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Abstract

This paper develops a model of a professional sports league with network externalities by integrating the theory of two-sided markets into a contest model. In professional team sports, the competition of the clubs functions as a platform that enables sponsors to interact with fans. In these club-mediated interactions, positive network effects operate from the fan market to the sponsor market, while positive or negative network effects operate from the sponsor market to the fan market. Clubs react to these network effects by charging higher (lower) prices to sponsors (fans). The size of these network effects also determines the level of competitive balance within the league. We further show that clubs benefit from stronger combined network effects through higher profits and that network externalities can mitigate the negative effect of revenue sharing on competitive balance. Finally, we derive implications for improving competitive balance by taking advantage of network externalities.

Keywords: Competitive balance, contest, multisided market, network externalities, team sports league

JEL Classification: L11, L13, L83, M21

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1 Introduction

The professional team sports industry has a unique organizational structure. It is the only industry in which production is organized by leagues. This unique organizational structure is the result of the industry-specific production and competition process. Industry outsiders often tend to regard individual teams as firms and treat them as production units. Unlike an automobile firm, however, an individual team cannot produce a marketable product. Each team needs at least one opponent to play a match. However, even a match between two teams is not an attractive product. The individual matches must be upgraded by integrating them into an organized championship race. This upgrade, which gives each individual match additional value within the larger context of the championship race, is managed by the league.

From a sports perspective, each team within a league wants to win as many games as possible. Economically, however, teams are not so much competitors but are rather complementors. The quality or economic value of the championship race depends to a large extent on the level of competitive balance. Unlike Toyota and Ford, which prefer weak competitors in their industry, sports teams like Real Madrid, the New York Yankees, and the Dallas Cowboys benefit from having strong opponents within their leagues. A more balanced league usually produces a more attractive - that is, economically more valuable - product.[1]

The clubs' competition provides the platform for the interaction of various market sides such as fans, advertisers and sponsors, the media, and merchandising companies. These interactions via an intermediary platform creates what is called a "multisided market." Each of the distinct market sides demands a specific good or service provided by the intermediary. Frequently, the market sides do not interact with each other directly; however, they exert network externalities on each other. These externalities influence the market's demand structure and the intermediary's pricing schemes.

Fans demand competition and the experience of a sports event, advertisers and sponsors demand an audience that they can inform about their products or services, the media demand an audience willing to pay for the use of their services, merchandising companies demand customers who want to buy their articles, etc. An interaction between two market sides only takes place because of the underlying sports event. Fans would hardly consume advertisement content if there were not a match taking place that featured their favorite

[1] According to the so-called "uncertainty of outcome" hypothesis (Rottenberg, 1956), fans prefer to attend games with an uncertain outcome and enjoy close championship races. For empirical contributions that analyze the relation between competitive balance and match attendance, see Downward and Dawson (2000), Borland and MacDonald (2003) and Szymanski (2003).

team. Merchandising companies would sell many fewer fan articles if their products were not linked to an active sports team, and so on. These examples underline the importance of the clubs' competition to act as a platform for the different market sides that interact and exert network externalities on each other.

We add to the literature by contributing to two different strands of literature: on the one hand, the literature on multisided markets and on the other hand, the literature on analytical models of sports leagues. To the best of our knowledge, we are the first to integrate the theory of two-sided markets into a contest model of a professional team sports league. Our model can then be used as a basic framework to analyze the effect of different cross-subsidization schemes in team sports leagues.

In particular, this paper develops a model of a professional sports league with network externalities by integrating the theory of two-sided markets into a two-stage contest model. In professional team sports, the competition of the clubs functions as a platform that enables sponsors to interact with fans. In these club-mediated interactions, positive network effects operate from the fan market to the sponsor market, while positive or negative network effects operate from the sponsor market to the fan market.[2] Clubs react to these network externalities by charging lower prices to fans and, under certain conditions, higher prices to sponsors. Our analysis shows that the size of these network externalities determines the level of competitive balance within the league. Depending on the market potential of the sponsors, competitive balance increases (small market potential) or decreases (large market potential) with stronger combined network effects. Traditional models that do not take network externalities into account, thus under- or overestimate the actual level of competitive balance, which may lead to wrong policy implications. Moreover, we show that clubs benefit from the presence of network externalities because club profits increase with stronger combined network effects.

The paper is of interest to policy-makers in a professional team sports league because we derive recommendations of how to improve competitive balance by taking advantage of network externalities. Our model suggests that an increase in the market potential of sponsors produces a more balanced league because the small club will increase its talent investments more than the large club in equilibrium. Finally, we show that network externalities can mitigate the negative effect of revenue sharing on competitive balance.

The paper is structured as follows. Section 2 reviews the related literature. In Section 3, we present our model with its notation and main assumptions. We specify fan and sponsor demand, the quality of the competition and club profits. In Section 4, we solve

[2] See Becker and Murphy (1993) for a discussion on advertisements as a good or bad. For further analysis of advertisements see, e.g., Depken and Wilson (2004) and Reisinger et al. (2009).

the two-stage game, derive the subgame-perfect equilibria and discuss the results. Section 5 highlights policy implications regarding competitive balance and revenue sharing. Finally, Section 6 points out possible extensions and concludes the paper.

2 Literature Review

Economists have designed various models of sports leagues. In an early contribution, El-Hodiri and Quirk (1971) developed a dynamic decision-making model of a professional sports league and incorporated certain fundamental features of the North American sports industry such as the reserve clause, player drafts and the sale of player contracts among teams. They show that revenue sharing does not influence competitive balance and thus confirm the "invariance proposition".[3] Fort and Quirk (1995) derive similar results in an updated, static version of the El-Hodiri and Quirk model. Atkinson et al. (1988) contradict the invariance proposition and show that revenue sharing can improve competitive balance. In their model, Atkinson et al. adopt a pool-sharing arrangement and a club revenue function that depends on the team's performance and on the performance of all other teams. Their result is supported by Marburger (1997), who builds his model on the assumption that fans care about the relative and absolute quality of teams. Vrooman (1995) shows that sharing the winning-elastic revenue does not affect competitive balance, while sharing the winning-inelastic revenue does improve competitive balance. Késenne (2000*b*) develops a two-team model consisting of a large- and a small-market club and shows that a payroll cap, defined as a fixed percentage of league revenue divided by the number of teams, will improve competitive balance as well as the distribution of player salary within the league (c.f. Késenne, 2007).

The recent sports economics literature has suggested modeling a team sports league by making use of contest theory.[4] In his seminal article, Szymanski (2003) applied Tullock's (1980) rent-seeking contest to ascertain the optimal design of sports leagues. Based on a model of two profit-maximizing clubs and a club revenue function that depends on the relative quality of the home team, Szymanski and Késenne (2004) show that gate revenue sharing decreases competitive balance. This result is driven by the so-called "dulling effect." The dulling effect describes the well-known fact in sports economics that revenue sharing reduces the incentive to invest in playing talent. Dietl and Lang (2008) confirm this finding and further show that gate revenue sharing increases social welfare.

[3] The "invariance proposition" goes back to Rottenberg (1956) and states that the distribution of playing talent between clubs in professional sports leagues does not depend on the allocation of property rights to players' services. See also Vrooman (1996).

[4] The first approaches in contest theory were made by Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983).

As this brief literature review shows, analytical models in sports are mainly focused on the effect of cross-subsidization schemes such as reserve clauses, revenue sharing and salary caps on competitive balance without taking into account that the competition of the clubs provides the platform for the interaction of various market sides (fans, sponsors, advertisers and the media). These club-mediated interactions of different market sides create a "multisided market."

Research related to multisided markets is flourishing and has been conducted on a broad range of topics and industries. For instance, software platforms (Evans et al., 2004), payment systems (Rochet and Tirole, 2002; Schmalensee, 2002; Wright, 2003, 2004), the Internet (Baye and Morgan, 2001; Caillaud and Jullien, 2003) and media markets (Crampes et al., 2009; Reisinger et al., 2009). More general models have been proposed by Rochet and Tirole (2003), Armstrong (2006), Armstrong and Wright (2007) and Belleflamme and Toulemonde (2009). Despite this large variety of applications, the theory of multisided markets has not yet been applied to sports leagues. This paper tries to fill this gap.

3 Model Setup

We model a professional team sports league with two clubs, denoted as 1 and 2. The clubs are asymmetric with respect to their market size - that is, there is one large-market club and one small-market club. Each club $i \in \{1, 2\}$ invests independently a certain amount $x_i \in \mathbb{R}_0^+$ in playing talent to maximize its profits. Talent is measured in perfectly divisible units that can be hired at a competitive labor market.

In our model, the competition of the clubs provides the platform that serves as the intermediary between two groups: fans and sponsors. Fans can consume sports competition by watching a match, while sponsors are attracted to the competition because sports events draw large crowds of potential customers and help to build a positive corporate image. The size of the crowd can then be leveraged through media coverage - an effect that we model indirectly. The attractiveness of a sports event for sponsors increases with the number of fans watching. The presence of sponsors, in turn, may have a negative effect on the attractiveness of the event for the fans. These indirect effects are modeled as network externalities in the sponsor and fan demand functions.

The timing of the model features a two-stage structure:

1. Stage: Clubs invest independently in playing talent with the objective of maximizing their own profits. Talent investments determine the win percentages and thus the quality of the competition of the two clubs.

2. Stage: Given a certain quality of competition, fans and sponsors make their decisions taking into account the network externalities that operate from one market side to the other. Each club then generates its own revenues dependent on the decisions of fans and sponsors.

In the sections that follow we derive the demand functions of fans and sponsors under network externalities and specify the quality of the competition. Finally, we derive club revenues, costs and profits.

3.1 Demand of fans and sponsors under network externalities

We assume that the fans of club $i \in \{1, 2\}$ demand the quantity $q_i^f \in \mathbb{R}_0^+$ given by[5]

$$q_i^f = m_i^f - \frac{p_i^f}{\theta_i} + n^s q_i^s, \tag{1}$$

while the amount of advertising $q_i^s \in \mathbb{R}_0^+$ that sponsors place at club $i \in \{1, 2\}$ is given by[6]

$$q_i^s = m^s - \frac{p_i^s}{\theta_i} + n^f q_i^f. \tag{2}$$

The price fans have to pay to be able to watch a match, is denoted by $p_i^f \in \mathbb{R}_0^+$, while $p_i^s \in \mathbb{R}_0^+$ stands for the price sponsors have to pay for advertisements. Clubs can charge fans for watching the match by selling tickets and also, indirectly, by collectively or individually selling media rights. Through ticket sales, clubs directly generate revenues from fan attendance. Through media rights sales, clubs indirectly generate revenues from fans by the sale of the rights to a broadcasting company, which in turn charges its viewers for the broadcast. In a first approach, our model includes the media indirectly as a lever for higher fan attendance. In further research, the media sector could be modeled as a third market side.

The parameter $m_i^f \in \mathbb{R}^+$ characterizes the market size of club i. Without loss of generality, we assume that club 1 is the large-market club, with a higher drawing potential, and as a result, a bigger fan base than the small-market club 2, such that $m_1^f > m_2^f$. Furthermore, the parameter $m^s \in \mathbb{R}^+$ represents the total market potential of the sponsors, or, in the case of a binding quota for sponsoring defined by the league, the sponsors' bounded market

[5]See, e.g., Armstrong (2006) who utilizes similar linear demand functions in a two-sided market with network externalities.

[6]For the sake of completeness, we define the demand function of the sponsors q_i^s to be zero in the case that there are no fans, i.e., $q_i^f = 0$. However, note that $q_i^f = 0$ will never be an equilibrium outcome.

potential.[7]

The network externalities that operate from the fan market to the sponsor market are referred to as "fan-related network externalities" and are denoted by $n^f \in [0, 1)$. We assume that the fan-related network externalities are positive because more fans imply more publicity and thus have a positive effect on the demand in the sponsor market. On the other hand, the network externalities that operate from the sponsor market to the fan market are referred to as "sponsor-related network externalities" and are denoted by $n^s \in (-1, 1)$. Thus, we allow for positive or negative sponsor-related network externalities. However, we assume that the positive fan-related network externalities are at least as strong as the sponsor-related network externalities in absolute values, i.e., $n^f \geq |n^s|$. The possibly positive (even though small) effect of advertising on consumers (see, e.g., Nelson, 1974 and Kotowitz and Mathewson, 1979) reduces the negative sponsor-related network externalities such that the assumption $n^f \geq |n^s|$ is reasonable.[8]

In general, network externalities can be illustrated by a displacement of the demand functions q_i^f and q_i^s. In this respect, stronger network externalities induce stronger displacement of the corresponding demand functions. The combined network effects from fans and sponsors, denoted by η are given by $\eta \equiv n^f + n^s$. A higher n^f implies that the positive fan-related network externalities are relatively more important than the sponsor-related network externalities, such that the combined network effects increase. Similarly, a higher n^s (i.e., either weaker negative or stronger positive sponsor-related network externalities) results in stronger combined network effects. By assuming that $n^f \geq |n^s|$ the combined network effects η are not smaller than zero - i.e., $\eta \in [0, 2)$. Consequently, $\eta > 0$ describes a situation with positive combined network effects in which the positive fan-related network externalities are stronger than the sponsor-related network externalities in absolute values. If $\eta = 0$ then the combined network effects equal zero. In this case, we have either a situation without network externalities (i.e., $n^f = n^s = 0$) or a situation with equalized network externalities in which both individual network externalities are equal in terms of absolute values (i.e., $n^f = -n^s$).

Finally, the parameter $\theta_i \in \mathbb{R}^+$ denotes the quality of the competition between club i against club j and is specified below by equation (5). We assume that a higher quality of the

[7]Note that the parameter m^s has no subscript, because we assume that there is only one homogeneous group of sponsors in the league offering advertisements to the two types of clubs. The introduction of a club-specific sponsor with market potential m_i^s at club i would not change the results qualitatively. Moreover, under a quota on sponsoring one can imagine restrictions on where advertisements may be placed or on the specific types of companies that are allowed to appear as sponsors in a league.

[8]A potentially negative externality derived from advertisements could be that fans want to watch sports events, not advertisements. In the case where the actual sports event is adapted to commercial requirements, e.g., special advertisement breaks, this aspect becomes even more obvious. For further discussion of this aspect, see Becker and Murphy (1993), Depken and Wilson (2004) and Reisinger et al. (2009).

the event (competition of the clubs) has a positive effect on fan demand, but at the same time, it has also a positive impact on sponsor demand (i.e., $\partial q_i^s/\partial\theta_i = p_i^s/\theta_i^2 + n^f(\partial q_i^f/\partial\theta_i) > 0$): there is a positive effect $\partial q_i^f/\partial\theta_i > 0$ through more fans and a positive leverage effect $p_i^s/\theta_i^2 > 0$, because a high quality event draws a larger audience. The media serve as an additional lever, increasing sponsors' exposure to consumers. Consequently, sponsors' demand increases through a higher quality via more media exposure (Borland and MacDonald, 2003 and Farrelly and Quester, 2003).

3.2 The quality of the competition

Following Dietl and Lang (2008) and Dietl et al. (2009), we assume that the quality of the competition θ_i depends on two factors: the probability of club i's success and the uncertainty of outcome. Furthermore, we assume that both factors enter the quality function as a linear combination with equal weights, that is, the quality of the competition is represented by the combination of the win percentage and the uncertainty of outcome.[9]

We measure the probability of club i's success by the win percentage w_i of this club. The win percentage is characterized by the contest-success function (CSF), which maps the vector (x_i, x_j) of talent investment into probabilities for each club. We apply the logit approach, which is the most widely used functional form of a CSF in sporting contests, and define the win percentage w_i of club i as[10]

$$w_i(x_i, x_j) = \frac{x_i}{x_i + x_j}, \tag{3}$$

where $x_i \geq 0$ characterizes the talent investments of club $i = \{1, 2\}$. We define $w_i(x_i, x_j) := 1/2$ if $x_i = x_j = 0$. Given that the win percentages must sum up to unity, we obtain the adding-up constraint: $w_j = 1 - w_i$ with $i, j \in \{1, 2\}$ and $i \neq j$. Following Szymanski (2004), we adopt the "Contest-Nash conjectures" and compute the derivative of equation (3) as $\partial w_i/\partial x_i = x_j/(x_i + x_j)^2$.[11]

The uncertainty of outcome is measured by the competitive balance in the league. Following Szymanski (2003), Dietl and Lang (2008), and Vrooman (2008), we specify competitive

[9] We will see below that this specification of the quality function gives rise to a quadratic revenue function widely used in the sports economic literature.

[10] The logit CSF was generally introduced by Tullock (1980). It was subsequently axiomatized by Skaperdas (1996) and Clark and Riis (1998). An alternative functional form would be the probit CSF (Lazear and Rosen, 1981; Dixit, 1987), the difference-form CSF (Hirshleifer, 1989) and the value weighted CSF (Runkel, 2006). See Dietl, Franck and Lang (2008) and Fort and Winfree (2009) for analyses concerning the effect of the discriminatory power in the CSF.

[11] See Szymanski (2004).

balance CB by the product of the win percentages, i.e.,

$$CB(x_i, x_j) = w_i(x_i, x_j) \cdot w_j(x_i, x_j) = \frac{x_i x_j}{(x_i + x_j)^2}, \tag{4}$$

with $i, j \in \{1, 2\}$ and $i \neq j$. Note that competitive balance attains its maximum of $1/4$ for a completely balanced league in which both clubs invest the same amount in talent such that $w_1 = w_2 = 1/2$. A less balanced league is then characterized by a lower value of CB.

With the specification of the win percentage given by equation (3) and competitive balance given by equation (4), club i's quality function θ_i as described above is derived as

$$\theta_i(x_i, x_j) = w_i(x_i, x_j) + w_i(x_i, x_j) \cdot w_j(x_i, x_j) = \frac{x_i(x_i + 2x_j)}{(x_i + x_j)^2}, \tag{5}$$

with $i, j \in \{1, 2\}$ and $i \neq j$. A higher win percentage w_i of club i induces the quality of the competition θ_i to increase, albeit with a decreasing rate, which reflects the impact of competitive balance on the quality of the competition, i.e., $\partial\theta_i/\partial w_i > 0$ and $\partial^2\theta_i/\partial w_i^2 < 0$.[12]

3.3 Derivation of club revenues, costs and profits

Each club generates its own revenues such that total revenue R_i of club i is given by the sum of fan-related revenue $p_i^f q_i^f$ and sponsor-related revenue $p_i^s q_i^s$:

$$R_i = p_i^f q_i^f + p_i^s q_i^s = \left[\left(m_i^f - q_i^f + n^s q_i^s\right) q_i^f + \left(m^s - q_i^s + n^f q_i^f\right) q_i^s\right] \cdot \theta_i, \tag{6}$$

with $\theta_i = 2w_i(x_i, x_j) - w_i(x_i, x_j)^2$. This club-specific revenue function, which is quadratic in the win percentage, is widely used in the sports economics literature. For instance, our revenue is consistent with the revenue functions used in Szymanski (2003, p. 1164). Moreover, note that club i's revenues increase with the quality of the competition θ_i.

By assuming a competitive labor market and following the sports economic literature, the market clearing cost of a unit of talent, denoted by c, is the same for every club. The cost function of club $i \in \{1, 2\}$ is thus given by $C(x_i) = cx_i$, where c is the marginal unit cost of talent.

The profit function of club i is then given by revenues minus costs and yields

$$\pi_i(x_i, x_j) = R_i(w_i(x_i, x_j)) - C(x_i). \tag{7}$$

[12] For analyses of competitive balance in sports leagues, see, e.g., Fort and Lee (2007) and Fort and Quirk (2009).

with $i, j \in \{1,2\}$ and $i \neq j$.

4 Equilibrium Analysis

In the first stage, clubs decide on their investments in playing talent, considering the cost of talent and its effect on their win percentage. In the second stage, given the quality of the competition as determined in stage 1, fans and sponsors make their decisions taking into account the network externalities. We apply backward induction to solve for the subgame-perfect equilibria in this two-stage game.

4.1 Stage 2

In this subsection, we characterize the point at which the pricing for fans and sponsors under network externalities is optimal such that clubs obtain maximum revenue. Clubs will take into account the relatedness of the fan and sponsor market and thus consider the consequences of the two distinct network externalities on the pricing decisions and demand functions. Formally, club $i = \{1,2\}$ maximizes its revenue $R_i = p_i^f q_i^f + p_i^s q_i^s$ in stage 2 by taking the investment decisions made in stage 1 as given. Note that we assume that marginal costs for sponsors and fans are zero. The equilibrium in prices and quantities in stage 2 is derived in the next lemma:

Lemma 1 *In stage 2, equilibrium prices and quantities for fans and sponsors of club* $i = \{1,2\}$ *are given by*

$$(\widehat{p}_i^f, \widehat{q}_i^f) = \left(\frac{m_i^f\left(2 - n^f\eta\right) + m^s(n^s - n^f)}{(2-\eta)(2+\eta)}\theta_i, \frac{2m_i^f + m^s\eta}{(2-\eta)(2+\eta)} \right), \tag{8}$$

$$(\widehat{p}_i^s, \widehat{q}_i^s) = \left(\frac{m_i^f(n^f - n^s) + m^s\left(2 - n^s\eta\right)}{(2-\eta)(2+\eta)}\theta_i, \frac{m_i^f\eta + 2m^s}{(2-\eta)(2+\eta)} \right). \tag{9}$$

Proof. See Appendix A.1. ■

In equilibrium, fans will demand the quantity represented by $\widehat{q}_i^f$ and are willing to pay the price represented by $\widehat{p}_i^f$. Correspondingly, the sponsors will demand $\widehat{q}_i^s$ and pay $\widehat{p}_i^s$ for each unit of advertisement in equilibrium.[13]

In order to build the intuition, we consider a scenario in which the sponsors and the fans of club i have symmetric market potential - i.e., $m^s = m_i^f = m_i > 0$. In this scenario,

[13] Note that if the market potential of the sponsors is larger than that of the fans of club i, i.e., $m^s > m_i^f$, we must bound m^s from above such that $m^s < \overline{m}^s \equiv \frac{m_i^f\left(2-n^f\eta\right)}{n^f-n^s}$ in order to guarantee that $\widehat{p}_i^f > 0$.

equilibrium prices and quantities for sponsors and fans of club $i = \{1, 2\}$ are given by

$$\widehat{q}_i^s = \widehat{q}_i^f = \frac{m_i}{2-\eta} \text{ and } \widehat{p}_i^s = \frac{m_i(1-n^s)}{2-\eta}\theta_i,\ \widehat{p}_i^f = \frac{m_i(1-n^f)}{2-\eta}\theta_i.$$

Note that due to the symmetry of the two markets, sponsors and fans of club i demand an equal quantity $\widehat{q}_i^f = \widehat{q}_i^s$ in equilibrium. We derive that stronger combined network effects η yield higher quantities for both fans and sponsors in equilibrium. This follows because an increase in n^s (i.e., either weaker negative or stronger positive sponsor-related network externalities) yields increased combined network effects and thus leads to an increase in the demand of fans. In combination with the positive fan-related network externalities, this induces an increase in demand on the part of sponsors. In contrast to the equilibrium quantities, the equilibrium prices differ between fans and sponsors. Sponsors pay a higher price in equilibrium than do fans - i.e., $\widehat{p}_i^s > \widehat{p}_i^f$ for all $n^f > |n^s|$. Note that the price $\widehat{p}_i^f$ for fans ($\widehat{p}_i^s$ for sponsors) is lower (higher), the stronger are the positive fan-related network externalities n^f, whereas the price $\widehat{p}_i^f$ for fans ($\widehat{p}_i^s$ for sponsors) is lower (higher), the lower is n^s.

Comparative statics for the general case with asymmetric market potential of fans and sponsors lead to the following proposition:

Lemma 2 *(i) Equilibrium quantities for fans and sponsors of club i increase with n^f and n^s, i.e., $\partial\widehat{q}_i^\mu/\partial n^f > 0$ and $\partial\widehat{q}_i^\mu/\partial n^s > 0$ for $\mu \in \{f, s\}$. (ii) Given a certain quality of the competition θ_i equilibrium prices for fans (sponsors) of club i decrease (increase) with stronger fan-related network externalities, i.e., $\partial\widehat{p}_i^f/\partial n^f < 0$ and $\partial\widehat{p}_i^s/\partial n^f > 0$.*

Proof. See Appendix A.2. ■

Part (i) of the lemma shows that the stronger are the positive fan-related network externalities n^f, the higher is the equilibrium quantity demanded by fans and sponsors. If there is a disutility of the sponsors' advertisements for fans ($n^s < 0$), then the equilibrium quantities demanded by fans and sponsors decrease with stronger, i.e., more negative sponsor-related network externalities. If, on the other hand, $n^s > 0$, then the equilibrium quantities demanded by fans and sponsors increase with stronger, i.e., more positive sponsor-related network externalities.

It follows that the equilibrium demands $\widehat{q}_i^s$ and $\widehat{q}_i^f$ are higher in a situation in which the combined network effects are positive than in a situation in which the combined network effects are zero. The intuition is as in the case with symmetric market potential above. Ceteris paribus, an increase in n^s leads to an increase in fan demand and consequently, due to positive fan-related network externalities, to an increase in the demand of the sponsors.

Note that fans of club i demand a higher quantity in equilibrium if their market potential is larger than that of the sponsors, i.e., $\widehat{q}_i^f > \widehat{q}_i^s \Leftrightarrow m_i^f > m^s$.

Part (ii) of the lemma shows that given a certain quality of the competition θ_i the equilibrium price $\widehat{p}_i^f$ for the fans of club i is lower, the stronger are the positive fan-related network externalities n^f, whereas the opposite holds true for the equilibrium price $\widehat{p}_i^s$ for the sponsors. This result is in accordance with the special case of symmetric market potentials. Relatively stronger fan-related network externalities induce an increase in the demand function of the sponsors and yield, ceteris paribus, a decrease in the prices for sponsors. Thus, if club i decreases the price for the market with the stronger positive network externalities (in our model the fan market), it enhances the positive effect on revenues. It follows that due to the positive network externalities exerted by the fans on the sponsors, a revenue-maximizing club has an incentive to keep prices low on the market with the positive network externalities, whereas in the market with relatively weaker positive or even negative network externalities (the sponsor market), it has an incentive to charge higher prices.

Whether the equilibrium price for fans is higher than that for the sponsors depends on the relationship between the market potential of fans and sponsors and the particular network externalities. Formally, we derive $\widehat{p}_i^f < \widehat{p}_i^s \Leftrightarrow m_i^f/m^s < (1-n^s)\,/\left(1-n^f\right)$. Thus, as long as the market potential of the fans relative to that of the sponsors is smaller than $(1-n^s)\,/\left(1-n^f\right)$, prices are higher in the sponsor market than in the fan market. Ceteris paribus, a decrease in the fan-related network externalities renders the fan market less important (due to its weaker network externalities) and the right-hand side of the inequality decreases such that the inequality may not be satisfied anymore. In this case, equilibrium prices on the fan market may be higher than on the sponsor market. Note that if the market potential of the sponsor market is higher than the market potential of the fan market for club i (i.e. $m^s > m_i^f$) then independent of the network externalities, prices will be higher in the sponsor market because $(1-n^s)\,/\left(1-n^f\right) > 1$ for all $1 > n^f > |n^s| \geq 0$.

Furthermore, we derive from (8) and (9) that in a situation without network externalities (i.e., $n^f = n^s = 0$), club i maximizes its revenue by making the quantity sold to fans directly proportional to the quantity sold to sponsors with $\widehat{q}_i^f = (m_i^f/m^s)\widehat{q}_i^s$.[14] Finally, we see that equilibrium prices for fans (sponsors) are lower (higher) in a situation with positive combined network effects than in a situation in which combined network effects equal zero.

[14] Note that this relationship holds true also in a situation in which combined network effects are zero.

By substituting equilibrium prices and quantities of fans and sponsors from (8) and (9) in the revenue function (6), we compute the revenue of club i as[15]

$$\widehat{R}_i = \kappa_i \cdot \theta_i = \kappa_i \frac{x_i(x_i + 2x_j)}{(x_i + x_j)^2}, \tag{10}$$

with

$$\kappa_i \equiv \frac{(m_i^f)^2 + (m^s)^2 + m_i^f m^s \eta}{(2-\eta)(2+\eta)}, \ i = \{1, 2\}. \tag{11}$$

In the next lemma, we derive some useful properties of the function κ_i which will be exploited in the subsequent analysis:

Lemma 3 *We consider $\kappa_i(\eta)$ as a function of the combined network externalities η and derive the following properties: $\kappa_1(\eta) > \kappa_2(\eta)$ and $\partial\kappa_1(\eta)/\partial\eta > \partial\kappa_2(\eta)/\partial\eta > 0$.*

Proof. Straightforward and therefore omitted. ■

It follows from Lemma 3 that given a certain quality of competition equal for both clubs - i.e., $\theta_1 = \theta_2$ - the revenue of the large club will be higher than the revenue of the small club. Moreover, revenues for both types of clubs increase with stronger combined network effects, where the increase is stronger for the large club than for the small club.

4.2 Stage 1

In stage 1, club i maximizes its profits by anticipating the decisions made in stage 2. By substituting club revenues (10) into the profit function (7), we derive the maximization problem of club $i = \{1, 2\}$ in stage 1 as

$$\max_{x_i \geq 0} \left\{ \pi_i = \widehat{R}_i(x_i, x_j) - cx_i \right\} = \left(\frac{(m_i^f)^2 + (m^s)^2 + m_i^f m^s \eta}{(2-\eta)(2+\eta)} \right) \frac{x_i(x_i + 2x_j)}{(x_i + x_j)^2} - cx_i, \tag{12}$$

The first-order conditions for this maximization problem yield[16]

$$\frac{\partial \pi_i}{\partial x_i} = \left(\frac{(m_i^f)^2 + (m^s)^2 + m_i^f m^s \eta}{(2-\eta)(2+\eta)} \right) \frac{2x_j^2}{(x_i + x_j)^3} - c = 0.$$

[15] The revenue function given by (10) satisfies the properties of the revenue function proposed by Szymanski and Késenne (2004, p. 168).

[16] It is easy to verify that the second-order conditions for a maximum are satisfied.

Solving this system of equations, yields the equilibrium talent investments of clubs $i = \{1, 2\}$ in stage 1 as

$$\widehat{x}_i = \frac{2\kappa_i\kappa_j\left[\kappa_i(\kappa_i + 3\kappa_j) - (\kappa_i\kappa_j)^{1/2}(3\kappa_i + \kappa_j)\right]}{c(\kappa_i - \kappa_j)^3}, \tag{13}$$

with $\kappa_i = \frac{(m_i^f)^2+(m^s)^2+m_i^f m^s \eta}{(2-\eta)(2+\eta)}$ and $i, j = \{1, 2\}, i \neq j$. Both types of clubs invest a positive amount $\widehat{x}_i > 0$ in playing talent. Moreover, the large club invests more in talent than does the small club (i.e., $\widehat{x}_1 > \widehat{x}_2$) because the marginal revenue of talent investments is higher for the former type of club due to the larger market potential of its fans.[17] Note that the investments of both clubs are influenced by the network externalities exerted by fans and sponsors. Again, the extent to which fans and sponsors indirectly influence each other determines the decision of each club to invest in playing talent.

Substituting the equilibrium investments (13) in the CSF function (3) yields the following equilibrium win percentages:

$$(\widehat{w}_1, \widehat{w}_2) = \left(\frac{\kappa_1}{\kappa_1 + (\kappa_1\kappa_2)^{1/2}}, \frac{\kappa_2}{\kappa_2 + (\kappa_1\kappa_2)^{1/2}}\right). \tag{14}$$

By analyzing the impact of network externalities on the win percentages, we can establish the following proposition:

Proposition 1 *Stronger combined network effects η induce the large (small) club to decrease (increase) its win percentage in equilibrium and thus produce a more balanced league if and only if the market potential of the sponsors is sufficiently small. Formally, $\partial\widehat{w}_1/\partial\eta < 0$ and $\partial\widehat{w}_2/\partial\eta > 0 \Leftrightarrow m^s < \widehat{m}^s \equiv (m_1^f m_2^f)^{1/2}$.*

Proof. See Appendix A.5. ■

The proposition shows that with a sufficiently small market potential of the sponsors, the win percentage of the large (small) club is lower (higher), the stronger the positive network externalities that operate from fans to sponsors are. A lower disutility or a higher utility of the sponsors' advertisements for the fans yields the same result. The intuition behind this proposition follows: The difference in market sizes for the two clubs regarding their fan base yields that sponsor-related revenues are relatively more important to the small club than to the large club. To attract sponsors, the small-market club increases its investment in talent as combined network effects increase, thereby increasing its win percentage. The potentially negative impact of more sponsors on the attractiveness of the match to the fans is less important to the small club. For the large-market club, the opposite rationale holds. Fan-related revenues are relatively more important because of the larger market size. In equilibrium, it

[17]See Buraimo et al. (2007) who analyze how closely playing success is linked to market size in practice.

is optimal for the large-market club to invest less in talent, as the revenue impact of less sponsors overcompensates the potentially decreasing attractiveness of the match to the fans. Consequently, with stronger combined network externalities competitive balance increases. Thus, a league in which the positive fan-related network externalities are stronger than the sponsor-related network externalities (in absolute value) may be characterized by a higher degree of competitive balance than a league in which combined network effects are zero. For a sufficiently large market potential of the sponsors, the opposite holds true. In this case, competitive balance decreases when combined network effects increase.

Furthermore note that the quality of the competition $\widehat{\theta}_i$ in equilibrium can be expressed in terms of κ_i as $\widehat{\theta}_i = \widehat{w}_i + \widehat{w}_i\widehat{w}_j = \frac{\kappa_i(2\kappa_j+(\kappa_i\kappa_j)^{1/2})}{(\kappa_i+(\kappa_i\kappa_j)^{1/2})(\kappa_j+(\kappa_i\kappa_j)^{1/2})}$. A direct consequence of Proposition 1 is that stronger network effects imply a lower (higher) quality of competition for the large (small) club if and only if the market potential of the sponsors is sufficiently small. Formally, $(\partial\widehat{\theta}_1/\partial\eta < 0$ and $\partial\widehat{\theta}_2/\partial\eta > 0) \Leftrightarrow m^s < \widehat{m}^s$.[18]

The impact of network externalities on club profits is established in the following proposition:

Proposition 2 *Stronger combined network effects yield an increase in profits for the small and the large club.*

Proof. See Appendix A.6. ■

The proposition shows that the profits of the small and the large club increase if the positive network externalities that operate from the fan market to the sponsor market increase (or equivalently, through a weaker impact by sponsors' advertisements on the fans). Thus, the two types of clubs benefit from stronger network effects. To see the intuition behind this result, remember that the profits of club i in equilibrium are given by $\widehat{\pi}_i = \kappa_i\widehat{\theta}_i - c\widehat{x}_i$, and thus, the partial derivatives with respect to combined network effects η yield $\partial\widehat{\pi}_i/\partial\eta = (\partial\kappa_i/\partial\eta)\widehat{\theta}_i + \kappa_i(\partial\widehat{\theta}_i/\partial\eta) - c(\partial\widehat{x}_i/\partial\eta)$. Through stronger combined network effects, both types of clubs face higher costs due to a higher investment level in playing talent. On the other hand, stronger combined network effects have a positive effect on equilibrium quantities $(\widehat{q}_i^f, \widehat{q}_i^s)$ and prices $(\widehat{p}_i^f, \widehat{p}_i^s)$ such that club revenues for both types of clubs increase. The higher club revenues compensate for the higher costs, and thus, club profits increase. Note that the positive effect on club revenues due to stronger combined network effects holds true even though the quality of the competition $\widehat{\theta}_i$ will decrease for the large (small) club if the market potential m^s of the sponsors is sufficiently small (large).

[18]Note that the match quality for the large (small) market club decreases (increases) if and only if the league becomes more balanced. As we know from Proposition 1, a more balanced (unbalanced) league emerges in the case of sufficiently low (high) market potential on the part of the sponsors.

5 Further Implications and Discussion

5.1 Competitive balance and network externalities

Research on competitive balance has not considered the influence of network effects so far, i.e., the parameter η has been assumed to be zero. By integrating the existence of network effects into models of sports leagues, new policy measures for leagues and their governing bodies emerge. For example, Proposition 1 suggests that network externalities potentially affect competitive balance when there is a limit on sponsoring activities. In particular, if sponsors only dispose of a limited quota for advertisements $m^s < \widehat{m}^s$, competitive balance increases through stronger network externalities that operate from fans to sponsors (or equivalently, through a weaker impact by sponsors' advertisements on the fans).

The league and its clubs cannot manipulate the strength of the network externalities. However, by controlling the market potential of the sponsors, they can make sure that the network externalities operate in the desired direction. The market potential of the sponsors is thus a crucial parameter to control the competitive balance in our league model. This result will be emphasized in the next proposition.

Proposition 3 *Competition in the league becomes more balanced when the market potential m^s of the sponsors increases.*

Proof. See Appendix A.7. ■

The proposition shows that a possible measure for improving competitive balance is to increase the market potential of the sponsors. For this to hold, however, the market potential of the sponsors has to remain below the threshold given in Proposition 1, i.e., $m^s < \widehat{m}^s$. Otherwise, stronger network effects would have a negative impact on the competitive balance in the league, and thus mitigate the positive effect of an increased m^s. An increase in the market potential of the sponsors could be achieved, for instance, through an increase in the quota for the amount of advertisements set by the league.

The intuition behind the result in Proposition 3 is that clubs generate revenues from fans and sponsors, where the amount of sponsorship revenues also depends on the amount of fans affiliated with a certain club (see Lemma 1). In equilibrium, the revenues generated from the sponsors' advertisements are higher for the large club than for the small club due to the larger market potential from the fans of the large club. An increase in the quota for the amount of advertising for the sponsors increases both clubs' revenues. Due to the decreasing returns to scale of sponsors' advertising, the increase in revenues, however, is stronger for the small club than for the large club. It follows that the incentives to invest in playing talent are higher for the small club than for the large club. This relative difference causes the former type of club to increase its equilibrium talent investments more than the latter type of club. As a result, the win percentage of the large (small) club decreases (increases) and a more balanced league emerges.

5.2 Revenue sharing and network externalities

In this section, we analyze the effect of revenue sharing in the presence of network externalities. The sharing of revenues plays an important role in the redistribution of revenues and has long been accepted as an exemption from antitrust law.[19] The basic idea of such a cross-subsidization policy is to guarantee a reasonable competitive balance in the league by redistributing revenues from large-market clubs to small-market clubs because large-market clubs have a higher revenue-generating potential than do small-market clubs (Atkinson et al., 1988; Késenne, 2000, Szymanski and Késenne, 2004; Dietl, Lang and Rathke, 2010). The current revenue-sharing schemes vary widely among professional sports leagues all over the world. The most prominent is possibly that operated by the National Football League (NFL), in which the visiting club receives 40% of the locally earned television and gate receipt revenue. Major League Baseball (MLB) has a revenue-sharing agreement whereby all the clubs in the American League put 34% of their locally generated revenue (gate, concessions, television, etc.) into a central pool, which is then divided equally among all the clubs.

We introduce a pool revenue-sharing arrangement into our model. Under a pool-sharing arrangement, club i receives an α-share of its revenue R_i and an $(1-\alpha)/2$-share of a league revenue pool $R_i + R_j$. The after-sharing revenues of club i, denoted by R_i^*, can be written as:

$$R_i^* = \alpha \widehat{R}_i + \frac{(1-\alpha)}{2}(\widehat{R}_i + \widehat{R}_j),$$

with $\alpha \in (0,1]$ and $i,j \in \{1,2\}$, $i \neq j$. Note that a higher parameter α represents a league with a lower degree of redistribution. Thus, the limiting case of $\alpha = 1$ describes a league without revenue-sharing.

The maximization problem of club i is thus given by

$$\max_{x_i \geq 0} \left\{ R_i^*(x_i, x_j) = \alpha \widehat{R}_i(x_i, x_j) + (1-\alpha)\widehat{R}_j(x_i, x_j) - cx_i \right\}, \tag{15}$$

with $i,j \in \{1,2\}$ and $i \neq j$. By solving this maximization and analyzing the effect of α on the equilibrium win percentages, we can establish the following proposition:

[19] Professional team sports leagues often find themselves under antitrust surveillance (Flynn and Gilbert, 2001). Most revenue-sharing arrangements, however, have not been challenged in the courts because revenue sharing is supposed to enhance competitive balance, and thus, is in the interest of the consumer (Szymanski, 2003).

Proposition 4 *In the presence of network externalities, revenue sharing always decreases the competitive balance in the league. Network externalities, however, mitigate the negative effect of revenue sharing on competitive balance if and only if the market potential m^s of the sponsors is sufficiently small with $m^s < \widehat{m}^s$.*

Proof. See Appendix A.8. ■

In accordance to other contest models of sports leagues with profit-maximizing clubs (e.g., Szymanski and Késenne, 2004; Grossmann, Dietl and Lang, 2010), the proposition shows that revenue sharing produces a less balanced competition in a league in which positive network externalities operate from the fan market to the sponsor market, while negative network externalities operate from the sponsor market to the fan market. A higher degree of revenue sharing, i.e., a lower parameter α, results in a higher win percentage for the large club and a lower win percentage for the small club.

The intuition behind this result is as follows. Revenue sharing has a negative effect on marginal revenue of both clubs in equilibrium. This so-called "dulling effect" is more pronounced for the underdog (small-market club) than for the dominant team (large-market club). Due to the logit formulation of the CSF, the (positive) marginal impact on the dominant team's revenues of a decrease in talent investment by the underdog is greater than the (positive) marginal impact on the underdog's revenues of a decrease in talent investment by the dominant team. As a result, the small club will reduce its investment level relatively more than the large club such that the league becomes less balanced through revenue sharing.

Network externalities, however, can mitigate the negative effect of revenue sharing on competitive balance. In particular, if the market potential of the sponsors is sufficiently small such that $m^s < \widehat{m}^s$ then stronger combined network effects reduce the differences between clubs in terms of win percentages and thus reduce the negative effect of revenue sharing on competitive balance. In the opposite case, i.e., $m^s > \widehat{m}^s$, network externalities even reinforce the dulling effect such that the negative impact of revenue sharing on competitive balance augments.

6 Conclusion

In this paper, we have developed a contest model of a professional team sports league with two market sides. The competition of the clubs is the platform between fans on one market side and sponsors on the other market side. Positive network externalities operate from the fan market to the sponsor market, and ambiguous network externalities operate from the sponsor market to the fan market.

Our analysis shows that a revenue-maximizing club has an incentive to keep prices low in the market with relatively stronger positive network externalities and charge a higher price in a market with relatively weaker positive or negative network externalities. In our model, low prices on the fan market enhance the positive effect on club revenues due to the positive network externalities that operate from the fan market to the sponsor market. An increase in the demand in the fan market leads (through positive fan-related network externalities) to an increase in the demand on the sponsor market. High prices in the market with positive network externalities would inhibit the positive effect on club revenues.

We further derive that network externalities may crucially affect competitive balance in a sports league. In particular, we show that stronger combined network effects induce both clubs to increase their talent investments in equilibrium. If the market potential of the sponsors is sufficiently small, the increase in talent investments of the small club will be stronger than that of the large club because the small club benefits more from stronger network effects than the large club. As a result, the win percentage of the small club increases and the win percentage of the large club decreases in equilibrium, yielding a more balanced league. With the introduction of a revenue sharing arrangement, our model shows that network externalities can mitigate the negative effect of revenue sharing on competitive balance.

We conclude that it is important to incorporate network externalities into the analysis of team sports leagues. Depending on the market potential of the sponsors, traditional analyses of sports leagues that do not take network externalities into account may under- or overestimate the actual level of competitive balance in a league. Based on these predictions, traditional analyses may therefore suggest the wrong policy implications. For instance, they may suggest the implementation of measures to increase competitive balance, which may not be necessary because the league may already be sufficiently balanced. Finally, our model suggests that both types of clubs benefit from the presence of network externalities because club profits always increase with stronger combined network effects. This result holds true even though costs increase for both types of clubs due to higher talent investments. The higher club revenues, however, compensate for the higher costs, such that club profits always increase.

Taking a closer look at major team sports leagues worldwide, one can find a number of phenomena that may be explained by our model. For example, the differences in match attendance and average ticket prices between national leagues in European football are accompanied by strong divergences in sponsor-related revenues. While match-day income (e.g., ticket sales and the like) makes up a higher percentage of revenues in the English Premier League than in the German Bundesliga, sponsorship is far more important in the latter.

This fact may mirror the trade-off between fan-related and sponsor-related revenues. The quota for sponsorship in many North American major leagues represents another example; even though teams might be able to obtain higher revenues by increasing the amount of sponsoring/advertisements, the majority of teams refrains from posting advertisements on jerseys.[20]

Our model serves as a basic framework for the analysis of network effects in team sports leagues. There is a broad range of further applications and model extensions. For instance, an interesting avenue for further research could be the application of our model to a league that operates with restrictions (caps and floors) on player salaries. Payroll restrictions to improve competitive balance and control costs are common in professional team sports leagues all around the world. The implementation of such payroll restrictions in the model with network externalities could yield further implications for the governance of team sports leagues.

[20]Note that teams in the National Football League (NFL) are allowed to post a sponsor on their jerseys. Only a small proportion of teams, however, makes use of this opportunity.

A Appendix

A.1 Proof of Lemma 1

In stage 2, club $i \in \{1,2\}$ maximizes its revenue $R_i = p_i^f q_i^f + p_i^s q_i^s$, by taking the investment decisions made in stage 1 as given. Formally, club i solves the following maximization problem:[21]

$$\max_{(q_i^f, q_i^s)\geq 0} R_i = p_i^f q_i^f + p_i^s q_i^s = \left[(m_i^f - q_i^f + n^s q_i^s) q_i^f + (m^s - q_i^s + n^f q_i^f) q_i^s\right] \theta_i. \tag{16}$$

The reaction functions of fans and sponsors are derived as

$$q_i^f\left(q_i^s\right) = \frac{1}{2}\left(m_i^f + (n^f + n^s) q_i^s\right) \text{ and } q_i^s\left(q_i^f\right) = \frac{1}{2}\left(m^s + (n^f + n^s) q_i^f\right).$$

Note that there is a positive relationship between the quantities demanded by sponsors and fans in equilibrium because if the combined network effects are positive, i.e., $n^f + n^s > 0$.

Solving this system of reaction functions, yields the following equilibrium quantities for club i

$$\widehat{q}_i^f = \frac{2m_i^f + m^s\eta}{(2-\eta)(2+\eta)} \text{ and } \widehat{q}_i^s = \frac{m_i^f\eta + 2m^s}{(2-\eta)(2+\eta)}.$$

Substitution into prices $p_i^f = \left(m_i^f - \widehat{q}_i^f + n^s \widehat{q}_i^s\right)\theta_i$ and $p_i^s = \left(m^s - \widehat{q}_i^s + n^f \widehat{q}_i^f\right)\theta_i$ yields

$$\widehat{p}_i^f = \frac{m_i^f\left(2 - n^f\eta\right) + m^s(n^s - n^f)}{(2-\eta)(2+\eta)}\theta_i \text{ and } \widehat{p}_i^s = \frac{m_i^f(n^f - n^s) + m^s\left(2 - n^s\eta\right)}{(2-\eta)(2+\eta)}\theta_i.$$

This completes the proof of the lemma.

A.2 Proof of Lemma 2

(i) In order to show that equilibrium quantities $(\widehat{q}_i^f, \widehat{q}_i^s)$ for fans and sponsors of club i increase (decrease) with stronger fan (sponsor) network effects, we compute

$$\frac{\partial \widehat{q}_i^f}{\partial n^f} = \frac{\partial \widehat{q}_i^f}{\partial n^s} = \frac{4m_i^f\eta + m^s(4+\eta^2)}{\left[(2-\eta)(2+\eta)\right]^2} > 0 \text{ and } \frac{\partial \widehat{q}_i^s}{\partial n^f} = \frac{\partial \widehat{q}_i^s}{\partial n^s} = \frac{4m^s\eta + m_i^f(4+\eta^2)}{\left[(2-\eta)(2+\eta)\right]^2} > 0,$$

for all $m_i^f > 0$, $m^s > 0$, $1 > n^f \geq |n^s| \geq 0$ and $\eta \in [0, 2)$.

(ii) In order to show that, given a certain quality of the competition θ_i, equilibrium prices

[21] In our setting it is an equivalent approach if clubs first maximize revenues with respect to quantities and then derive equilibrium prices or vice versa.

$\widehat{p}_i^f$ for fans ($\widehat{p}_i^s$ for sponsors) of club i decrease (increase) with stronger fan-related network effects, we compute

$$\begin{aligned}
\frac{\partial \widehat{p}_i^f}{\partial n^f} &= \frac{m_i^f\left(n^s\eta^2 - 4n^f\right) + m^s\left[4 + \eta(n^f - 3n^s)\right]}{\left[(2-\eta)(2+\eta)\right]^2} < 0, \\
\frac{\partial \widehat{p}_i^s}{\partial n^f} &= \frac{m_i^s\left(4n^f - n^s\eta^2\right) + m_i^f\left[4 + \eta(n^f - 3n^s)\right]}{\left[(2-\eta)(2+\eta)\right]^2} > 0,
\end{aligned}$$

for all $m_i^f > 0$, $m^s > 0$, $1 > n^f \geq |n^s| \geq 0$ and $\eta \in [0, 2)$. This completes the proof of the lemma.

A.3 Proof of Proposition 1

To prove that stronger network effects induce the large (small) club to decrease (increase) its win percentage in equilibrium if and only if the market potential of the sponsors is sufficiently small, we proceed as follows. We write $\frac{\partial \kappa_i(\eta)}{\partial \eta} = \kappa_i'(\eta)$. According to Lemma 3, we know that $\kappa_1(\eta) > \kappa_2(\eta)$ and $\kappa_1'(\eta) > \kappa_2'(\eta) > 0$. Thus, we compute $\widehat{w}_1/\widehat{w}_2 = \kappa_1(\eta)/\left[\kappa_1(\eta)\kappa_2(\eta)\right]^{1/2} > 1$. Now, we will show that $\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial \eta} < 0$ and thus $\frac{\partial \widehat{w}_1}{\partial \eta} < 0$ and $\frac{\partial \widehat{w}_2}{\partial \eta} > 0$:

$$\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial \eta} = \frac{\kappa_1(\eta)\left[\kappa_1'(\eta)\kappa_2(\eta) - \kappa_1(\eta)\kappa_2'(\eta)\right]}{2\left[\kappa_1(\eta)\kappa_2(\eta)\right]^{3/2}} < 0 \Leftrightarrow \frac{\kappa_1(\eta)}{\kappa_2(\eta)} > \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)}.$$

With $\kappa_i(\eta)$ given by (11), it holds

$$\frac{\kappa_1(\eta)}{\kappa_2(\eta)} = \frac{(m_1^f)^2 + (m^s)^2 + m_1^f m^s \eta}{(m_2^f)^2 + (m^s)^2 + m_2^f m^s \eta} \text{ and } \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)} = \frac{\left(m_1^f\eta + 2m^s\right)\left(2m_1^f + m^s\eta\right)}{\left(m_2^f\eta + 2m^s\right)\left(2m_2^f + m^s\eta\right)}.$$

We conclude that $\frac{\kappa_1(\eta)}{\kappa_2(\eta)} > \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)} \Leftrightarrow m^s < \widehat{m}^s$. This completes the proof of the proposition.

A.4 Proof of Proposition 2

For expositional sake, we provide a formal proof for a linear revenue function. The proof for a quadratic revenue function is mathematically equivalent but notational very cumbersome. We therefore stick to the case of linear revenues. In case of linear revenues, the profit function of club i is given by $\pi_i = \kappa_i w_i - x_i$, such that the equilibrium investments $\widehat{x}_i$ and win percentages $\widehat{w}_i$ yield

$$(\widehat{x}_1, \widehat{x}_2) = \left(\frac{\kappa_1^2\kappa_2}{(\kappa_1+\kappa_2)^2}, \frac{\kappa_1\kappa_2^2}{(\kappa_1+\kappa_2)^2}\right) \text{ and } (\widehat{w}_1, \widehat{w}_2) = \left(\frac{\kappa_1}{\kappa_1+\kappa_2}, \frac{\kappa_2}{\kappa_1+\kappa_2}\right).$$

Equilibrium profits $\widehat{\pi}_i$ of club i are thus computed as $\widehat{\pi}_i = \frac{\kappa_i^2}{\kappa_1+\kappa_2}$. The derivative with respect to network effects η is given by

$$\frac{\partial \widehat{\pi}_i}{\partial \eta} = \frac{\kappa_i(\eta)\left[(\kappa_i(\eta) + 2\kappa_j(\eta))\,\kappa_i'(\eta) - \kappa_i(\eta)\kappa_j'(\eta)\right]}{(\kappa_i(\eta) + \kappa_j(\eta))^2}.$$

We derive $\frac{\partial \widehat{\pi}_1}{\partial \eta} > 0 \Leftrightarrow \kappa_1(\eta) > \kappa_2(\eta)$ and $\kappa_1'(\eta) > \kappa_2'(\eta) > 0$, whereas $\frac{\partial \widehat{\pi}_2}{\partial \eta} > 0 \Leftrightarrow \frac{2\kappa_1(\eta)+\kappa_2(\eta)}{\kappa_2(\eta)} > \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)}$. However, one can show that the last inequality is always fulfilled for κ_i given by (11), in combination with $m_1^f > m_2^f > 0$, $m^s > 0$ and $\eta \in [0, 2)$. This completes the proof of the proposition.

A.5 Proof of Proposition 3

To prove that a larger market potential m^s of the sponsors increases the competitive balance in the league, we proceed as follows. We consider $\kappa_i(m^s) = \frac{(m_i^f)^2+(m^s)^2+m_i^f m^s \eta}{(2-\eta)(2+\eta)}$ as a function of m^s and write $\frac{\partial \kappa_i(m^s)}{\partial m^s} = \kappa_i'(m^s)$. We derive the following properties:

$$\begin{aligned}
\kappa_1(m^s) - \kappa_2(m^s) &= \frac{\left(m_1^f - m_2^f\right)\left(m_1^f + m_2^f + m^s\eta\right)}{(2-\eta)(2+\eta)} > 0, \\
\kappa_i'(m^s) &= \frac{m_i^f\eta + 2m^s}{(2-\eta)(2+\eta)} > 0, \text{ and } \kappa_1'(m^s) > \kappa_2'(m^s).
\end{aligned}$$

for all $m_1^f > m_2^f > 0, m^s > 0$ and $\eta \in [0, 2)$. We know that competitive balance can be expressed in terms of $\kappa_i(m^s)$ as $\frac{\widehat{w}_1}{\widehat{w}_2} = \frac{\kappa_1(m^s)}{[\kappa_1(m^s)\kappa_2(m^s)]^{1/2}} > 1$. Now, we will show that $\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial m^s} < 0$ and thus $\frac{\partial \widehat{w}_1}{\partial m^s} < 0$ and $\frac{\partial \widehat{w}_2}{\partial m^s} > 0$:

$$\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial m^s} = \frac{\kappa_1(m^s)\left[\kappa_1'(m^s)\kappa_2(m^s) - \kappa_1(m^s)\kappa_2'(m^s)\right]}{2\left[\kappa_1(m^s)\kappa_2(m^s)\right]^{3/2}} < 0 \Leftrightarrow \frac{\kappa_1(m^s)}{\kappa_2(m^s)} > \frac{\kappa_1'(m^s)}{\kappa_2'(m^s)}$$

We derive

$$\frac{\kappa_1(m^s)}{\kappa_2(m^s)} = \frac{(m_1^f)^2 + (m^s)^2 + m_1^f m^s\eta}{(m_2^f)^2 + (m^s)^2 + m_2^f m^s\eta} \text{ and } \frac{\kappa_1'(m^s)}{\kappa_2'(m^s)} = \frac{m_1^f\eta + 2m^s}{m_2^f\eta + 2m^s}$$

and can show that $\frac{\kappa_1(m^s)}{\kappa_2(m^s)} > \frac{\kappa_1'(m^s)}{\kappa_2'(m^s)}$ holds for all $m^s > 0$. We conclude that competitive balance increases with a larger market potential of the sponsors, i.e., $\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial m^s} < 0$. This completes the proof of the proposition.

A.6 Proof of Proposition 4

The first-order conditions of the maximization problem (15) are derived as

$$\frac{\partial R_i^*}{\partial x_i} = \alpha \frac{\partial \widehat{R}_i}{\partial w_i}\frac{\partial w_i}{\partial x_i} + \frac{1-\alpha}{2}\left(\frac{\partial \widehat{R}_i}{\partial w_i}\frac{\partial w_i}{\partial x_i} + \frac{\partial \widehat{R}_j}{\partial w_j}\frac{\partial w_j}{\partial x_i}\right) - c = 0,$$

with $i, j \in \{1, 2\}$ and $i \neq j$. It is easy to verify that the second-order conditions for a maximum are satisfied. By combining the first-order conditions of club i and j, and using the adding-up constraint $\frac{\partial w_i}{\partial x_i} = -\frac{\partial w_j}{\partial x_i}$, we obtain

$$\left[\alpha \frac{\partial \widehat{R}_i}{\partial w_i} - \frac{1-\alpha}{2}\left(\frac{\partial \widehat{R}_j}{\partial w_j} - \frac{\partial \widehat{R}_i}{\partial w_i}\right)\right]\frac{\partial w_i}{\partial x_i} = \left[\alpha \frac{\partial \widehat{R}_j}{\partial w_j} - \frac{1-\alpha}{2}\left(\frac{\partial \widehat{R}_i}{\partial w_i} - \frac{\partial \widehat{R}_j}{\partial w_j}\right)\right]\frac{\partial w_j}{\partial x_j},$$

with $i, j \in \{1, 2\}$ and $i \neq j$. By using (3) and (10), and after some rearrangements, we find that in equilibrium $(\widehat{x}_1^*, \widehat{x}_2^*)$ it must hold

$$\widehat{x}_1^* = \frac{(1-\alpha)(\kappa_1 - \kappa_2) + \left[(1-\alpha)^2(\kappa_1^2 + \kappa_2^2) + 2\kappa_1\kappa_2(1 + \alpha(6+\alpha))\right]^{1/2}}{2(1+\alpha)\kappa_2}\widehat{x}_2^*,$$

with κ_i given by (11) and $i \in \{1, 2\}$. It follows that the equilibrium win percentage of club i is given by

$$\widehat{w}_i^* = \frac{\kappa_i(1+3\alpha) + \kappa_j(1-\alpha) - \left[(1-\alpha)^2(\kappa_1^2 + \kappa_2^2) + 2\kappa_1\kappa_2(1 + \alpha(6+\alpha))\right]^{1/2}}{4\alpha(\kappa_i - \kappa_j)},$$

with κ_i given by (11) and $i, j \in \{1, 2\}$, $i \neq j$.

We further compute the partial derivative of $\widehat{w}_1^*$ with respect to α at $\alpha = 1$ as $\left.\frac{\partial \widehat{w}_1^*}{\partial \alpha}\right|_{\alpha=1} = -\frac{\kappa_1 + \kappa_2 - 2\sqrt{\kappa_1\kappa_2}}{4(\kappa_1 - \kappa_2)} < 0$, because $\kappa_1 > \kappa_2$. We conclude that a higher degree of revenue sharing (i.e., a lower α) increases the win percentage of the large-market club 1 and consequently decreases the win percentage of the small-market club 2. As a result, competitive balance decreases which proves part (i) of the proposition.

To prove part (ii), we proceed as follows. We define $F(\eta)$ as the partial derivative of $\widehat{w}_1^*$ with respect to α at $\alpha = 1$, i.e., $F(\eta) := \frac{\kappa_1(\eta) + \kappa_2(\eta) - 2\sqrt{\kappa_1(\eta)\kappa_2(\eta)}}{4(\kappa_1(\eta) - \kappa_2(\eta))}$, and we show

$$F'(\eta) = \frac{\left[\kappa_1(\eta) + \kappa_2(\eta) - 2\sqrt{\kappa_1(\eta)\kappa_2(\eta)}\right]\left[\kappa_1(\eta)\kappa_2'(\eta) - \kappa_1'(\eta)\kappa_2(\eta)\right]}{4\sqrt{\kappa_1(\eta)\kappa_2(\eta)}\left[\kappa_1(\eta) - \kappa_2(\eta)\right]^2} > 0 \Leftrightarrow \frac{\kappa_1(\eta)}{\kappa_2(\eta)} > \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)}$$

As we know from the proof of Proposition 1, the last inequality is satisfied if and only if

$m^s < \widehat{m}^s$. We conclude that stronger combined network effects η mitigate the negative effect of revenue sharing on competitive balance if and only if $m^s < \widehat{m}^s$. Note that numerical simulations have shown that parts (i) and (ii) of the proposition hold for all parameters $\alpha \in (0, 1]$.

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