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The Effect of Gate Revenue-Sharing on Social Welfare

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Abstract

This paper provides a theoretical model of a team sports league based on contest theory and studies the welfare effect of gate revenue-sharing. It derives two counter-intuitive results. First, it challenges the "invariance proposition" by showing that revenue-sharing reduces competitive balance and thus produces a more unbalanced league. Second, the paper concludes that a lower degree of competitive balance compared with the non-cooperative league equilibrium yields a higher level of social welfare and club profits. Combining both results, we conclude that gate revenue-sharing increases social welfare and club profits in our model.

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1 Introduction

According to the "uncertainty of outcome" hypothesis a certain degree of (competitive) balance is necessary to maintain a successful sporting contest. One of the most common means of improving competitive balance within a professional sports league is gate revenue-sharing. In its simplest form gate revenue-sharing allows the visiting club to retain a share of the home club's gate revenues.

The current revenue-sharing arrangements differ widely among professional leagues all over the world. In 1876 the Major League Baseball (MLB) introduced a 50-50 split of gate receipts that was reduced over time. Since 2003 all clubs in the American League have put 34% of their locally-generated revenue (gate, concession, television, etc.) into a central pool which is then divided equally among clubs. The current revenue-sharing arrangement of the National Football League (NFL) secures the visiting team 40% of the gate receipts (revenues from luxury boxes, parking and concessions are excluded from this sharing arrangement). In the Australian Football League (AFL) gate receipts were split evenly between the home and the visiting team. This 50-50 split was finally abolished in 2000. In Europe there is less gate revenue-sharing. The soccer leagues have adopted various forms of gate revenue-sharing in their history. In England, until the early 1980s up to 20% of the gate receipts were given to the visiting teams in league matches. In the German soccer league (DFL) the home team receives 94% of the gate receipts with the other 6% going to the league. Gate revenue-sharing is quite common in most Cup competitions with a knock-out system. In addition some leagues have adopted other means of increasing competitive balance, such as salary caps, rookie draft systems and luxury taxes.

The effect of gate revenue-sharing on competitive balance has been challenged by the so-called "invariance proposition", which states that revenue-sharing does not affect the distribution of talent between clubs. The invariance proposition has remained highly controversial even up until today and no consensus has emerged so far. Most of the existing controversy on the effect of revenue-sharing on competitive balance stems from the different approaches, the different models and the different methodology used in the literature.

El-Hodiri and Quirk (1971), Fort and Quirk (1995) and Rascher (1997) conclude that revenue-sharing will not affect the distribution of talent between profit-maximizing clubs on the assumption that only the win percentage of the home team affects club revenue.[1] Vrooman (1995) shows that the sharing of winning-elastic revenue does not affect competitive balance whereas the sharing of winning-inelastic revenue improves competitive balance. Atkinson et al. (1988) challenge the invariance proposition by showing that revenue-sharing can improve competitive balance if clubs maximize profits. In their model Atkinson et al. adopt a pool sharing arrangement and a club revenue function that depends on a team's own performance and on the performance of all other teams. Their result is supported by Marburger (1997) and Késenne (2000)), who build their models on the assumption that fans care about the relative and absolute quality of teams, and Rascher (1997) and Késenne (2000), who both consider an objective function which includes the maximization of wins ("utility maximization").

The most counter-intuitive result is presented by Szymanski and Késenne (2004). From a model of two profit-maximizing clubs and a club revenue function that depends on the relative quality of the home team, they show that gate revenue-sharing decreases competitive balance. This result is driven by the so-called "dulling effect." The dulling effect describes the well-known result in sports economics that revenue-sharing reduces the incentive to invest in playing talent. This dulling effect is stronger for the small-market club than for the large-market club. In effect, the difference in talent investments between both clubs increases.

In our opinion, the major drawback of the literature analyzing the effect of revenue-sharing is the implicit assumption that competitive balance is socially desirable. On this assumption the revenue-sharing arrangement that maximizes competitive balance is optimal. We show that this assumption does not hold true in our model. Maximizing competitive balance does not maximize social welfare. In particular, we derive club-specific demand, revenue and profit functions from a general fan utility function and develop a contest model of a sports league with heterogeneous clubs. From the (consumer) utility and (club) profit functions we are able to analyze the welfare effects of alternative gate revenue-sharing arrangements. We arrive at two counter-intuitive results. First, we reproduce the result of Szymanski and Késenne (2004) that gate revenue-sharing decreases competitive balance. Second, we show that a lower degree of competitive balance com-

[1] Note that Fort and Quirk (1995) derive this result on the assumption that only gate revenues are shared. Moreover, they argue that the sharing of locally-generated television revenues can improve competitive balance when teams earn revenue from the gate and from local television contracts.

pared with the non-cooperative league equilibrium actually yields a higher level of social welfare and club profits. Combining the two results, we conclude that gate revenue-sharing increases both social welfare and club profits in our model.

Of course, our paper is not the first to integrate consumer preferences in economic models of sports leagues. To our knowledge Cyrenne (2001) was the first to model explicitly consumer preferences and perform a welfare analysis in a sports league. He develops a quality-of-play model which captures consumer preferences and shows under which conditions the clubs' demand for talented players are strategic complements or substitutes. Falconieri et al. (2004) investigate the conditions under which the collective sale of broadcasting rights is preferred from a social welfare point of view compared with their sale individually by teams. In their model, they derive the demand and the price for a match with a given quality via consumer preferences. To the best of our knowledge, however, we are the first to analyze the welfare effects of gate revenue-sharing on the basis of a general fan utility function.

The remainder of the paper is organized as follows. Section 2 outlines the basic model. In Section 3 we investigate the non-cooperative equilibrium and in Section 4 the social welfare optimum and league optimum. Section 5 presents a comparison between the outcomes. Section 6 concludes the study.

2 Model Specification

We consider a two-club league[2] which awards an exogenously-given league prize P awarded to the winner of the championship.[3] In addition to this performance-related exogenous league prize, each club i generates its own revenues R_i stemming from gate receipts of the match played at the ground of club i against club j. These revenues R_i are assumed to depend on the gate price p_i and club i's fan demand $d(m_i, p_i, q_i)$ for the match between club i and club j. Fan demand in turn depends on the market size m_i (or drawing power) of the club i, the gate price p_i and the quality q_i of the match between club i and club j. Moreover, the gate revenues from the home and away match are distributed among

[2] According to Vrooman (1995) the "strength of the two-team model derives from its simplicity and efficiency in dealing with the questions of talent polarization." See also Szymanski and Késenne (2004) who conduct their analysis in a two-club league. Not all results from a two-club model, however, hold for a n-club model. Therefore caution is necessary if policy implications are derived from a two-club model.

[3] We take this exogenous prize as a proxy for all revenues which are performance-related (e.g. marketing and sponsorship income).

clubs according to a (gate) revenue-sharing arrangement with $\alpha \in [\frac{1}{2}, 1]$ characterizing the share assigned to the home team. Note that a high parameter α represents a league with a low degree of redistribution, i.e. $\alpha = 1$ characterizes a league without revenue-sharing, while $\alpha = 1/2$ characterizes a league with full revenue-sharing.

In order to derive the price and the fan demand for a match with quality q_i we follow the approach in Falconieri et al. (2004): we assume a continuum of fans that differ in their willingness to pay for a match between club i and club j with a given quality q_i. Every fan k has a certain preference for match quality which is measured by θ_k. For simplicity, the fan types θ_k are assumed to be uniformly distributed in $[0, 1]$, i.e. the measure of potential fans amounts to 1. The net-utility of fan k with type θ_k is specified as:

$$\max\{\theta_k q_i - p_i, 0\}$$

By assuming an interior solution, at price p_i the fan type which is indifferent is given by $\theta^* = \frac{p_i}{q_i}$. Hence the measure of fans that purchase at p_i is derived as $1 - \theta^* = \frac{q_i - p_i}{q_i}$. By assuming that each club has a certain market size or drawing potential given by $m_i > 0$, the aggregate demand function for club $i = 1, 2$ is now defined as

$$d(m_i, p_i, q_i) := m_i \frac{q_i - p_i}{q_i} = m_i \left(1 - \frac{p_i}{q_i}\right)$$

Note that match quality q_i has a positive, but decreasing, marginal effect on demand, i.e. $\frac{\partial d}{\partial q_i} > 0$ and $\frac{\partial^2 d}{\partial^2 q_i} < 0$. Moreover, the market size or drawing potential m_i has a positive effect on demand, i.e. $\frac{\partial d}{\partial m_i} > 0$. For a given set of parameters (p_i, q_i), the club with the larger market size or drawing potential generates higher demand. Without loss of generality, we assume throughout this chapter that club 1 is the "large-market club" and club 2 the "small-market club" with $m_1 \geq m_2$. By normalizing the costs of hosting a match to zero, we find that gate revenues are derived as $R_i = p_i d(m_i, p_i, q_i)$. The club maximizes the revenues R_i and thus fixes the price of a match with quality q_i to $p_i^* = \frac{q_i}{2}$. Hence, gate revenues of club $i = 1, 2$ are derived as:

$$R_i = \frac{m_i}{4} q_i$$

In accordance with the literature, we assume that the match quality q_i depends on two factors: the probability of club i's success and the uncertainty of outcome.[4]

We measure the probability of club i's success by the win percentage w_i of this club. The win percentage is characterized by the contest-success function (CSF) and depends on the proportion of playing talent hired by each club. We apply the logit approach, which is the most widely used functional form of a CSF in sporting contests. The win percentage of club i is given by:[5]

$$w_i(t) = \frac{t_i}{t_i + t_j} \quad (i, j = 1, 2,\ i \neq j)$$

Given that the win percentages must sum up to unity, we obtain the adding-up constraint: $w_j = 1 - w_i$. In our model, we allow that the supply of talent may be fixed or elastic. Furthermore, we adopt the usual "Contest-Nash conjecture" $\frac{dt_i}{dt_j} = 0$ and compute the derivative of the win percentages as $\frac{\partial w_i}{\partial t_i} = \frac{t_j}{(t_i+t_j)^2}$.[6] Owing to the adding-up constraint we derive:

$$\frac{\partial w_i}{\partial t_i} = -\frac{\partial w_j}{\partial t_i} \tag{1}$$

The uncertainty of outcome is measured by the competitive balance in the league. Following Hoehn and Szymanski (1999) and Szymanski (2003), we specify competitive balance as $w_i w_j$.

In order to deduce explicitly the gate revenues, we use the following specific formulation of the quality q_i from a match played between club i and club j:[7]

$$q_i(w_i, w_j) := \mu w_i + (1 - \mu) w_i w_j \quad (i, j = 1, 2,\ i \neq j) \tag{2}$$

with $\frac{\partial q_i}{\partial w_i} = 1 - 2(1 - \mu)w_i$. The parameter $\mu \in [0, 1]$ represents the weight in the

[4] For the sake of simplicity, we extrapolate from the possibility that match quality also depends on aggregate talent. This is a restrictive assumption but can be justified by a focus on North America, where all available talent plays in the major leagues.

[5] The logit CSF was generally introduced by Tullock (1980) and subsequently was axiomatized by Skaperdas (1996) and Clark and Riis (1998). An alternative functional form would be the probit CSF (e.g. Lazear and Rosen (1981), Dixit (1987)) and the difference-form CSF (e.g. Hirshleifer (1989)).

[6] According to Szymanski (2004) "it makes no sense to talk of any conjectural variation other than zero." Only the Contest-Nash conjectures are consistent with the concept of Nash equilibrium in a static game. Moreover, note that the assumption of fixed or elastic supply only affects the equilibrium price of talent in our model.

[7] Note that this specification of the quality function differs fundamentally from the quality function used in Falconieri et al. (2004). Moreover, we will see below that the gate revenues which are derived from our specification of the quality function give rise to the revenue functions widely used in the sports economic literature.

quality function between fans' preference for "own team winning" and for competitive balance. When $\mu = 1$ then the match quality only depends on the win percentage of the home team, while when $\mu = 0$ the match quality only depends on the degree of competitive balance. If the relative preference for "own team winning" is equal or bigger than 1/2, then the match quality increases in the win percentage of the home team for all $w_i \in [0, 1]$. Whereas, if the relative preference for "own team winning" is smaller than 1/2, then match quality increases in the win percentage if $w_i < \frac{1}{2(1-\mu)} \leq 1$.

With this specification of the quality function, gate revenues R_i of club $i = 1, 2$ are now given by

$$R_i = \frac{m_i}{4} q_i = \frac{m_i}{4}(w_i - (1-\mu)w_i^2) \tag{3}$$

This club-specific revenue function is consistent with the revenue functions used e.g. in Hoehn and Szymanski (1999), Szymanski (2003) and Szymanski and Késenne (2004). In contrast, however, with the articles quoted, we have derived our revenue function from consumer preferences and thus are able to perform a welfare analysis with respect to gate revenue-sharing.

3 Non-Cooperative Equilibrium

In this section we consider the competitive equilibrium in the league. Both clubs participate in a non-cooperative game and choose independently a level of talent in order to maximize (expected) profits.[8] We assume that talent is measured in perfectly divisible units that can be hired in a competitive market for talent at a wage rate c per unit. Hence, club i's investment costs $C(t_i)$ for talent are given by $C(t_i) = ct_i$. The expected pay-off of club $i = 1, 2$ is determined by the following (expected) profit function:

$$\begin{aligned} E(\Pi_i) &= w_i P + \alpha R_i + (1-\alpha) R_j - C(t_i) \\ &= w_i P + \alpha \left(\frac{m_i}{4}(w_i - (1-\mu)w_i^2) \right) + (1-\alpha) \left(\frac{m_j}{4}(w_j - (1-\mu)w_j^2) \right) - ct_i \end{aligned}$$

[8] The clubs in the US major leagues are commonly considered to be profit maximizers whereas in Europe clubs are usually considered to be win maximizers. The situation in Europe is changing, however, as many examples (Manchester United and Liverpool) demonstrate. For a discussion about the clubs' objective function see e.g. Sloane (1971), Késenne (2000), Fort and Quirk (2004) and Késenne (2006).

with $i, j = 1, 2, i \neq j$. With probability w_i club i wins the championship given club i's and club j's investment level t_i and t_j, respectively, and receives the exogenous league prize P. From the home match club i obtains share α of the gate revenues $R_i = \frac{m_i}{4}(w_i - (1-\mu)w_i^2)$ and from the away match share $(1-\alpha)$ of the gate revenues $R_j = \frac{m_j}{4}(w_j - (1-\mu)w_j^2)$. The investment costs are determined by ct_i.

The corresponding first-order conditions are derived as:

$$
\begin{aligned}
\frac{\partial E(\Pi_1)}{\partial t_1} &= \frac{\partial w_1}{\partial t_1}P + \alpha\frac{\partial R_1}{\partial w_1}\frac{\partial w_1}{\partial t_1} + (1-\alpha)\frac{\partial R_2}{\partial w_2}\frac{\partial w_2}{\partial t_1} - c = 0 \\
\frac{\partial E(\Pi_2)}{\partial t_2} &= \frac{\partial w_2}{\partial t_2}P + \alpha\frac{\partial R_2}{\partial w_2}\frac{\partial w_2}{\partial t_2} + (1-\alpha)\frac{\partial R_1}{\partial w_1}\frac{\partial w_1}{\partial t_2} - c = 0
\end{aligned}
$$

By combining the first-order conditions and using the adding-up constraint (1), we obtain

$$
\left(P + \alpha\frac{\partial R_1}{\partial w_1} - (1-\alpha)\frac{\partial R_2}{\partial w_2}\right)\frac{\partial w_1}{\partial t_1} = \left(P + \alpha\frac{\partial R_2}{\partial w_2} - (1-\alpha)\frac{\partial R_1}{\partial w_1}\right)\frac{\partial w_2}{\partial t_2}
$$

and compute

$$
\frac{\frac{\partial w_2}{\partial t_2}}{\frac{\partial w_1}{\partial t_1}} = \frac{t_1^*}{t_2^*} = \frac{P + \alpha\frac{\partial R_1}{\partial w_1} - (1-\alpha)\frac{\partial R_2}{\partial w_2}}{P + \alpha\frac{\partial R_2}{\partial w_2} - (1-\alpha)\frac{\partial R_1}{\partial w_1}} \tag{4}
$$

We are not able to solve explicitly the equilibrium investments (t_1^*, t_2^*). Instead, we establish the following relationship which must hold in equilibrium between club 1's and club 2's investment level t_1^* and t_2^*, respectively:

$$
t_1^* = \psi(\alpha)t_2^*
$$

In the following lemma, we specify the function $\psi(\alpha)$ and derive some useful properties of it by assuming that the exogenous league prize P is sufficiently high:

Lemma 1 *(i) The function $\psi(\alpha)$ which describes the relationship in the non-cooperative equilibrium between t_1^* and t_2^* is given by:*[9]

$$
\psi(\alpha) := \frac{1}{2\lambda_2}\left(\rho + \sqrt{\rho^2 + 4\lambda_1\lambda_2}\right)
$$

with $\rho := (1 - 2\alpha(1-\mu))(m_1 - m_2)$, $\lambda_1 := 4P + \alpha m_1 - (1-\alpha)(2\mu - 1)m_2$ and $\lambda_2 := 4P + \alpha m_2 - (1-\alpha)(2\mu - 1)m_1$.

[9] Note that ψ is a function of $(\alpha, \mu, m_1, m_2, P)$. For notational clarity we only write $\psi(\alpha)$.

(ii) $\psi(\alpha)$ is an increasing function in revenue-sharing, i.e. a decreasing function in α such that $\frac{\partial\psi(\alpha)}{\partial\alpha} < 0$.

(iii) $\psi(\alpha)$ is equal to or larger than unity, i.e. $\psi(\alpha) = 1 \Leftrightarrow \mu = 0 \vee m_1 = m_2$ and $\psi(\alpha) > 1 \Leftrightarrow \mu > 0 \wedge m_1 > m_2 \ \forall\alpha \in [\frac{1}{2}, 1]$.

Proof. See Appendix A.1. ■

Unless otherwise stated, we assume in the subsequent analysis that fans besides competitive balance also care about their own team winning, i.e. $\mu \in (0, 1)$ and clubs are heterogeneous with respect to their market size, i.e. $m_1 > m_2$.

The win percentages in the non-cooperative equilibrium of club 1 and club 2 can be expressed in terms of $\psi(\alpha)$ by

$$w_1^*(\alpha) = \frac{\psi(\alpha)}{\psi(\alpha) + 1} \text{ and } w_2^*(\alpha) = \frac{1}{\psi(\alpha) + 1}$$

with the derivatives given by

$$\frac{\partial w_1^*(\alpha)}{\partial\alpha} = \frac{\partial\psi(\alpha)}{\partial\alpha}\frac{1}{(\psi(\alpha)+1)^2} \text{ and } \frac{\partial w_2^*(\alpha)}{\partial\alpha} = -\frac{\partial\psi(\alpha)}{\partial\alpha}\frac{1}{(\psi(\alpha)+1)^2}$$

The next proposition summarizes the main results in this section:

Proposition 1 *(i) The investment level of the large-market club 1 is higher than the investment level of the small-market club 2.*

(ii) Equilibrium investments decrease in revenue-sharing, i.e. increase in the parameter α such that $\frac{dt_1^*}{d\alpha} > 0$ and $\frac{dt_2^*}{d\alpha} > 0$.

(iii) The win percentage of club 1 (club 2) increases (decreases) in revenue-sharing, i.e. decreases (increases) in the parameter α such that $\frac{\partial w_1^*(\alpha)}{\partial\alpha} < 0$ and $\frac{\partial w_2^*(\alpha)}{\partial\alpha} > 0$.

(iv) Revenue-sharing reduces competitive balance and produces a more unbalanced league.

Proof. See Appendix A.2. ■

A direct consequence of part (i) is that in the non-cooperative equilibrium, the large-market club 1 is the dominant team yielding a win percentage of more than $\frac{1}{2}$ and the small-market club is the subordinate team yielding a win percentage of less than $\frac{1}{2}$, independent of the revenue-sharing arrangement. The reason for this result is that the marginal impact of an additional win on gate revenues is higher for the large-market club than for the small-market club. Moreover, the difference in win percentages between club 1 and club 2 in the non-cooperative equilibrium is given by $w_1^*(\alpha) - w_2^*(\alpha) = \frac{\psi(\alpha)-1}{\psi(\alpha)+1} > 0$.

Part (ii) reflects the "dulling effect" of revenue-sharing.[10] The dulling effect describes the well-known result in sports economics that revenue-sharing reduces the incentive to invest into playing talent. This result follows from the fact that the marginal benefit of own investment has to be shared with the other club through the revenue-sharing arrangement.

Part (iii) states that a higher degree of redistribution in the league (more revenue-sharing) yields a higher probability of the large-market club and a lower probability of the small-market club to win the championship. This shows that the dulling effect is stronger for the small-market club than for the large-market club. The reason for this result is a form of "free-riding". When gate revenues are shared, the clubs' investment behaviour is such that they take into account the impact of their investment on gate revenues for both their home games and their away games. Owing to the logit formulation of the contest success function, the (positive) marginal impact on the large-market club's gate revenues of a decrease in talent investments by the small-market club is greater than the (positive) marginal impact on the small-market club's gate revenues of a decrease in talent investments by the large-market club, the small-market club will reduce its investment level more than the large-market club.

Part (iv) stating that revenue-sharing decreases competitive balance represents the central result in this section and proves to be counter-intuitive. League authorities established restrictive arrangements such as revenue-sharing in order to improve competitive balance. The basic idea of revenue-sharing was to redistribute revenues form the rich (large-market) clubs to the poor (small-market) clubs because the non-cooperative equilibrium was assumed to produce a level of competitive balance that is too low. The branch of theoretical literature challenging the invariance proposition and stating that revenue-sharing improves competitive balance is in line with this argumentation.[11] Our analysis, however, reveals that revenue-sharing has the opposite effect on competitive balance.[12] The intuition behind this result is the following: Part (ii) and part (iii) of

[10] The notion "dulling effect" was introduced by Szymanski and Késenne (2004).

[11] See e.g. Atkinson et al. (1988), Marburger (1997) and Késenne (2000).

[12] Our result is sustained by Szymanski and Késenne (2004). Moreover, Szymanski (2004) comes to the same result by assuming that the supply of talent is fixed. This shows that revenue sharing can lead to a more unbalanced league in fixed talent supply models and flexible talent supply models.

this proposition have revealed that the dulling effect of revenue-sharing is stronger for the small-market club than for the large-market club. Since the large-market club is the dominant team and the small-market club the subordinate team (see part (i)), a higher degree of revenue-sharing increases the difference between the clubs' win percentages in equilibrium. This produces a more unbalanced league and thus decreases competitive balance.

4 Social Welfare Optimum and League Optimum

Social welfare is given by the sum of aggregate consumer (fan) surplus, aggregate club profit and total player utility.

Aggregate consumer surplus is computed by summing up the consumer surplus from fans of club 1 and club 2. The consumer surplus CS_i from fans of club $i = 1, 2$ in turn corresponds to the integral of the demand function $d(m_i, p_i, q_i)$ from the equilibrium price $p^* = \frac{q}{2}$ to the maximal price $\overline{p}_i = q_i$ which fans are willing to pay for quality q_i:

$$CS_i = \int_{p_i^*}^{\overline{p}_i} d(m_i, p_i, q_i) dp_i = \int_{\frac{q_i}{2}}^{q_i} m_i \frac{q_i - p_i}{q_i} dp_i = \frac{m_i}{8} q_i$$

Aggregate club profit is derived by summing up the profits of club 1 and club 2:

$$\Pi = P + \frac{m_1}{4} q_1 + \frac{m_2}{4} q_2 - c \cdot (t_1 + t_2)$$

Note that the league optimum is characterized by the maximum of aggregate club profit.

If we assume that the players' utility corresponds to their salary, total players' utility is given by the aggregate salary payments $ct_1 + ct_2$ in the league.

Addition of aggregate consumer surplus, aggregate club profit and aggregate salary payments, produces social welfare as

$$\begin{aligned} W &= P + \frac{3}{8}(m_1 q_1 + m_2 q_2) \\ &= P + \frac{3}{8}(m_1(w_1 - (1-\mu)w_1^2) + m_2(w_2 - (1-\mu)w_2^2)) \end{aligned} \tag{5}$$

The players' utility does not influence social welfare since the only costs faced by the clubs are player salaries. That is, player salaries are only a transfer from clubs to players.

Moreover, social welfare is independent of the revenue-sharing parameter α since the aggregate club profit is independent of α. Hence, social welfare only depends on the market size m_i, the match quality q_i and the exogenous league prize P.

In the following proposition we maximize aggregate club profit and social welfare and derive the corresponding optimal win percentages:

Proposition 2 *The welfare optimal and league optimal win percentages coincide and are given by*

$$w_1^\kappa = \frac{m_1 + m_2(1-2\mu)}{2(m_1+m_2)(1-\mu)} > \frac{1}{2} \text{ and } w_2^\kappa = \frac{m_1(1-2\mu) + m_2}{2(m_1+m_2)(1-\mu)} < \frac{1}{2} \; (\kappa \in \{LO, WO\}) \quad (6)$$

Proof. See Appendix A.3. ■

The proposition shows that the relative performances in the welfare optimum and the league optimum coincide in our model. The absolute level of talent investment need not, however, coincide. A league planner who wants to maximize joint club profits will choose the minimal necessary investment level consistent with (6).[13] This investment level chosen by the league planner will necessarily maximize social welfare since only the relative level of talent investment between both clubs is crucial for social welfare. The reverse does not hold true. Not every welfare optimal investment level maximizes joint club profits. In other words, a continuum of investment levels consistent with (6) maximizes social welfare whereas there is a unique investment level consistent with (6) which maximizes aggregate club profit.

Similar to the non-cooperative equilibrium, we further conclude that in the welfare optimum and the league optimum the large-market club is the dominant team and the small-market club the subordinate team with $w_1^\kappa > \frac{1}{2}$ and $w_2^\kappa < \frac{1}{2}$. The corresponding difference between the win percentages in the welfare/league optimum is given by $w_1^\kappa - w_2^\kappa = \frac{(m_1-m_2)\mu}{(m_1+m_2)(1-\mu)} > 0$ with $\kappa \in \{LO, WO\}$. The difference increases in the preference parameter μ of the quality function. In other words, if fans care more for their own team winning, then the welfare/league optimal degree of competitive balance decreases. Furthermore, the welfare/league optimal win percentage of club i increases in its own market size m_i and decreases in the market size m_j of the other club j.[14] A bigger

[13] One can think of this minimal necessary investment level as the minimal amount which has to be invested in order to maintain the league's operation.

[14] Since $\frac{\partial w_i^\kappa}{\partial m_i} = \frac{m_j \mu}{(m_i+m_j)^2(1-\mu)} > 0$ and $\frac{\partial w_i^\kappa}{\partial m_j} = -\frac{m_i \mu}{(m_i+m_j)^2(1-\mu)} < 0$ for $\mu \in (0,1)$ and $\kappa \in \{LO, WO\}$.

market size of the large (small) market club causes $w_1^\kappa - w_2^\kappa$ to increase (decrease) and thus a more unbalanced (balanced) league becomes desirable from the perspective of a league planner and from a social welfare point of view.

5 Comparison of the outcomes

So far, our analysis has shown that the dulling effect of revenue-sharing in the non-cooperative case is stronger for the small-market club than for the large-market club. As a consequence, increased revenue-sharing reduces competitive balance and produces a more unbalanced league. But how does revenue-sharing influence social welfare and aggregate club profit?

By comparing the non-cooperative equilibrium with the welfare optimum and the league optimum, we derive the following results:

Proposition 3 *(i) The league is more unbalanced in the welfare/league optimum compared with the non-cooperative equilibrium.*

(ii) Revenue-sharing increases social welfare and aggregate club profit.

Proof. See Appendix A.4 ■

According to part (i), the difference between the clubs' win percentages is larger in the welfare/league optimum than in the non-cooperative equilibrium. Thus, from the perspective of a league planner and from a social welfare point of view, the degree of competitive balance in the non-cooperative equilibrium is too high. A more imbalanced league is desirable. In the non-cooperative equilibrium, the small-market club wins too often and the large-market club does not win often enough. Formally, $w_1^\kappa > w_1^*(\alpha) > \frac{1}{2}$ and $w_2^\kappa < w_2^*(\alpha) < \frac{1}{2}$ $\forall \alpha \in [\frac{1}{2}, 1]$ and $\kappa \in \{LO, WO\}$. This is a surprising result since it is usually argued that if playing talent can be freely traded in the market the outcome will be such that the large-market club obtains too much talent and the small-market club too little talent. The proposition shows, however, that the distribution of playing talent in the non-cooperative equilibrium is still too balanced. As a consequence, measures that decrease, not increase competitive balance will increase social welfare and aggregate club profit in our league. In this respect, gate revenue-sharing proves to be an appropriate measure of decreasing competitive balance and increasing social welfare and club profit.

What is the intuition behind this result? Each club imposes a negative externality through own talent investments on the other club's expected revenue. Because of the asymmetric market size, the small-market club imposes a larger externality on the large-market club than vice versa.[15] None of the clubs, however, internalizes this negative externality. As a consequence, in the non-cooperative equilibrium, the marginal revenue of talent is equalized between the two clubs, but not the marginal revenue of a win. More precisely, the marginal revenue of a win is larger for the large-market club than for the small-market club. As a consequence, a decrease in the win percentage of the small-market club and an increase in the win percentage of the large-market club in the non-cooperative equilibrium results in higher social welfare and larger club profits. The maximum degree of competitive "imbalance" and therefore the highest levels of social welfare and club profit are obtained in a league with full revenue-sharing ($\alpha = \frac{1}{2}$).

Moreover, the consumers/fans also benefit from a higher degree of revenue-sharing in the league since the aggregate consumer surplus is also maximized for the welfare optimal win percentages (w_1^{WO}, w_2^{WO}).[16] Hence, analogous to social welfare, revenue-sharing increases the aggregate consumer surplus and thus benefits consumers.

In the following corollary, we determine under which conditions the social optimum and the non-cooperative equilibrium coincide:

Corollary 1 *Social welfare is maximized in the non-cooperative equilibrium and the league is perfectly balanced with* $w_i^{WO} = w_i^*(\alpha) = \frac{1}{2}\ \forall \alpha \in [\frac{1}{2}, 1]$ $(i = 1, 2)$ *if at least one of the following conditions is satisfied:*

(i) Clubs are homogeneous with respect to their market size $(m_1 = m_2 = m)$.

(ii) Fans only care for competitive balance $(\mu = 0)$.

Proof. See Appendix A.5. ■

In a league of homogeneous clubs (case (i)), both clubs invest the same amount in the non-cooperative equilibrium obtaining a perfectly balanced league. In this case, the symmetric investment level in the non-cooperative equilibrium is given by $t_1^*(\alpha) = t_2^*(\alpha) =$

[15] This is because of the fact that the increase in revenue for a given increase in win percentage is higher for the large-market club than for the small-market club.

[16] It is straightforward to prove this claim. Compare $CS_1 + CS_2 = \frac{1}{8}(m_1 q_1 + m_2 q_2)$ with social welfare $W = P + \frac{3}{8}(m_1 q_1 + m_2 q_2)$ and note that the exogenous league prize P does not influence the maximization problem.

$\frac{1}{4c}(P + \frac{m(2\alpha-1)\mu}{4})$.[17] Social welfare is derived as $W = P + \frac{3}{8}m(q_1 + q_2)$ and is maximized for each symmetric investment level in a perfectly balanced league. In a league in which fans only care for competitive balance (case (ii)), the symmetric investment level in the non-cooperative equilibrium is given by $t_1^*(\alpha) = t_2^*(\alpha) = \frac{P}{4c}$. In this case, the match quality is equal for both clubs (i.e. $q_1 = q_2 = q$) and is maximized for each symmetric investment level. Since social welfare $W = P + \frac{3}{8}q(m_1 + m_2)$ is proportional to the total quality, W is maximized in a perfectly balanced league.[18]

6 Conclusion

In this paper we develop a theoretical model of a team sports league based on contest theory in order to study the welfare effect of alternative gate revenue-sharing arrangements. We derive club-specific demand and revenue from a general fan utility function by assuming that a fan's willingness to pay depends on the fan-type, win-percentage of the home team and competitive balance. Using this approach, we are able to extend the literature by providing an integrated framework which analyzes the effect of gate revenue on social welfare. The existing literature is focused on the effect of revenue-sharing on competitive balance and implicitly assumes that competitive balance is socially desirable without explaining the underlying assumptions regarding consumer preferences.

Our analysis challenges the "invariance proposition" by showing that gate revenue-sharing decreases competitive balance and produces a more unbalanced league. This result is driven by the dulling effect of revenue-sharing. The dulling effect is revealed to be stronger for the small-market club than for the large-market club. Moreover, we show that a lower degree of competitive balance than in the non-cooperative league equilibrium yields a higher level of social welfare and aggregate club profit. Combining both results, we conclude that in order to increase social welfare and club profits, arrangements which decrease, not increase, competitive balance should be implemented. In this respect, gate revenue-sharing proves to be an appropriate measure of decreasing competitive balance and increasing social welfare.

[17] Note that the assumption of fixed or elastic supply affects the equilibrium price of talent. Moreover, note that in a league of full revenue sharing ($\alpha = 1/2$), equilibrium investments are independent of the club's drawing potential m and the preference parameter μ.

[18] Note that the league optimal win percentages in case (i) and (ii) are also given by $w_1^{LO} = w_2^{LO} = \frac{1}{2}$. As already mentioned, however, the investment level in the league optimum is given by an infinitesimally small amount consistent with $w_1^{LO} = w_2^{LO} = \frac{1}{2}$ and therefore does not coincide with the welfare optimum.

A Appendix: Proofs

A.1 Proof of Lemma 1

ad (i) Equation (4) is given by $\frac{t_1}{t_2} = \frac{P+\alpha\frac{\partial R_1}{\partial w_1}-(1-\alpha)\frac{\partial R_2}{\partial w_2}}{P+\alpha\frac{\partial R_2}{\partial w_2}-(1-\alpha)\frac{\partial R_1}{\partial w_1}}$, we compute

$$\frac{t_1}{t_2} = \frac{4P(t_1+t_2)+\alpha m_1(t_1(2\mu-1)+t_2)-(1-\alpha)(t_1+t_2(2\mu-1))}{4P(t_1+t_2)+\alpha m_2(t_1+t_2(2\mu-1))-(1-\alpha)(t_1(2\mu-1)+t_2)} \tag{7}$$

By arranging (7) such that $t_1 = \psi(\alpha)t_2$, we formally obtain two solutions for the function $\psi(\alpha)$ which characterizes the relationship between t_1 and t_2 in the non-cooperative equilibrium:

$$\psi_1(\alpha) = \frac{1}{2\lambda_2}\left(\rho + \sqrt{\rho^2+4\lambda_1\lambda_2}\right) \text{ and } \psi_2(\alpha) = \frac{1}{2\lambda_2}\left(\rho - \sqrt{\rho^2+4\lambda_1\lambda_2}\right)$$

with $\rho := (1-2\alpha(1-\mu))(m_1-m_2), \lambda_1 := 4P+\alpha m_1-(1-\alpha)(2\mu-1)m_2$ and $\lambda_2 := 4P+\alpha m_2-(1-\alpha)(2\mu-1)m_1$. However, the negative solution $\psi_2(\alpha)$ can be ruled out because in case of a sufficiently high exogenous league prize P it yields negative equilibrium payoffs and therefore does not ensure the existence of a Nash-equilibrium in pure strategies.[19]

We will show that $\psi_2(\alpha)$ always yields a negative solution:

To prove this claim we assume that the exogenous league prize P is sufficiently high with $P > P^* := \frac{(1-\alpha)(2\mu-1)m_2-\alpha m_1}{4}$. In this case we obtain $\lambda_1 > 0$ and $\lambda_2 > 0$ for all $\mu \in [0,1]$. (a) Suppose $\mu \in [0, \frac{1}{2}]$. Let $\alpha \leq \frac{1}{2(1-\mu)}$ then $\rho > 0$ and we derive $\psi_1(\alpha) > 0$ and $\psi_2(\alpha) < 0$. Let $\alpha > \frac{1}{2(1-\mu)}$ then $\rho < 0$ and we derive $\psi_1(\alpha) > 0$ and $\psi_2(\alpha) < 0$. (b) Suppose $\mu \in (1/2, 1]$ then $\rho \geq 0$ $\forall \alpha \in [1/2, 1]$. Also in this case we derive $\psi_1(\alpha) > 0$ and $\psi_2(\alpha) < 0$.

Hence, only the positive solution $\psi_1(\alpha)$ yields non-negative equilibrium payoffs and thus ensures the existence of a Nash-equilibrium in pure strategies.[20]

ad (ii) We claim that $\psi(\alpha)$ is an increasing function in revenue-sharing, i.e. $\frac{\partial\psi(\alpha)}{\partial\alpha} < 0$

[19] The existence of Nash-equilibria in the Tullock contest is discussed in the rent-seeking literature e.g. in Lockard and Tullock (2001). In our case we can show that if the negative solution $\psi_2(\alpha)$ is not ruled out, the FOCs and SOCs fail to characterize the global maximum. Nevertheless, there exists a symmetric mixed-strategy equilibrium, since the conditions for the corresponding existence theorem in Dasgupta and Maskin (1986) are satisfied. The case of mixed-strategies in a discrete choice set is analysed for a rent-seeking setting e.g. by Baye et al. (1994).

[20] Note that in the subsequent analysis we write $\psi(\alpha)$ instead of $\psi_1(\alpha)$.

$\forall\alpha \in [\frac{1}{2}, 1]$. It suffices to show that: $s(\alpha) := \frac{\partial\psi(\alpha)}{\partial\alpha} < 0\ \forall\alpha \in [\frac{1}{2}, 1]$.

(a1) $s(\alpha)$ is a continuous function for all $\alpha \in \mathbb{R}$

(b1) There exists only one α where $s(\alpha) = 0$: $s(\alpha) = 0 \Leftrightarrow \alpha^* = \frac{m_1(2\mu-1)-4P}{m_1(2\mu-1)+m_2}$. We derive that α^* is smaller than $\frac{1}{2}$ for all $P > 0$ if $\mu \in [0, \frac{1}{2}]$ and for all $P > P^{**} := \frac{m_1(2\mu-1)-m_2}{4}$ if $\mu \in (\frac{1}{2}, 1]$.

(c1) Evaluation of the function $s(\alpha)$ for $\alpha > \alpha^*$ yields $s(\alpha) < 0$. For example, evaluation of $s(\alpha)$ for $\alpha = \frac{1}{2}$ yields $s(\frac{1}{2}) = -\frac{8\mu^2(m_1-m_2)(m_1(2P+m_2(1-\mu))+2m_2P)}{8P+m_1(1-\mu)+m_2(1-\mu)} < 0$ for $m_1 > m_2$ and $\mu > 0$.

From (a1),(b1) and (c1), we conclude that the continuous function $s(\alpha)$ is always smaller than zero on the compact interval $\alpha \in [\frac{1}{2}, 1]$ and thus $\psi(\alpha)$ is an increasing function in revenue-sharing, i.e. $\frac{\partial\psi(\alpha)}{\partial\alpha} < 0\ \forall\alpha \in [\frac{1}{2}, 1]$. This proves the claim.

ad (iii) We claim that $\psi(\alpha) = 1 \Leftrightarrow \mu = 0 \vee m_1 = m_2$ and $\psi(\alpha) > 1 \Leftrightarrow \mu > 0 \wedge m_1 > m_2$ $\forall\alpha \in [\frac{1}{2}, 1]$.

It is straightforward to show that $\psi(\alpha) = 1 \Leftrightarrow \mu = 0 \vee m_1 = m_2$ which proves the first part of the claim.

In the next step, we set $\mu > 0 \wedge m_1 > m_2$ and prove that $\psi(\alpha) > 1\ \forall\alpha \in [\frac{1}{2}, 1]$. It suffices to show that $r(\alpha) := \psi(\alpha) - 1 > 0\ \forall\alpha \in [\frac{1}{2}, 1]$.

(a2) $r(\alpha)$ is a continuous function for all $\alpha \in \mathbb{R}$.

(b2) There exists only one $\alpha \in \mathbb{R}$ where $r(\alpha) = 0$: $r(\alpha) = 0 \Leftrightarrow \alpha^* = \frac{m_1(2\mu-1)-4P}{m_1(2\mu-1)+m_2}$. Analogous to above, we derive that α^* is smaller than $\frac{1}{2}$ for all $P > 0$ if $\mu \in [0, \frac{1}{2}]$ and for all $P > P^{**}$ if $\mu \in (\frac{1}{2}, 1]$.

(c2) Evaluation of the function $r(\alpha)$ for $\alpha > \alpha^*$ yields $r(\alpha) > 0$. For example, evaluation of $r(\alpha)$ for $\alpha = \frac{1}{2}$ yields $r(\frac{1}{2}) = 2\mu(m_1 - m_2) > 0$ for $m_1 > m_2$ and $\mu > 0$.

From (a2), (b2) and (c2) we derive that the continuous function $r(\alpha)$ is always larger than zero on the compact interval $\alpha \in [\frac{1}{2}, 1]$ and thus $\psi(\alpha)$ is always larger than unity on the same interval, i.e. $\psi(\alpha) > 1\ \forall\alpha \in [\frac{1}{2}, 1]$. This proves the claim.

A.2 Proof of Proposition 1

We assume that fans care besides competitive balance also for own team winning ($\mu > 0$) and clubs are heterogeneous with respect to their market size ($m_1 > m_2$).

ad (i) We claim that in the non-cooperative equilibrium the investment level of the large-market club 1 is higher than the investment level of the small-market club 2, i.e.

$t_1^*(\alpha) > t_2^*(\alpha)\ \forall\alpha \in [\frac{1}{2}, 1]$. It suffices to show that

$$t_1^*(\alpha) > t_2^*(\alpha) \Leftrightarrow w_1^*(\alpha) > w_2^*(\alpha)\ \forall\alpha \in [\frac{1}{2}, 1]$$

The win percentages in equilibrium are given by $w_1^*(\alpha) = \frac{\psi(\alpha)}{\psi(\alpha)+1}$ and $w_2^*(\alpha) = \frac{1}{\psi(\alpha)+1}$. Hence, $\frac{w_1^*(\alpha)}{w_2^*(\alpha)} = \psi(\alpha)$ and according to lemma 1, we know that $\psi(\alpha) > 1\ \forall\alpha \in [\frac{1}{2}, 1]$. We conclude $w_1^*(\alpha) > w_2^*(\alpha)$ and thus obtain $t_1^*(\alpha) > t_2^*(\alpha)$ which proves the claim.

ad (iii) We claim that the win percentage of the large (small) market club 1 (club 2) is an increasing (decreasing) function in revenue-sharing, i.e. $\frac{\partial w_1^*(\alpha)}{\partial\alpha} < 0$ and $\frac{\partial w_2^*(\alpha)}{\partial\alpha} > 0$ $\forall\alpha \in [\frac{1}{2}, 1]$. From lemma 1 we know that $\frac{\partial\psi(\alpha)}{\partial\alpha} < 0\ \forall\alpha \in [\frac{1}{2}, 1]$ and derive

$$\frac{\partial w_1^*(\alpha)}{\partial\alpha} = \frac{\partial\psi(\alpha)}{\partial\alpha}\frac{1}{(\psi(\alpha)+1)^2} < 0 \text{ and } \frac{\partial w_2^*(\alpha)}{\partial\alpha} = -\frac{\partial\psi(\alpha)}{\partial\alpha}\frac{1}{(\psi(\alpha)+1)^2} > 0.$$

This proves the claim.

ad (iv) We claim that revenue-sharing reduces competitive balance and produces a more unbalanced league. In other words, a lower parameter α of the revenue-sharing arrangement increases the difference $w_1^*(\alpha) - w_2^*(\alpha) = \frac{\psi(\alpha)-1}{\psi(\alpha)+1}$ between the win percentages of club 1 and club 2. To prove the claim it suffices to show that $\frac{\partial(w_1^*(\alpha)-w_2^*(\alpha))}{\partial\alpha} < 0$. Since club 1 is the dominant team and club 2 is the subordinate team, we obtain $w_1^*(\alpha) - w_2^*(\alpha) > 0$ and compute

$$\frac{\partial(w_1^*(\alpha) - w_2^*(\alpha))}{\partial\alpha} = \frac{\partial\psi(\alpha)}{\partial\alpha}\frac{2}{(\psi(\alpha)+1)^2}$$

From lemma 1, we know that $\frac{\partial\psi(\alpha)}{\partial\alpha} < 0$ and thus $\frac{\partial(w_1^*(\alpha)-w_2^*(\alpha))}{\partial\alpha} < 0$. Hence, decreasing the parameter α of the revenue-sharing arrangement, i.e. more revenue-sharing in the league, increases the difference between the win percentages of club 1 and club 2 and produces a more unbalanced league. This proves the claim.

ad (ii) We claim that equilibrium investments decrease in revenue-sharing, i.e. $\frac{\partial t_i^*}{\partial\alpha} > 0$. To prove this claim, we follow Szymanski and Késenne (2004) and derive the total differential of the first-order conditions $\frac{\partial E(\Pi_1)}{\partial t_1} = 0$ and $\frac{\partial E(\Pi_2)}{\partial t_2} = 0$ which yields:

$$\begin{aligned}
\frac{\partial^2 E(\Pi_1)}{\partial t_1^2}dt_1 + \frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial t_2}dt_2 + \frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial\alpha} &= 0 \\
\frac{\partial^2 E(\Pi_2)}{\partial t_2 \partial t_1}dt_1 + \frac{\partial^2 E(\Pi_2)}{\partial t_2^2}dt_2 + \frac{\partial^2 E(\Pi_2)}{\partial t_2 \partial\alpha} &= 0
\end{aligned}$$

This system of equations can also be written as

$$\begin{bmatrix} \frac{\partial^2 E(\Pi_1)}{\partial t_1^2} & \frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial t_2} \\ \frac{\partial^2 E(\Pi_2)}{\partial t_2 \partial t_1} & \frac{\partial^2 E(\Pi_2)}{\partial t_2^2} \end{bmatrix} \begin{bmatrix} dt_1 \\ dt_2 \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial \alpha} \\ -\frac{\partial^2 E(\Pi_2)}{\partial t_2 \partial \alpha} \end{bmatrix} d\alpha$$

Applying Cramer's Rule we derive

$$\frac{dt_1}{d\alpha} = \frac{\frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial t_2}\frac{\partial^2 E(\Pi_2)}{\partial t_2 \partial \alpha} - \frac{\partial^2 E(\Pi_2)}{\partial t_2^2}\frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial \alpha}}{\frac{\partial^2 E(\Pi_1)}{\partial t_1^2}\frac{\partial^2 E(\Pi_2)}{\partial t_2^2} - \frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial t_2}\frac{\partial^2 E(\Pi_2)}{\partial t_2 \partial t_1}} \tag{8}$$

According to the stability condition in Dixit (1986) the denominator of equation (8) is assumed to be positive. A sufficient condition for stability is therefore

$$\frac{\partial^2 E(\Pi_1)}{\partial t_1^2}\frac{\partial^2 E(\Pi_2)}{\partial t_2^2} > \frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial t_2}\frac{\partial^2 E(\Pi_2)}{\partial t_2 \partial t_1}$$

since the second order conditions $\frac{\partial^2 E(\Pi_1)}{\partial t_1^2}$ and $\frac{\partial^2 E(\Pi_2)}{\partial t_2^2}$ are negative, given the assumptions about the revenue function. Moreover, we compute

$$\begin{aligned} \frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial \alpha} &= \left(\frac{\partial R_1}{\partial w_1} + \frac{\partial R_2}{\partial w_2}\right)\frac{\partial w_1}{\partial t_1} > 0 \\ \frac{\partial^2 E(\Pi_2)}{\partial t_2 \partial \alpha} &= \left(\frac{\partial R_2}{\partial w_2} + \frac{\partial R_1}{\partial w_1}\right)\frac{\partial w_2}{\partial t_2} > 0 \end{aligned}$$

for all $w_i \in [0,1]$ if $\mu \in (\frac{1}{2}, 1]$ and for all $w_i < \frac{1}{2(1-\mu)} < 1$ if $\mu \in [0, \frac{1}{2}]$.

The expression $\frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial t_2}$ characterizes the slope of club 1's reaction function. Since club 1 is the large-market club its reaction function slopes upward and therefore we obtain $\frac{\partial^2 E(\Pi_1)}{\partial t_1 \partial t_2} > 0$. Hence, also the numerator of (8) is positive and we derive that $\frac{dt_1^*}{d\alpha} > 0$.

From part (iv) of this proposition we know that revenue-sharing reduces competitive balance. Now, if revenue-sharing induces club 1 to reduce its investment level then it must also be the case for club 2, i.e. $\frac{dt_2^*}{d\alpha} > 0$. This proves the claim.

A.3 Proof of Proposition 2

We assume that fans care besides competitive balance also for own team winning ($\mu > 0$) and clubs are heterogeneous with respect to their market size ($m_1 > m_2$). We first derive the social welfare optimum and then the league optimum.

A.3.1 Social welfare optimum

Social welfare W is given by

$$W = P + \frac{3}{8}(m_1 q_1 + m_2 q_2) = P + \frac{3}{8}(m_1(w_1 - (1-\mu)w_1^2) + m_2(w_2 - (1-\mu)w_2^2))$$

The corresponding first-order conditions are computed as:[21]

$$\begin{aligned}
\frac{\partial W}{\partial t_1} &= \frac{3t_2(m_1(t_2 - t_1(1-2\mu)) + m_2(t_2(1-2\mu) - t_1))}{8(t_1+t_2)^3} = 0 \\
\frac{\partial W}{\partial t_2} &= \frac{3t_1(m_1(t_1(1-2\mu) - t_2) + m_2(t_1 - t_2(1-2\mu))}{8(t_1+t_2)^3} = 0
\end{aligned}$$

Since we are not able to explicitly solve for the welfare optimal investment levels (t_1^{WO}, t_2^{WO}), we establish similar to lemma 1 the following relationship which must hold in the welfare optimum:

$$t_1^{WO} = \psi^{WO} t_2^{WO} \text{ with } \psi^{WO} = \frac{m_1 + m_2(1-2\mu)}{m_1(1-2\mu) + m_2}$$

By assuming that $\mu < \frac{m_1+m_2}{2m_1}$ we guarantee an interior solution. In this case, the corresponding win percentages $w_i^{WO} = \frac{t_i^{WO}}{t_i^{WO}+t_j^{WO}}$ in the welfare optimum are given by:

$$w_1^{WO} = \frac{m_1 + m_2(1-2\mu)}{2(m_1+m_2)(1-\mu)} \text{ and } w_2^{WO} = \frac{m_1(1-2\mu) + m_2}{2(m_1+m_2)(1-\mu)}$$

It is straightforward to show that $w_1^{WO} > \frac{1}{2}$ and $w_2^{WO} < \frac{1}{2}$ for all $m_1 > m_2$ and $\mu > 0$. This proves the claim.

Moreover, we claim that each investment level (t_1, t_2) which satisfies $t_1^{WO} = \psi^{WO} t_2^{WO}$ maximizes social welfare: We define $(t_1^{(k)}, t_2^{(k)})$ as a sequence which is consistent with $t_1^{WO} = \psi^{WO} t_2^{WO}$. For example, define $t_2^{(k)} := \frac{1}{k}$ and $t_1^{(k)} := \frac{m_1+m_2(1-2\mu)}{m_1(1-2\mu)+m_2}\frac{1}{k}$. We derive that $\frac{\partial W(t_1^{(k)}, t_2^{(k)})}{\partial t_1} = 0$ and $\frac{\partial W(t_1^{(k)}, t_2^{(k)})}{\partial t_2} = 0$ for all $k \in \mathbb{N}$. Moreover, $w_1(t_1^{(k)}, t_2^{(k)}) = \frac{m_1+m_2(1-2\mu)}{2(m_1+m_2)(1-\mu)}$ and $w_2(t_1^{(k)}, t_2^{(k)}) = \frac{m_1(1-2\mu)+m_2}{2(m_1+m_2)(1-\mu)}$. Hence, $(t_1^{(k)}, t_2^{(k)})$ maximizes social welfare for all $k \in \mathbb{N}$ and the claim is proved.

[21] The second-order conditions for a maximum are satisfied.

A.3.2 League Optimum

In order to maximize aggregate club profit Π, a league planner has to solve the following maximization problem:[22]

$$\max_{(t_1,t_2)}\{P+\frac{m_1}{4}q_1+\frac{m_2}{4}q_2-c\cdot(t_1+t_2)\}$$

Analogous to A.3.1, we derive the following relationship which must hold in the league optimum:

$$t_1^{LO}=\psi^{LO}t_2^{LO} \text{ with } \psi^{LO}=\frac{m_1+m_2(1-2\mu)}{m_1(1-2\mu)+m_2} \tag{9}$$

Hence, the league optimal win percentages are given by

$$w_1^{LO}=\frac{m_1+m_2(1-2\mu)}{2(m_1+m_2)(1-\mu)}>\frac{1}{2} \text{ and } w_2^{LO}=\frac{m_1(1-2\mu)+m_2}{2(m_1+m_2)(1-\mu)}<\frac{1}{2} \tag{10}$$

and coincide with the welfare optimal win percentages. However, in contrast to the welfare optimum, not every investment level (t_1,t_2) which satisfies $t_1^{LO}=\psi^{LO}t_2^{LO}$ maximizes the aggregate club profit. This is due to the fact that aggregate costs $c\cdot(t_1+t_2)$ are now included in aggregate profits. As a consequence, an infinitesimal small amount consistent with $t_1^{LO}=\psi^{LO}t_2^{LO}$ (such that (10) is satisfied) maximizes aggregate club profit. To see this, consider a monotone decreasing sequence $(t_1^{(k)},t_2^{(k)})$ with limit 0 such that $t_1^{(k)}=\psi^{LO}t_2^{(k)}$ for all $k\in\mathbb{N}$. Hence, $(t_1^{(k)},t_2^{(k)})$ satisfies (10) and thus maximizes aggregate gate revenues $\frac{m_1}{4}q_1+\frac{m_2}{4}q_2$ for all $k\in\mathbb{N}$. Moreover, aggregate club profit Π can be increased by decreasing the investment level, i.e. $\Pi(t_1^{(k+1)},t_2^{(k+1)})>\Pi(t_1^{(k)},t_2^{(k)})$. Without restrictions on the minimal amount of talent which has to be invested, the league planner would spend in the league optimum an infinitesimal small amount still consistent with (9). However, in a league in which a minimal amount $T>0$ of talent investment is necessary in order to maintain the league's operation, a league planner who wants to maximize aggregate club profit will exactly invest this minimal amount such that $t_1^{LO}=\psi^{LO}t_2^{LO}$ and $T=t_1^{LO}+t_2^{LO}$.

[22] We assume that the league planner has no influence on the equilibrium price $p_i^*=\frac{q_i}{2}$ and hence acts as a price taker.

A.4 Proof of Proposition 3

We assume that fans care besides competitive balance also for own team winning ($\mu > 0$) and clubs are heterogeneous with respect to their market size ($m_1 > m_2$).

ad (i) We claim that our two-club league is more unbalanced in the welfare optimum and the league optimum compared to the non-cooperative equilibrium independent of the revenue-sharing parameter α, i.e. $|w_1^\kappa - w_2^\kappa| > |w_1^*(\alpha) - w_2^*(\alpha)|$ $\forall \alpha \in [\frac{1}{2}, 1]$ and $\kappa \in \{WO, LO\}$.

We define $g(\alpha) := w_1^\kappa - w_1^*(\alpha)$ and derive the following properties of $g(\alpha)$:

(a) $g(\alpha)$ is a continuous function for all $\alpha \in \mathbb{R}$.

(b) There exists only one $\alpha \in \mathbb{R}$ where $g(\alpha) = 0$:

$$g(\alpha) = 0 \Leftrightarrow \alpha^{**} = \frac{1}{2} - \frac{(m_1 + m_2)P}{m_1 m_2 \mu} < \frac{1}{2}$$

(c) Evaluation of the function $g(\alpha)$ for $\alpha > \alpha^{**}$ yields that $g(\alpha) > 0$. For example, evaluation of $g(\alpha)$ for $\alpha = \frac{1}{2}$ and $\kappa \in \{WO, LO\}$ yields

$$g(\frac{1}{2}) = w_1^\kappa - w_1^*(\frac{1}{2}) = \frac{4\mu P(m_1 - m_2)}{(1-\mu)(m_1+m_2)(8P + (1-\mu)(m_1+m_2))} > 0$$

From (a),(b) and (c) we derive that the continuous function $g(\alpha)$ is always larger than zero on the compact interval $\alpha \in [\frac{1}{2}, 1]$ and thus $w_1^\kappa > w_1^*(\alpha)$ $\forall \alpha \in [\frac{1}{2}, 1]$ and $\kappa \in \{WO, LO\}$. Moreover, we know that the large-market club 1 is the dominant team, i.e. $w_1^*(\alpha) > \frac{1}{2}$ $\forall \alpha \in [\frac{1}{2}, 1]$. By using the adding-up constraint: $w_j = 1 - w_i$ we conclude that $1 - w_1^\kappa = w_2^\kappa < w_2^*(\alpha) = 1 - w_1^*(\alpha) < \frac{1}{2}$ $\forall \alpha \in [\frac{1}{2}, 1]$ and $\kappa \in \{WO, LO\}$. Hence, the following inequality holds true:

$$w_1^\kappa - w_2^\kappa > w_1^*(\alpha) - w_2^*(\alpha) > 0 \ \forall \alpha \in [\frac{1}{2}, 1] \text{ and } \kappa \in \{WO, LO\}.$$

This proves the claim.

ad (ii) Part (i) of this proposition has shown that the league is more unbalanced in the welfare and league optimum than in the non-cooperative equilibrium. This implies $w_1^\kappa > w_1^*(\alpha) > \frac{1}{2}$ and $w_2^\kappa < w_2^*(\alpha) < \frac{1}{2}$ $\forall \alpha \in [\frac{1}{2}, 1]$ and $\kappa \in \{WO, LO\}$. A more imbalanced league is socially desirable and also desirable from the league planner's point of view. Moreover, according to proposition 1, the win percentage of the large (small) market club 1 (club 2) is an increasing (decreasing) function in revenue-sharing. Thus, by decreasing the parameter α (more gate revenue-sharing), the win percentage of the large-market club 1 increases and the win percentage of the small-market club 2 decreases. This causes the degree of competitive balance to decrease which in turn increases social welfare and aggregate club profit (due to the fact that the welfare optimal and league optimal win percentages are approached). Social welfare and aggregate club profit increase until the maximal level of revenue-sharing is reached in a league with full revenue-sharing ($\alpha = \frac{1}{2}$)

A.5 Proof of Corollary 1

We claim that social welfare is maximized in the non-cooperative equilibrium and the league is perfectly balanced iff (i) clubs are homogeneous with respect to their market size ($m_1 = m_2$) or (ii) the fan's preference is such that they only care for competitive balance ($\mu = 0$).

If $m_1 = m_2$ or $\mu = 0$ we derive that

(i) in the non-cooperative equilibrium holds $\psi(\alpha) = 1$ and thus the corresponding win percentages are given by $w_1^*(\alpha) = w_2^*(\alpha) = \frac{1}{2}\ \forall \alpha \in [\frac{1}{2}, 1]$.

(ii) in the social optimum the win percentages are given by $w_1^{WO} = w_2^{WO} = \frac{1}{2}$ according to (6).

Comparing (i) and (ii) proves the claim.

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