma^* + \frac{\partial g_2}{\partial \tau} d\tau &= 0.
\end{align*}
\]

The total differential can also be written as

\[
\left[ \begin{array}{cc} g_{1a} & g_{1\gamma} \\
g_{2a} & g_{2\gamma} \end{array} \right] \left[ \begin{array}{c} da^* \\ d\gamma^* \end{array} \right] = \left[ \begin{array}{c} -g_{1\tau} \\
-g_{2\tau} \end{array} \right] d\tau,
\]

where

\[
\begin{align*}
g_{1a} &= \frac{\partial g_1}{\partial a^*} = c''(a^*), \\
g_{1\gamma} &= \frac{\partial g_1}{\partial \gamma^*} = -(1 - \tau), \\
g_{1\tau} &= \frac{\partial g_1}{\partial \tau} = \gamma^*, \\
g_{2a} &= \frac{\partial g_2}{\partial a^*} = -\left[ \tau c''(a^*) + c'''(a^*)(a^* \tau + (1 - \tau)^2 \gamma^*) \right], \\
g_{2\gamma} &= \frac{\partial g_2}{\partial \gamma^*} = -(1 - \tau) [1 + \rho(1 - \tau)c''(a^*)], \\
g_{2\tau} &= \frac{\partial g_2}{\partial \tau} = \gamma^* - 1 - c''(a^*) [a^* - 2\gamma^* \rho(1 - \tau)].
\end{align*}
\]

Applying Cramer’s Rule to (14), we derive

\[
\begin{align*}
d\gamma^* = \frac{g_{1\tau}g_{2\tau} - g_{2\gamma}g_{1\tau}}{g_{1a}g_{2\gamma} - g_{1\gamma}g_{2a}} d\tau \\
d\tau = \frac{g_{2a}g_{1\tau} - g_{1a}g_{2\tau}}{g_{1a}g_{2\gamma} - g_{1\gamma}g_{2a}} d\gamma^*.
\end{align*}
\]

Plugging (15) into (16), we obtain

\[
\begin{align*}
\frac{da^*}{d\tau} &= c''(a^*) [(1 - \tau)\gamma^* \rho - a^*] - 1, \\
\mu &= \frac{g_{1\tau}g_{2\tau} - g_{2\gamma}g_{1\tau}}{g_{1a}g_{2\gamma} - g_{1\gamma}g_{2a}}, \\
\frac{d\gamma^*}{d\tau} &= \gamma^* \left[ a^* \tau + \rho(1 - \tau)^2 \gamma^* c''(a^*) - c''(a^*) [1 + (1 - \tau)\gamma^* + (a^* - 2\rho(1 - \tau)\gamma^*) c''(a^*)] \right], \\
\frac{d\gamma^*}{d\tau} &= \frac{g_{2a}g_{1\tau} - g_{1a}g_{2\tau}}{g_{1a}g_{2\gamma} - g_{1\gamma}g_{2a}} d\gamma^*.
\end{align*}
\]

with \( \mu \equiv c''(a^*) [1 + \tau + (1 - \tau)c''(a^*)] + c''(a^*) [a^* \tau + (1 - \tau)^2 \gamma^* \rho] \). We derive

\[
\frac{d\gamma^*}{d\tau} = 0 \iff \rho = \rho_\gamma \equiv \frac{c''(a^*) (1 - (1 + \tau)\gamma^* + c''(a^*) a^*) - \gamma^* c''(a^*) a^*}{c'(a^*) [2c''(a^*)^2 + c'(a^*) c''(a^*)]}.
\]

It is easy to show that \( \frac{d|d\gamma^*/d\rho|}{d\rho} > 0 \), i.e., \( \frac{d\gamma^*}{d\tau} \) is a monotonically increasing function in \( \rho \).
It follows that

\[
\frac{d\gamma^*}{d\tau} > 0 \iff \rho > \rho_\gamma.
\]

This proves part (i) of the proposition.
Regarding the agent’s optimal effort $a^*$, with $c''(a^*) > \Psi$, it formally holds

$$\frac{da^*}{d\tau} \geq 0 \Leftrightarrow \rho \geq \rho_a \equiv \frac{1 + c''(a^*)a^*}{c'(a^*)c''(a^*)}. \quad (17)$$

At first glance, given that the risk parameter is sufficiently large, i.e., $\rho > \rho_a$, it is possible that the agent exerts more effort in equilibrium if the bonus tax increases. However, extensive numerical simulations for different (convex) cost functions $c(a)$ have shown that $\rho > \rho_a$ can never be satisfied in equilibrium.\(^{18}\)

To formally prove this claim, we provide a proof by contradiction to show that the agent always reduces his effort with a higher bonus tax. Suppose that $\frac{da^*}{d\tau} \geq 0$. This assumption directly implies that $\frac{d\gamma^*}{d\tau} > 0$ because $c'(a^*) = (1 - \tau)\gamma^*$.\(^{19}\) By using the participation constraint and the incentive compatibility constraint, we rearrange the principal’s first-order condition and obtain\(^{20}\)

$$1 - \gamma^* = \left[ \begin{array}{c} \frac{c''(a^*)}{\text{not dec. in } \tau} \rho \frac{c'(a^*)}{\text{not dec. in } \tau} + \frac{a^*}{\text{not dec. in } \tau} \cdot \frac{\tau}{(1 - \tau)} \end{array} \right]. \quad (18)$$

Under the assumption $\frac{da^*}{d\tau} \geq 0$, the rhs of (18) increases with a higher bonus tax $\tau$. It follows that $\gamma^*$ on the lhs, which is by definition in the interval $(0, 1)$, must decrease (i.e., $\frac{d\gamma^*}{d\tau} < 0$) to guarantee also an increase of the lhs. This result, however, contradicts the assumption $\frac{da^*}{d\tau} \geq 0$, which implies $\frac{d\gamma^*}{d\tau} > 0$. Hence, our assumption was wrong and it must be the case that $\frac{da^*}{d\tau} < 0$. This proves part (iii) of the proposition.

Part (ii). Based on the optimality condition regarding the fixed salary $\delta^* = \hat{u} - c'(a^*)a^* + \frac{c'(a^*)^2}{2} \rho + c(a^*)$, we compute

$$\frac{d\delta^*}{d\tau} = \frac{da^*}{d\tau} c''(a^*) [\rho c'(a^*) - a^*].$$

With $\frac{da^*}{d\tau} < 0$, we derive

$$\frac{d\delta^*}{d\tau} \leq 0 \Leftrightarrow \rho \geq \rho_\delta \equiv \frac{a^*}{c'(a^*)}.$$  

This proves part (ii) and completes the proof of the proposition.

**A.4 Proof of Lemma 1**

According to Proposition 1, the existence and uniqueness of the equilibrium is guaranteed for the quadratic cost function $c(a) = (b/2) \cdot a^2$. Substituting this cost function into the optimality conditions of Proposition 1, we compute the optimal compensation package

\(^{18}\)Note that the optimal effort $a^*$ on the rhs of (17) also depends on the risk parameter $\rho$.

\(^{19}\)A higher bonus tax decreases the rhs of this equation and a necessary condition to guarantee an increase of the lhs is $\frac{da^*}{d\tau} > 0$.

\(^{20}\)See also the discussion after Proposition 2.
\((\gamma^*, \delta^*)\) as
\[
\gamma^* = \frac{1}{1 + \tau + b\rho(1 - \tau)} \quad \text{and} \quad \delta^* = \hat{u} - \frac{(1 - b\rho)(1 - \tau)^2}{2b[1 + \tau + b\rho(1 - \tau)]^2}.
\]
In equilibrium, the agent’s effort \(a^*\) yields
\[
a^* = \frac{1 - \tau}{b[1 + \tau + b\rho(1 - \tau)]}.
\]

Parts (i) and (ii). We compute the partial derivatives of \(\gamma^*\) and \(\delta^*\) with respect to \(\tau\) as
\[
\frac{d\gamma^*}{d\tau} = -\frac{1 - bp}{[1 + \tau + b\rho(1 - \tau)]^2} \leq 0 \iff \rho \geq \frac{1}{b} \quad \text{and}
\]
\[
\frac{d\delta^*}{d\tau} = \frac{2(1 - \tau)(1 - bp)}{b[1 + \tau + b\rho(1 - \tau)]^3} \leq 0 \iff \rho \geq \frac{1}{b}.
\]

If \(\rho > \frac{1}{b}\), then a higher tax rate \(\tau\) increases \(\gamma^*\) and decreases \(\delta^*\). If \(\rho < \frac{1}{b}\), then a higher tax rate \(\tau\) decreases \(\gamma^*\) and increases \(\delta^*\). This shows the substitution effect between the fixed salary and the bonus rate. Moreover, we compute
\[
\frac{d^2\gamma^*}{d\tau^2} = \frac{2(1 - bp)^2}{[1 + \tau + b\rho(1 - \tau)]^3} > 0.
\]

Hence, the principal increases (decreases) \(\gamma^*\) with an increasing (decreasing) rate if \(\rho > 1/b\) (\(\rho < 1/b\)).

Part (iii). We compute the first and second partial derivatives of \(a^*\) with respect to \(\tau\) as
\[
\frac{da^*}{d\tau} = -\frac{2}{b[1 + \tau + b\rho(1 - \tau)]^2} < 0 \quad \text{and} \quad \frac{d^2a^*}{d\tau^2} = \frac{4(1 - bp)}{b[1 + \tau + b\rho(1 - \tau)]^3} \leq 0 \iff \rho \geq \frac{1}{b}.
\]

### A.5 Proof of Proposition 3

Part (i). The partial derivative of the expected bonus \(E[BP^*] = \gamma^*a^*\) (paid by the principal) with respect to \(\tau\) is given by
\[
\frac{\partial E[BP^*]}{\partial \tau} = \frac{\tau - 3 + b\rho(1 - \tau)}{b[1 + \tau + b\rho(1 - \tau)]^3} = 0 \iff \tau = \tau^{BP} \equiv \frac{b\rho - 3}{b\rho - 1}.
\]

If \(\rho < 1/b\), then \(E[BP^*]\) is a convex function in \(\tau\) and hence it has a minimum at \(\tau = \tau^{BP}\). If \(\rho \in (\frac{1}{b}, \frac{5 - \tau}{b(1 - \tau)})\), then \(E[BP^*]\) is a concave function in \(\tau\) and hence it has a maximum at \(\tau = \tau^{BP}\). Moreover, we derive that for \(\rho \in (1/b, 3/b)\) the tax rate \(\tau^{BP}\) is negative and for \(\rho < 1/b\) or \(\rho > 3/b\), \(\tau^{BP}\) is positive. It follows that the risk parameter \(\rho\) has to be in the interval \((\frac{1}{b}, \frac{5 - \tau}{b(1 - \tau)})\) to guarantee that the bonus paid by the principal increases with a
higher tax rate until the maximum is reached for $\tau = \tau^{BP}$.

Part (ii). The partial derivative of the expected bonus $(1 - \tau)E[BP^*]$ (received by the agent) with respect to $\tau$ is given by

$$\frac{\partial ((1 - \tau)E[BP^*])}{\partial \tau} = -\frac{4(1 - \tau)}{b[1 + \tau + \rho(1 - \tau)]^3} < 0.$$  

Hence, the bonus received by the agent always decreases with a higher bonus tax.
References


