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Abstract

This paper develops a contest model to compare social welfare in homogeneous leagues in which all clubs maximize identical objective functions with mixed leagues in which clubs maximize different objective functions. We show that homogeneous leagues in which all clubs are profit-maximizers dominate all other leagues. Mixed leagues in which small-market clubs are profit- and large-market clubs are win-maximizers (type-I mixed leagues) are dominated by all other leagues. From a welfare perspective, large-market clubs win too often in (purely) win-maximizing and type-I mixed leagues, whereas small-market clubs win too many games in (purely) profit-maximizing leagues and in mixed leagues in which large-market clubs are profit- and small-market clubs are win-maximizers (type-II mixed leagues). These results have important policy implications: Social welfare will increase if clubs are reorganized from non-profit members associations to profit-maximizing corporations. Moreover, we show that revenue sharing decreases (increases) social welfare in mixed (homogeneous) leagues.

Keywords: Social welfare, team sports leagues, objective functions, mixed leagues, competitive balance

JEL Classification: M21, L83

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1 Introduction

Welfare analysis is the heart of economics. There is a huge body of literature devoted to the welfare effects of regulations, institutions, policies and the like. Surprisingly, there are hardly any welfare analyses in the professional team sports industry. We believe that the lack of welfare analysis in professional team sports is caused by the confusion created by the so-called uncertainty of outcome hypothesis (see Rottenberg 1956 and Neale 1964). According to this hypothesis, fans prefer to attend games with an uncertain outcome and enjoy close championship races. The uncertainty of outcome hypothesis describes one of the unique economic characteristics of the team sports industry. Unlike Toyota, Microsoft, and Wal-Mart, who benefit from weak competitors in their respective industries, Real Madrid and the New York Yankees need strong competitors to maximize their revenues. In sports, a weak team produces a negative externality on its stronger competitors.

Based on the uncertainty of outcome hypothesis, professional team sports leagues have introduced a variety of measures to increase competitive balance. Two of the most prominent measures are reserve clauses[1] and revenue-sharing arrangements. Whether these measures actually increase competitive balance or not is the most disputed question in the sports economics literature. According to Rottenberg's invariance proposition,[2] the distribution of playing talent between clubs in professional sports leagues does not depend on the allocation of property rights to players' services. In particular, changes in property rights, such as the introduction of a reserve clause, will not alter the allocation of players and therefore have no impact on competitive balance. Quirk and El-Hodiri (1974), Fort and Quirk (1995), and Vrooman (1995) extended this invariance proposition to gate revenue sharing. Invariance propositions provide economists with tough challenges, both theoretically and empirically. Theoretically, it is important to identify the exact assumptions under which such propositions hold. The empirical challenge is to show whether these assumptions actually hold and lead to the predicted results. So far, the empirical challenge proved to be too tough, because apart from the problems of measuring

[1] The reserve clause binds a player to his club beyond the expiration of his contract until the player is either released, retires, or is traded to another club. Although the player's obligation to play for his club, as well as the club's obligation to pay the player, were terminated, the player was not free to enter into another contract with another club.

[2] Rottenberg's invariance proposition is often regarded as a predecessor of the famous Coase Theorem (see e.g. Fort 2005).

competitive balance, it has been impossible to isolate the effect of single measures such as revenue sharing or free agency on competitive balance.

A number of authors have taken on the theoretical challenge. Their analysis can be grouped along two dimensions of assumptions: profit- versus win-maximization and fixed versus flexible supply of talent. Along the first dimension, club owners may be either modelled as profit- or win-maximizers. Profit-maximizers do not care about winning percentages unless they affect profits. Win-maximizers invest as much as they can into playing talent and are only constrained by zero profit. The second dimension concerns the elasticity of talent supply. Under the assumption of fixed supply, aggregate talent within the league is constant and the race for talent is a zero-sum game between owners. Under flexible supply, owners can hire as much talent as they want at a constant (exogenous) wage rate.

According to this categorization, the invariance proposition with regard to revenue sharing is derived under the assumptions of profit-maximization and fixed supply. There is wide agreement that the invariance proposition does not hold in leagues with either win-maximizing owners or flexible talent supply (see e.g. Atkinson, Stanley, and Tschirhart 1988; Késenne 2000, 2005; Vrooman 2008). There is disagreement, however, whether the invariance proposition holds in a league with profit-maximizing owners and fixed talent supply. Szymanski and Késenne (2004) argue that increased gate revenue sharing results in a more uneven distribution of talent between large- and small-market clubs. This result contradicts the invariance proposition with respect to gate revenue sharing.

Where does this disagreement come from? Obviously, Szymanski and Késenne work from the same assumptions as Quirk and El-Hodiri and others. The root of the disagreement lies in the underlying model conjectures. As Szymanski (2004) has shown, the assumption of fixed talent supply is often used to justify 'Walrasian fixed-supply conjectures' instead of 'Contest-Nash conjectures'. Under Walrasian fixed-supply conjectures, the quantity of talent hired by at least one club owner is determined by the choices of all the other club owners. In a two-club league, the Walrasian fixed-supply conjecture collapses the non-cooperative choice of talents into a choice of winning percentages by only one club owner. Under the Walrasian fixed-supply conjectures, the "game" between profit-maximizing owners loses its non-cooperative character and leads to results that are more in line with joint profit-maximization.

We believe that the invariance proposition and the related literature on competitive balance miss the point by raising the wrong question. In our view, it is much more important to analyze the *welfare effects* of different assumptions and issues of league design, such as club owner objectives and revenue sharing, than their effect on competitive balance. An exclusive focus on competitive balance would only be justified if the uncertainty of outcome hypothesis completely holds. If, on the other hand, social welfare does not monotonically increase as competitive balance increases, an exclusive focus on the effects of different assumptions and measures on competitive balance will result in inefficient policy conclusions.

There is strong evidence that competitive balance is not a good proxy for social welfare. Theoretically, a fully competitive league does not maximize social welfare if clubs differ with respect to market size. Large-market clubs have, on average, higher marginal revenues of wins than small-market clubs. As a result, league revenues (and profits) are maximized when the large-market clubs have higher winning percentages than their small-market rivals. Empirical evidence supports the assumption that match attendance is maximized when the home team's winning probability is about twice as large as the visiting team's (e.g. Forrest and Simmons 2002; for an overview see Borland and McDonald 2003).

Given this evidence, we present a model that analyzes the welfare effects of heterogeneous club objectives. So far, most models have assumed that leagues were homogenous in the sense that all clubs maximize identical objective functions. Traditionally, these objectives were either profit- or win-maximization. Exceptions are Rascher (1997) and Vrooman (1997, 2000), who introduced a league in which owners maximize a combination of profits and wins. This objective function is more general than the traditional assumptions because it allows club owners to trade-off profits for wins. Even with this more general objective function, however, the league is still modelled as homogeneous because all the club owners maximize identical objective functions.

Our major contribution in this respect is the introduction of heterogeneous objective functions. This extension allows us to compare mixed leagues in which club owners maximize different objective functions with homogeneous leagues in which all the club owners maximize identical objective functions. Mixed leagues have not yet been modelled in sports economics despite the fact that most major leagues are mixed leagues. For example, the most valuable team in 2008, according to Forbes, Manchester United, is fully owned by the Glazer family and may be regarded as a profit-maximizing club. In the prestigious Union of European Football Associations (UEFA) Champions League, Manchester United competes against clubs such as Real Madrid, F.C. Internazionale Milano, and F.C. Barcelona. Since these clubs are organized as (not-for-profit) members associations, they should be characterized as win-maximizing clubs.

Second, and most importantly, we explicitly integrate the consumer (fan) into our analysis in order to compare social welfare in homogeneous and mixed leagues. We derive club-specific demand and revenue from a general fan utility function by assuming that a fan's willingness to pay depends on fan type, on the preferred team's winning percentage, and on competitive balance.

Using this approach, we are able to extend the literature by providing an integrated framework to analyze welfare effects. In particular, we show that homogeneous leagues in which all clubs are profit-maximizers dominate all other leagues, whereas mixed leagues in which small-market clubs are profit- and large-market clubs are win-maximizers (type-I mixed leagues) are dominated by all other leagues. In addition, we show that, from a welfare perspective, large-market clubs win too often in (purely) win-maximizing and type-I mixed leagues, whereas small-market clubs win too many games in (purely) profit-maximizing leagues and in mixed leagues in which the large-market clubs are profit- and the small-market clubs are win-maximizers (type-II mixed leagues).

These results have important policy implications. For example, these results show that - contrary to prevailing claims - social welfare would increase if clubs were reorganized from win-maximizing non-profit members associations to profit-maximizing (public or private) corporations. Moreover, it is socially desirable to reorganize large-market clubs first, because in mixed leagues it is better if the large-market clubs maximize profits instead of the small-market clubs. Furthermore, the efficiency of measures that increase the competitiveness of small-market clubs depends on the league type. If the large-market clubs are profit-maximizers, for example, small-market clubs should win fewer rather than more games.

Finally, we derive new insights regarding the invariance proposition. Most importantly, the invariance proposition with respect to revenue sharing does not hold in any league. Revenue sharing affects both competitive balance and social welfare. In profit-maximizing leagues, revenue sharing decreases and in win-maximizing leagues it increases competitive balance. In both cases, the effect on social welfare is positive because profit-maximizing leagues have too much and win-maximizing leagues too little competitive balance without revenue sharing. In mixed leagues, on the other hand, revenue-sharing arrangements decrease competitive balance and social welfare. These results also have important policy implications because they show how the effect of revenue sharing differs with respect to the league type. Homogeneous leagues should introduce revenue sharing; mixed leagues should not.

The remainder of the paper is organized as follows. In the next section, we present the model framework and derive fan demand, club revenues, profits, and social welfare. In section 3, we consider homogeneous leagues in which all club owners are profit-maximizers or win-maximizers. In section 4, we consider mixed leagues and differentiate two cases: (i) large-market clubs are win-maximizers and small-market clubs are profit-maximizers and (ii) vice versa. Section 5 presents the welfare analysis where we compare the different types of league with respect to competitive balance and social welfare. In section 6, we analyze the effect of revenue sharing on competitive balance and social welfare in mixed leagues. Finally, section 7 summarizes the main results and their policy implications.

2 Model

We model a two-club league where both clubs participate in a non-cooperative game and invest independently a certain amount in playing talent. Each club $i = 1, 2$ generates its own revenues, denoted by R_i, according to a fan demand function depending on the match quality. We assume that there are two types of club: a large-market club with a high drawing potential and a small-market club with a low drawing potential. Talent investments, denoted by x_i for club i, determine the match quality and therefore, through fan demand, the revenue of both clubs.

Fan Demand, Club Revenues, and Profits

Fan demand for a match with quality q_i is derived as follows.[3] We assume a continuum of fans who differ in their willingness to pay for a match between club i and club j with quality q_i. Every fan k has a certain preference for match quality that is measured by θ_k. For simplicity, we assume that these preferences are uniformly distributed in $[0, 1]$, i.e. the measure of potential fans is one. Furthermore, we assume a constant marginal utility of quality and define the net utility of fan θ_k as $\max\{\theta_k q_i - p_i, 0\}$. At price p_i, the fan who is indifferent to the consumption of the product or not is given by $\theta^* = \frac{p_i}{q_i}$.[4] Hence, the measure of fans who purchase at p_i is derived as $1 - \theta^* = \frac{q_i - p_i}{q_i}$. The fan demand function of club $i = 1, 2$ is therefore given by[5]

$$d(m_i, p_i, q_i) := m_i \frac{q_i - p_i}{q_i} = m_i \left(1 - \frac{p_i}{q_i}\right),$$

where $m_i \in \mathbb{R}^+$ represents the market size parameter of club i. We assume that clubs are heterogeneous with respect to their market size or drawing potential. Without loss of generality, we assume throughout this paper that club 1 is the 'large-market' club with a high drawing potential and club 2 is the 'small-market' club with a low drawing potential, i.e. $m_1 > m_2$. As a consequence, the large club generates higher demand for a given set of parameters (p_i, q_i) than the small club. Since we consider a two-club league, we can normalize one market size parameter to unity such that $m_1 := m$ and $m_2 := 1$. We assume $m \in (\underline{m}, \overline{m})$ with $\underline{m} := 1$ and $\overline{m} := 2$.[6]

By normalizing all other costs (e.g. stadium and broadcasting costs) to zero, we see that club i's revenue is simply $R_i = p_i \cdot d(m_i, p_i, q_i)$. Then, the club will choose the profit-maximizing price $p_i^* = \frac{q_i}{2}$.[7] Given this profit-maximizing price, club i's revenue depends solely on the quality of the match and is derived as

$$R_i = \frac{m_i}{4} q_i. \tag{1}$$

[3] We follow the approach in Falconieri, Palomino, and Sákovics (2004), Dietl and Lang (2008), and Dietl, Lang, and Rathke (2009).

[4] The price p_i can, for example, be interpreted as the gate price or the subscription fee for TV coverage of the match.

[5] Note that fan demand increases in quality, albeit with a decreasing rate, i.e. $\frac{\partial d}{\partial q_i} > 0$ and $\frac{\partial^2 d}{\partial^2 q_i} < 0$.

[6] Note that we also have to bound m from above in order to guarantee positive equilibrium investments in the WM league and the type-I mixed league.

[7] Note that the optimal price is increasing in quality, i.e. $\frac{\partial p_i^*}{\partial q_i} > 0$.

Following Dietl and Lang (2008), we assume that match quality q_i depends on two factors: the probability of club i's success and the uncertainty of outcome. Furthermore, we assume that both factors enter the quality function as a linear combination with equal weights, i.e. quality = probability of success + uncertainty of outcome.[8]

We measure the probability of club i's success by the win percentage w_i of this club. The win percentage is characterized by the contest-success function (CSF), which maps the vector (x_1, x_2) of talent investment into probabilities for each club. We apply the logit approach, which is the most widely used functional form of a CSF in sporting contests.[9] The win percentage of club $i = 1, 2$ in this imperfectly discriminating contest is then given by

$$w_i(x_i, x_j) = \frac{x_i}{x_i + x_j}, \tag{2}$$

with $i, j = 1, 2$, $i \neq j$. Given that the win percentages must sum up to unity, we obtain the adding-up constraint: $w_j = 1 - w_i$. In our model, we adopt the 'Contest-Nash conjectures' $\frac{\partial x_i}{\partial x_j} = 0$ and compute the derivative of Equation 2 as $\frac{\partial w_i}{\partial x_i} = \frac{x_j}{(x_i+x_j)^2}$.[10]

The uncertainty of outcome is measured by the competitive balance in the league. Following Szymanski (2003), Dietl and Lang (2008), and Vrooman (2008), we specify competitive balance by the product of the winning percentages

$$CB(x_i, x_j) = w_i(x_i, x_j)w_j(x_i, x_j) = \frac{x_i x_j}{(x_i + x_j)^2} \tag{3}$$

with $i, j = 1, 2$, $i \neq j$. Note that competitive balance $CB(\cdot)$ attains its maximum of 1/4 for a completely balanced league in which both clubs invest the same amount in talent such that $w_1 = w_2 = 1/2$. A less balanced league is then characterized by a lower value of $CB(\cdot)$.

With the specification of the win percentage given by Equation 2 and competitive balance given by Equation 3, the quality function is derived as

$$q_i(x_i, x_j) = w_i(x_i, x_j) + w_i(x_i, x_j)w_j(x_i, x_j), \tag{4}$$

[8] We will see below that this specification of the quality function gives rise to a quadratic revenue function widely used in the sports economic literature.

[9] The logit CSF was generally introduced by Tullock (1980) and was subsequently axiomatized by Skaperdas (1996) and Clark and Riis (1998). An alternative functional form would be the probit CSF (e.g. Lazear and Rosen 1981; Dixit 1987) and the difference-form CSF (e.g. Hirshleifer 1989). For applications in sporting contests see e.g. Szymanski (2003), Szymanski and Késenne (2004), Dietl, Franck, and Lang (2008).

[10] See Szymanski (2004).

with $i, j = 1, 2$, $i \neq j$. Plugging Equation 4 into Equation 1 and noting that $w_j = 1 - w_i$, we derive the revenue function of club $i = 1, 2$ as

$$R_i(x_i, x_j) = \frac{m_i}{4} q_i(x_i, x_j) = \frac{m_i}{4} \left(2w_i(x_i, x_j) - w_i(x_i, x_j)^2\right), \tag{5}$$

with $m_1 = m$ and $m_2 = 1$. This 'well-behaved' club-specific revenue function is consistent with the revenue functions used e.g. in Hoehn and Szymanski (1999), Szymanski (2003), Szymanski and Késenne (2004), Késenne (2006, 2007), and Vrooman (2007, 2008). However, in contrast to the articles quoted, we have derived our revenue function from consumer preferences and thus are able to perform a welfare analysis.[11]

The cost function of club $i = 1, 2$ is given by $C(x_i) = cx_i$, where c is the marginal unit cost of talent.[12]

The profit function of club $i = 1, 2$ is then given by

$$\begin{aligned} \pi_i(x_i, x_j) &= R_i(x_i, x_j) - C(x_i) \\ &= \frac{m_i}{4} \left(\frac{x_i x_j}{(x_i + x_j)^2} + \frac{x_i}{x_i + x_j} \right) - cx_i, \end{aligned}$$

with $i, j = 1, 2$, $i \neq j$ and $m_1 = m$, $m_2 = 1$.

Social Welfare

Social welfare is given by the sum of aggregate consumer (fan) surplus, aggregate club profit, and aggregate player salaries. Aggregate consumer surplus is computed by summing up the consumer surplus from fans of club 1 and club 2. The consumer surplus CS_i from fans of club $i = 1, 2$ in turn corresponds to the integral of the demand function $d(m_i, p_i, q_i)$ from the equilibrium price $p^* = \frac{q}{2}$ to the maximal price $\overline{p}_i = q_i$ that fans are

[11] Note that the fans' preferences in the quality function (given by Equation 4) for own team winning and competitive balance is equal for both clubs. As a consequence, the parameters of the linear term w_i and the quadratic term w_i^2 in the revenue function (given by Equation 5) are equal for both clubs. Without this simplifying assumption, the relationship between competitive balance in the win-maximizing league and the profit-maximizing league would be indeterminate (see Fort and Quirk 2004 and Késenne 2007, p. 42).

[12] By assuming a competitive labor market, the market clearing cost of a unit of talent is the same for every club. Moreover, for the sake of simplicity, we do not take into account non-labor costs and normalize the fixed capital cost to zero. Note that the results derived from this cost function do not necessarily hold for a more general cost function. See Vrooman (1995) for a more general cost function where clubs have different marginal costs or Késenne (2007) for a cost function with a fixed capital cost.

willing to pay for quality q_i

$$CS_i = \int_{p_i^*}^{\overline{p}_i} d(m_i, p_i, q_i) dp_i = \int_{\frac{q_i}{2}}^{q_i} m_i \frac{q_i - p_i}{q_i} dp_i = \frac{m_i}{8} q_i,$$

with $m_1 = m$ and $m_2 = 1$. By assuming that the players' utility corresponds to their salary, the total players' utility is given by the aggregate salary payments $PS = cx_1 + cx_2$ in the league. Addition of the aggregate consumer surplus, aggregate club profit, and aggregate salary payments produces social welfare as

$$W(x_i, x_j) = \frac{3}{8} \left(mq_1(x_i, x_j) + q_2(x_i, x_j) \right). \tag{6}$$

Note that salary payments do not directly influence social welfare because salaries merely represent a transfer from clubs to players. As a consequence, social welfare depends only on the quality of the league.

In the next proposition we derive the welfare-maximizing win percentages:

Proposition 1

Social welfare is maximized for the following win percentages

$$(w_1^*, w_2^*) = \left(\frac{m}{m+1}, \frac{1}{m+1} \right). \tag{7}$$

Proof. See Appendix. ■

The proposition shows that a certain degree of imbalance is socially desirable. More precisely, from a welfare point of view, it is desirable that the large club wins more often than the small club, since $w_1^* > w_2^*$ with $m > 1$. From Equation 7, we compute the welfare optimal level of competitive balance as

$$CB^* = \frac{m}{(m+1)^2}.$$

Note that the welfare optimal level of competitive balance decreases in the market size parameter m, i.e. $\frac{\partial CB^*}{\partial m} < 0$. As a consequence, the higher the asymmetry between the two clubs in terms of market size, the higher the welfare optimal degree of imbalance in the league. In other words, the league should be more imbalanced with the large club winning more often than the small club, the bigger the difference between the clubs.

3 Homogeneous Leagues

In this section, we first consider a homogeneous league consisting of two clubs with pure profit-maximizing team owners (the so-called 'PM league') and then a league with pure win-percentage-maximizing team owners (the so-called 'WM league'). In the subsequent section, we consider a mixed league in which one team owner maximizes club profits and one team owner maximizes the club's win percentage (we call such a league a 'type-I' and a 'type-II' mixed league, respectively).

Profit-Maximizing Clubs

We assume that both clubs are profit-maximizers. The maximization problem of club $i = 1, 2$ is therefore given by $\max_{x_i \geq 0} \pi_i = R_i(x_i, x_j) - C(x_i)$. The corresponding first-order conditions yield[13]

$$\frac{\partial \pi_1}{\partial x_1} = \frac{m x_2^2}{2\left(x_1 + x_2\right)^3} - c = 0 \quad \text{and} \quad \frac{\partial \pi_2}{\partial x_2} = \frac{x_1^2}{2\left(x_1 + x_2\right)^3} - c = 0. \tag{8}$$

The equilibrium talent investments in a homogeneous league with pure profit-maximizing clubs are then computed as

$$\left(x_1^{PM}, x_2^{PM}\right) = \left(\frac{m^{3/2}}{2c(\sqrt{m}+1)^3}, \frac{m}{2c(\sqrt{m}+1)^3}\right). \tag{9}$$

Using Equations 2 and 9, we derive the following equilibrium win percentages in a PM league

$$\left(w_1^{PM}, w_2^{PM}\right) = \left(\frac{\sqrt{m}}{\sqrt{m}+1}, \frac{1}{\sqrt{m}+1}\right).$$

We derive $\frac{x_1^{PM}}{x_2^{PM}} = \sqrt{m} > 1 \; \forall m \in (\underline{m}, \overline{m})$. Thus, in equilibrium, the large club invests more in playing talent than the small club because the marginal revenue of talent investments is higher for the club with the larger market size. Moreover, a bigger difference between the two clubs in terms of market size induces both clubs to increase their investment level in equilibrium, i.e. $\frac{\partial x_i^{PM}}{\partial m} > 0$ for $i = 1, 2$. The large club, however, increases its investments in playing talent more than the small club, i.e. $\frac{\partial x_1^{PM}}{\partial m} > \frac{\partial x_2^{PM}}{\partial m}$. It follows that competitive balance $CB^{PM} = w_1^{PM} \cdot w_2^{PM}$ decreases in the market size parameter m, i.e. $\frac{\partial CB^{PM}}{\partial m} < 0$.

[13] It is easy to show that the second-order conditions for a maximum are satisfied.

Social welfare is computed in the next lemma:

Lemma 1

Social welfare in a PM league is given by $W^{PM} = \frac{3}{8}\left(mq_1^{PM} + q_2^{PM}\right)$ *with* $q_1^{PM} = 1 - \frac{1}{(1+\sqrt{m})^2}$ *and* $q_2^{PM} = \frac{1+2\sqrt{m}}{(1+\sqrt{m})^2}$.

Proof. See Appendix. ■

Win-Maximizing Clubs

In this section, both clubs are assumed to be win-maximizers. Each club chooses independently a level of talent in order to maximize the level of own talents subject to its budget constraint.[14] The maximization problem of club $i = 1, 2$ is then given by $\max_{x_i \geq 0} x_i$ subject to $cx_i \leq R_i(x_1, x_2)$. The first-order conditions for this maximization problem yield

$$1 - \lambda_i \left(c - \frac{\partial R_i}{\partial x_i}\right) \leq 0, \quad x_i \left(1 - \lambda_i \left(c - \frac{\partial R_i}{\partial x_i}\right)\right) = 0, \quad R_i - cx_i \geq 0, \quad \lambda_i(R_i - cx_i) = 0,$$

where λ_i is the Lagrange multiplier for club $i = 1, 2$. It follows that each club will spend all its revenue on playing talent such that the optimality conditions for this maximization problem are reduced to

$$\frac{m(x_1 + 2x_2)}{4(x_1 + x_2)^2} = c \text{ (for club 1)} \quad \text{and} \quad \frac{2x_1 + x_2}{4(x_1 + x_2)^2} = c \text{ (for club 2)}. \tag{10}$$

The equilibrium talent investments in a homogeneous league with pure win-maximizing clubs are computed from Equation 10 as

$$\left(x_1^{WM}, x_2^{WM}\right) = \left(\frac{3m(2m-1)}{4c(m+1)^2}, \frac{3m(2-m)}{4c(m+1)^2}\right). \tag{11}$$

In order to guarantee positive equilibrium investments, the difference between the two clubs in terms of market size must not be too large. Formally, the market size parameter m of club 1 has to be bounded from above such that $m \in (\underline{m}, \overline{m}) = (1, 2)$.[15]

[14] We choose this objective function since, according to Késenne (2006), 'win maximization is not an operational objective, because clubs cannot control their winning percentage. Clubs can only maximize the talents of the team. The best guarantee for a high winning percentage is fielding a performing team by attracting the best players'.

[15] For $m \geq 2$, the optimal choice for the small club is zero. Since we are not interested in a situation where a club is not participating, we choose to restrict the range of m to ensure positive equilibrium

Using Equations 2 and 11, we derive the following equilibrium win percentages in a WM league

$$\left(w_1^{WM}, w_2^{WM}\right) = \left(\frac{2m-1}{m+1}, \frac{2-m}{m+1}\right).$$

We derive $\frac{x_1^{WM}}{x_2^{WM}} = \frac{2m-1}{2-m} > 1 \ \forall m \in (\underline{m}, \overline{m})$. Thus, in equilibrium, the large club always invests more in playing talent than the small club. By comparing the equilibrium investments in a WM league with the corresponding investments in a PM league, we deduce that a large club in a WM league always invests more than the same large club in a PM league. The opposite is true for a small club but only if the market size parameter is sufficiently large, i.e. $x_1^{PM} < x_1^{WM} \ \forall m \in (\underline{m}, \overline{m})$ and $x_2^{PM} > x_2^{WM} \Leftrightarrow m > \widehat{m} \approx 1.61$. In a WM league, however, the large (small) club always has a higher (lower) win percentage in equilibrium than the same large (small) club in a PM league independent of the market size, i.e. $w_1^{WM} > w_1^{PM}$ and $w_2^{WM} < w_2^{PM} \ \forall m \in (\underline{m}, \overline{m})$.

Moreover, in a WM league, a bigger difference between the two clubs in terms of market size induces the large club to increase its investment level and the small club to decrease its investment level in equilibrium, i.e. $\frac{\partial x_1^{WM}}{\partial m} > 0$ and $\frac{\partial x_2^{WM}}{\partial m} < 0$.[16] The result of a bigger difference between the clubs is, similar to a PM league, a decrease of competitive balance $CB^{WM} = w_1^{WM} \cdot w_2^{WM}$ in the WM league, i.e. $\frac{\partial CB^{WM}}{\partial m} < 0$.

Social welfare is computed in the next lemma:[17]

Lemma 2
Social welfare in a WM league is given by $W^{WM} = \frac{3}{8}\left(mq_1^{WM} + q_2^{WM}\right)$ *with* $q_1^{WM} = \frac{3(2m-1)}{(1+m)^2}$ *and* $q_2^{WM} = \frac{3(2-m)m}{(1+m)^2}$.

Proof. See Appendix. ■

4 Mixed Leagues

In this section, we model a league in which clubs are heterogeneous with respect to their objective function. We differentiate two types of so-called 'mixed leagues'. A 'type-I'

investments.

[16]Note that the increase in talent investments of the large club is bigger than the decrease of the small club in absolute values, i.e. $\frac{\partial x_1^{WM}}{\partial m} > \left|\frac{\partial x_2^{WM}}{\partial m}\right|$.

[17]Note that in a WM league, aggregate club profits are zero because each club spends all its revenue on playing talent, i.e. $cx_i^{WM} = R_i^{WM}$, $i = 1, 2$.

mixed league is a league where the large-market club is a win-maximizer and the small-market club a profit-maximizer. A 'type-II' mixed league is analogously a league where the large-market club is a profit-maximizer and the small-market club is a win-maximizer.

Type-I Mixed League

We assume that the large club 1 is a win-maximizer and the small club 2 is a profit-maximizer. The maximization problem for club 1 is given by $\max_{x_1 \geq 0} x_1$ subject to $cx_1 \leq R_1(x_1, x_2)$ and for club 2 it is given by $\max_{x_2 \geq 0} \pi_2 = R_2(x_1, x_2) - cx_2$.

The first-order conditions for this problem are similar to the homogeneous league cases above. Consider Equation 10 for the large, win-maximizing club 1 and Equation 8 for the small, profit-maximizing club 2. Simple algebra and rearrangements of the first-order conditions yield the following equilibrium talent investments in a type-I mixed league[18]

$$\begin{aligned} x_1^{TypeI} &= \frac{m(\sqrt{m(16+m)}(4+m) - m(12+m))}{32c}, \\ x_2^{TypeI} &= \frac{m(-\sqrt{m(16+m)}(6+m) + m(14+m) + 16)}{32c}. \end{aligned} \tag{12}$$

Using Equations 2 and 12, we derive the following equilibrium win percentages in a type-I mixed league

$$\left(w_1^{TypeI}, w_2^{TypeI}\right) = \left(\frac{\sqrt{m(16+m)} - m}{4}, \frac{4 + m - \sqrt{m(16+m)}}{4}\right).$$

The relationship between the equilibrium investments is given by $\frac{x_1^{TypeI}}{x_2^{TypeI}} = \frac{4\sqrt{m}}{\sqrt{16+m} - 3\sqrt{m}} > 1$ $\forall\ m \in (\underline{m}, \overline{m})$. Thus, a large, win-maximizing club invests more in talents than a small, profit-maximizing club, independent of their market size, i.e. $x_1^{TypeI} > x_2^{TypeI}$ $\forall m \in (\underline{m}, \overline{m})$. This result is intuitive since both the larger market size and the win-maximizing behavior induce club 1 to invest more than club 2.

Similar to a WM league, equilibrium investments of the large, win-maximizing (small, profit-maximizing) club increase (decrease) in the market size parameter m, with $\frac{\partial x_1^{TypeI}}{\partial m} > \left|\frac{\partial x_2^{TypeI}}{\partial m}\right|$. As a consequence, competitive balance $CB^{TypeI} = w_1^{TypeI} \cdot w_2^{TypeI}$ decreases in the market size parameter m, i.e. $\frac{\partial CB^{TypeI}}{\partial m} < 0$.

[18]Note that $m < \overline{m}$ guarantees positive equilibrium investments of club 2.

Social welfare is computed in the next lemma:[19]

Lemma 3

Social welfare in a type-I mixed league is given by $W^{TypeI} = \frac{3}{8}\left(mq_1^{TypeI} + q_2^{TypeI}\right)$, *with* $q_1^{TypeI} = \frac{\sqrt{m(16+m)}(4+m)-m(12+m)}{8}$ *and* $q_2^{TypeI} = \frac{8-m(8+m-\sqrt{m(16+m)})}{8}$.

Proof. See Appendix. ■

Type-II Mixed League

In this last case, we assume that the large club 1 is a profit-maximizer and the small club 2 is a win-maximizer. The maximization problem for club 1 is given by $\max_{x_1 \geq 0} \pi_1 = R_1(x_1, x_2) - cx_1$ and for club 2 it is given by $\max_{x_2 \geq 0} x_2$ subject to $cx_2 \leq R_2(x_1, x_2)$.

For the first-order conditions, consider Equation 8 for club 1 and Equation 10 for club 2. The equilibrium investments are then computed as

$$\begin{aligned} x_1^{TypeII} &= \frac{m(16m+14)+1-\sqrt{16m+1}(6m+1)}{32cm^2}, \\ x_2^{TypeII} &= \frac{\sqrt{16m+1}(4m+1)-(1+12m)}{32cm^2}. \end{aligned} \tag{13}$$

Using Equations 2 and 13, we derive the following equilibrium win percentages in a type-II mixed league

$$\left(w_1^{TypeII}, w_2^{TypeII}\right) = \left(\frac{1+4m-\sqrt{16m+1}}{4m}, \frac{\sqrt{16m+1}-1}{4m}\right).$$

The relationship between the equilibrium investments is given by $\frac{x_1^{TypeII}}{x_2^{TypeII}} = \frac{\sqrt{16m+1}-3}{4} < 1$ $\forall\ m \in (\underline{m}, \overline{m})$. Thus, in a type-II mixed league, a small, win-maximizing club invests more in playing talent than a large, profit-maximizing club, independent of their market size, i.e. $x_1^{TypeII} < x_2^{TypeII}$ $\forall\ m \in (\underline{m}, \overline{m})$. We derive the following corollary:

Corollary 1

In a mixed league in which clubs differ with respect to their objective function, the win-maximizing club always invests more than the profit-maximizing club independent of the market size.

[19] Note that the win-maximizing club 1 spends all its revenue on playing talent, i.e. $cx_1^{TypeI} = R_1^{TypeI}$, such that it makes zero profits.

The fact that the large, profit-maximizing club invests less than the small, win-maximizing club shows that the clubs' objective functions have a stronger influence on the investment behavior than the market size. In particular, in a type-II mixed league the win-maximizing behavior of club 2 overcompensates for the positive effect on talent investments of club 1's larger market size.

Increasing the market size m in a type-II mixed league induces the large, profit-maximizing club to increase and the small, win-maximizing club to decrease their investment level in equilibrium, with $\frac{\partial x_1^{TypeII}}{\partial m} > \left|\frac{\partial x_2^{TypeII}}{\partial m}\right|$. As a consequence, competitive balance $CB^{TypeII} = w_1^{TypeII} \cdot w_2^{TypeII}$ increases in the market size parameter m, i.e. $\frac{\partial CB^{TypeII}}{\partial m} > 0$.

Social welfare is computed in the next lemma:

Lemma 4
Social welfare in a type-II mixed league is given by $W^{TypeII} = \frac{3}{8}\left(mq_1^{TypeII} + q_2^{TypeII}\right)$, *with* $q_1^{TypeII} = \frac{8m(m-1)-1+\sqrt{16m+1}}{8m^2}$ *and* $q_2^{TypeII} = \frac{\sqrt{16m+1}(4m+1)-(1+12m)}{8m^2}$.

Proof. See Appendix. ■

5 Welfare Analysis

In this section, we analyze how the clubs' objective functions influence social welfare. However, before proceeding with the welfare analysis, we provide a comparison between the leagues with respect to competitive balance:

Proposition 2
(i) The PM league dominates all other leagues with respect to competitive balance.
(ii) The type-I mixed league is dominated by all other leagues with respect to competitive balance.
(iii) The type-II mixed league dominates the WM league with respect to competitive balance if the market size of the large club is sufficiently large.

Proof. See Appendix. ■

The proposition shows that a league with two profit-maximizing clubs is the most balanced league, i.e. $CB^{PM} > CB^{\mu}$ for $\mu \in \{WM, TypeI, TypeII\}$, whereas a mixed league in which the large club is a win-maximizer and the small club a profit-maximizer proves to be the least balanced league, i.e. $CB^{TypeI} < CB^{\mu}$ for $\mu \in \{PM, WM, TypeII\}$. The homogeneous win-maximizing league and the type-II mixed league are somewhere in between, depending on the market size parameter. Moreover, if the difference between the large and the small club is sufficiently large, then the type-II mixed league is more balanced than the WM league, i.e. $CB^{TypeII} > CB^{WM} \Leftrightarrow m > m^{CB} \approx 1.3128$.[20]

Together with the results from sections 3 and 4, we derive from Proposition 2 that, in a type-II mixed league, the small (large) club always has a higher (lower) win percentage in equilibrium than the large (small) club in a PM league independent of the market size, i.e. $w_2^{TypeII} > w_1^{PM} > \frac{1}{2}$ and $w_1^{TypeII} < w_2^{PM} < \frac{1}{2}$ $\forall m \in (\underline{m}, \overline{m})$. The opposite is true with respect to a type-I mixed league: the small (large) club in a type-II mixed league always has a lower (higher) win percentage in equilibrium than the large (small) club in a type-I mixed league independent of the market size, i.e. $\frac{1}{2} < w_2^{TypeII} < w_1^{TypeI}$ and $\frac{1}{2} > w_1^{TypeII} > w_2^{TypeI}$ $\forall m \in (\underline{m}, \overline{m})$. The relationship between the win percentages in a type-II mixed league and a WM league, however, are dependent on the market size parameter. Formally: $w_2^{TypeII} < w_1^{WM}$ and $w_1^{TypeII} > w_2^{WM}$ if and only if $m > m^{CB}$. These results are depicted in Figure 1.

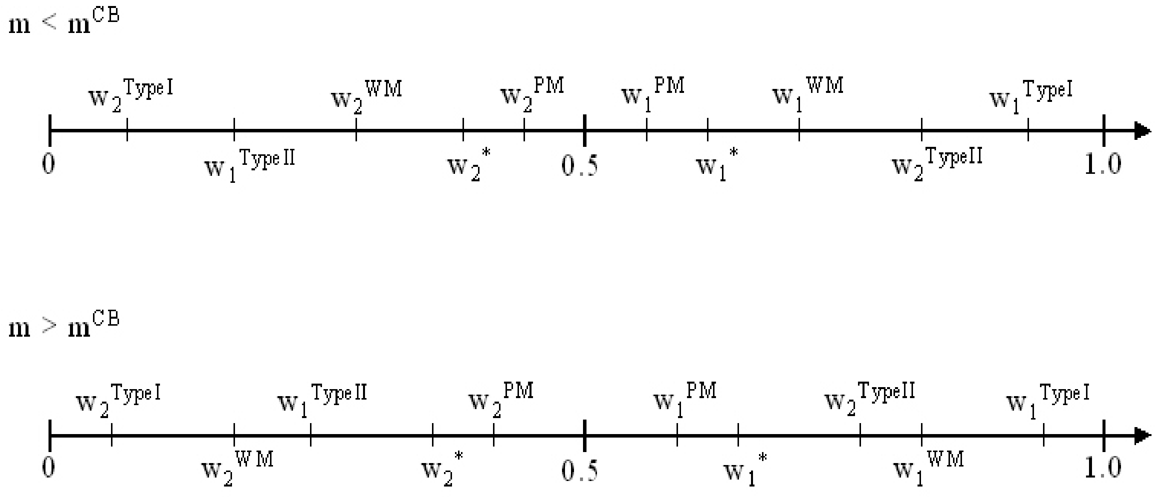

Figure 1: Equilibrium win percentages in the different leagues

We now start our welfare analysis and first compare the welfare-maximizing win percentages (w_1^*, w_2^*) and the respective equilibrium win percentages in the different leagues. The following results can be deduced:

[20]See also Késenne (2007, p. 46). In contrast to Késenne (2007), we derive, however, that the PM league is the most balanced league.

Proposition 3

From a welfare point of view, the small (large) club wins too often and the large (small) club does not win often enough in the PM league and the type-II mixed league (in the WM league and the type-I mixed league).

Proof. See Appendix. ■

The proposition shows that the degree of competitive balance is too high and a more unbalanced league is socially desirable in pure profit-maximizing leagues. A decrease in the win percentage of the small club and an increase in the win percentage of the large club would thus result in higher social welfare.[21]

The opposite is true in pure win-maximizing leagues and type-I mixed leagues: the degree of competitive balance is too low and a more balanced league is socially desirable. The large club wins too often and the small club does not win often enough. An increase in the win percentage of the small club and a decrease in the win percentage of the large club would thus result in higher social welfare in a WM league and a type-I mixed league.

In mixed leagues in which the large club is a profit- and the small club is a win-maximizer (type-II mixed leagues), the degree of competitive balance is also too low from a welfare perspective. The difference from the WM league and the type-I mixed league is, however, that the small club wins too often and the large club does not win often enough.[22] A decrease in the win percentage of the small club and an increase in the win percentage of the large club would result in higher social welfare in a type-II mixed league.

How do the different leagues compare to each other with respect to social welfare? More precisely, which league approximates the welfare-maximizing win percentages from Proposition 1 in the best way? Similar to Proposition 2, there does not exist a unique ordering between the leagues regarding social welfare. However, we can derive the following results:

Proposition 4

(i) The PM league dominates all other leagues with respect to social welfare.

(ii) The type-I mixed league is dominated by all other leagues with respect to social welfare.

[21] See also Szymanski (2006) and Dietl and Lang (2008).

[22] Recall that in a type-II mixed league, the small club is the dominant team that always invests more than the large club independent of the market sizes.

(iii) The type-II mixed league dominates the WM league with respect to social welfare if the market size of the large club is sufficiently large.

Proof. See Appendix. ■

A homogeneous league in which both clubs are profit-maximizers (PM league) generates the highest level of social welfare, i.e. $W^{PM} > W^{\mu}$ for $\mu \in \{TypeI, TypeII, WM\}$. The PM league is the welfare dominating league not because it is the most balanced league but because the equilibrium win percentages $\left(w_1^{PM}, w_2^{PM}\right)$ and competitive balance CB^{PM} in the PM league approximate the welfare-maximizing win percentages (w_1^*, w_2^*) and competitive balance CB^* in the best way.

On the other hand, a mixed league in which the small club is a profit- and the large club is a win-maximizer (type-I mixed league) generates the lowest level of social welfare, i.e. $W^{TypeI} < W^{\mu}$ for $\mu \in \{TypeII, PM, WM\}$. In this league the equilibrium win percentages $(w_1^{TypeI}, w_2^{TypeI})$ and competitive balance CB^{TypeI} are furthest away from the corresponding welfare optimal win percentages and competitive balance.[23]

The mixed league in which the large club is a profit- and the small club is a win-maximizer (type-II mixed leagues) and the homogeneous league in which both clubs are win-maximizers (WM league) are somewhere in between, depending on the market size parameter. More precisely, social welfare is higher in the type-II mixed league than the WM league if the difference between the clubs in terms of market size is sufficiently large, i.e. $W^{TypeII} > W^{WM} \Leftrightarrow m > m^W \approx 1.72$.

6 Revenue Sharing

In this section, we analyze the welfare effect of revenue sharing. In our model, the share of revenues that is assigned to the home team is given by the parameter $\alpha \in [\frac{1}{2}, 1]$, while $(1-\alpha)$ is assumed to be the share of revenues received by the away team. The after-sharing revenues of club i, denoted by R_i^*, are then given by $R_i^* = \alpha R_i + (1-\alpha) R_j$, with $\alpha \in [\frac{1}{2}, 1]$ and $i, j = 1, 2$, $i \neq j$. The impact of revenue sharing on competitive balance is stated in the following proposition:[24]

[23] See also Figure 1.

[24] Note that our results are robust with respect to pool revenue sharing, which is another popular form of revenue sharing in sports leagues. Under a pool-sharing arrangement each club receives an α-share of its revenue and an equal $(1-\alpha)$-share of a league revenue pool, where $\alpha \in [0, 1]$. In this case, the after-sharing revenues of club i are given by $R_i^* = \alpha R_i + \frac{(1-\alpha)}{2}(R_i + R_j)$.

Proposition 5

(i) Revenue sharing decreases competitive balance in a homogeneous league with profit-maximizing clubs, whereas it increases competitive balance in a homogeneous league with win-maximizing clubs.

(ii) Revenue sharing decreases competitive balance in mixed leagues where clubs differ with respect to their objective function.

Proof. See Appendix. ■

ad (i) The effect of revenue sharing on competitive balance in homogeneous leagues has been extensively analyzed in the literature. In a league with two profit-maximizing clubs, revenue sharing reduces the incentive to invest in playing talent for both clubs. This so-called 'dulling effect' of revenue sharing, however, is stronger for the small club than for the large club.[25] As a consequence, the small club will reduce its investment level more than the large club. Since the large club dominates the small club in terms of talent, a higher degree of revenue sharing produces a more unbalanced league and thus decreases competitive balance.[26]

However, in a league with two win-maximizing clubs, any revenue-sharing arrangement that transfers revenues from the large to the small club induces the small club to increase and the large club to decrease their talent investments. As a consequence, revenue sharing improves competitive balance by assuming again that the large club is the dominant team in terms of talent.[27]

ad (ii) In mixed leagues where clubs differ with respect to their objective function, revenue sharing has, similar to the PM league, a 'dulling effect' on the investment incentives. Both types of club reduce their investments in playing talent through a higher degree of revenue sharing. The profit-maximizing club, however, will reduce its investments more than the win-maximizing club.[28] Since, in both types of mixed leagues, the profit-maximizing club always invests less than the win-maximizing club independent of the market size (see Corollary 1), a higher degree of revenue sharing decreases competitive balance and produces a more unbalanced league.

The welfare implications of these results are stated in the following corollary:

[25] The notion 'dulling effect' was introduced by Szymanski and Késenne (2004).

[26] See e.g. Szymanski and Késenne (2004), Késenne (2005), Késenne (2007), and Dietl and Lang (2008).

[27] See e.g. Késenne (2006) and Késenne (2007).

[28] See Table 1.

Corollary 2

Revenue sharing increases social welfare in homogeneous leagues whereas it decreases social welfare in mixed leagues where clubs differ with respect to their objective function.

The first part of the corollary shows that revenue sharing is beneficial to social welfare in both types of homogeneous league. The mechanism behind this result, however, differs between the leagues. According to Proposition 3, in a PM league (WM league), the small club (large club) wins too often and the large club (small club) does not win often enough from a welfare point of view. In other words, the degree of competitive balance in a league with profit-maximizing (win-maximizing) clubs is too high (low). Since revenue sharing decreases (increases) competitive balance in a PM league (WM league), it will increase social welfare in both homogeneous leagues.

In contrast, the second part of the corollary states that revenue sharing is detrimental to social welfare in mixed leagues where clubs have different objective functions. The intuition is as follows: We know from Proposition 3 that, from a welfare perspective, in both types of mixed leagues, the profit-maximizing club does not win often enough and the win-maximizing club wins too often (independent of the market size). In a type-II mixed league, the small, win-maximizing club invests more than the large, profit-maximizing club. Revenue sharing decreases competitive balance by inducing the profit-maximizing club to decrease its investments more than the small, win-maximizing club. As a result, both clubs depart from the welfare-maximizing win percentages and social welfare declines. For a type-I mixed league, the reasoning is the same except for the fact that the win-maximizing club is the large club that already invests more than the small, profit-maximizing club.

7 Summary and Policy Implications

In this paper, we develop a contest model of a team sports league to compare social welfare in homogeneous leagues in which all clubs maximize identical objective functions and mixed leagues in which clubs maximize different objective functions. Starting from a general fan utility function, we provide an integrated framework to analyze welfare effects.

In particular, we show that there does not exist a unique ordering regarding social welfare: leagues in which all clubs are profit-maximizers dominate all other leagues whereas mixed leagues in which small-market clubs are profit- and large-market clubs are win-maximizers (type-I mixed leagues) are dominated by all other leagues. Mixed leagues in which the large-market clubs are profit- and the small-market clubs are win-maximizers (type-II mixed leagues) dominate a homogeneous league of win-maximizing clubs if the market size of the large-market clubs is sufficiently large.

In addition, we show that, from a welfare perspective, small-market clubs win too many games in (purely) profit-maximizing and type-II mixed leagues, whereas the large-market clubs win too often in (purely) win-maximizing and type-I mixed leagues. These results show that social welfare would increase if clubs were reorganized from non-profit members associations to profit-maximizing corporations. Some leagues, however, still try to prevent the transformation of professional sports clubs from not-for-profit members associations to profit-maximizing public or private corporations.

A typical example is the German Soccer League (DFL). The DFL allows its clubs to hive off their professional football units by forming a public or private corporation under the provision that 50% of the voting rights plus one voting right will remain with the members association. This 50% + 1 rule practically prevents the clubs from changing their overall objective from win- to profit-maximization. The major rationale for the 50% + 1 rule was to ensure that fans have a voice in their club's decision. Our results show that fans may benefit from a reorganization of their clubs into public or private corporations. If such a reorganization takes place, it is socially desirable to reorganize large-market clubs first, because in mixed leagues it is better if the large-market clubs maximize profits instead of the small-market clubs.

Furthermore, the efficiency of measures that increase the competitiveness of small-market clubs depends on the league type. If the large-market clubs are profit-maximizers, for example, small-market clubs should win fewer rather than more games. In such leagues, all measures in favor of small-market clubs, such as transfer restrictions and reverse-order drafts, can be dangerous because they may lead to a decrease instead of an increase in social welfare.

Finally, we derive new insights regarding the invariance proposition with respect to revenue sharing. In our model, the invariance proposition with respect to revenue sharing does not hold in any league. Revenue sharing affects both competitive balance and social welfare. In pure profit-maximizing leagues, revenue sharing decreases and in pure win-maximizing leagues it increases competitive balance. In both cases, the effect on social welfare is positive because profit-maximizing leagues have too much and win-maximizing leagues too little competitive balance without revenue sharing. In mixed leagues, on the other hand, revenue-sharing arrangements decrease competitive balance by inducing the profit-maximizing clubs, who already invest less, to decrease their investments more than the win-maximizing clubs. As a result, social welfare decreases in both types of mixed league through the introduction of revenue-sharing arrangements.

The current revenue-sharing arrangements differ widely among professional leagues all over the world. In 1876, Major League Baseball (MLB) in the United States introduced a 50-50 split of gate receipts that was reduced over time. Since 2003, all the clubs in the American League have to put 34% of their locally generated revenue (gate, concession, television, etc.) into a central pool which is then divided equally among clubs. The current revenue-sharing arrangement of the National Football League (NFL) in the United States secures the visiting team 40% of the gate receipts (revenues from luxury boxes, parking and concessions are excluded from this sharing arrangement). In the Australian Football League (AFL), gate receipts were split evenly between the home and the visiting team. This 50-50 split was finally abolished in 2000. In Europe, there is less gate revenue sharing. The soccer leagues have adopted various forms of gate revenue sharing in their history. In England, until the early 1980s, up to 20% of the gate receipts were given to the visiting teams in league matches. In the DFL the home team receives 94% of the gate receipts, with the other 6% going to the league. Gate revenue sharing, however, is quite common in most cup competitions with a knock-out system.

Our results suggest that the welfare effect of these revenue-sharing arrangements depends on the prevailing league type. In homogeneous leagues, such as the NFL or DFL, revenue sharing increases social welfare. In mixed leagues, such as the UEFA Champions League, the English Premier League, and many other leagues around the world, revenue sharing decreases social welfare.

Appendix

Proof of Proposition 1

Social welfare $W = \frac{3}{8}(mq_1 + q_2) = \frac{3}{8}(m(2w_1 - w_1^2) + (2w_2 - w_2^2))$ can be expressed in terms of w_1 only. Note that $q_2 = 2w_2 - w_2^2 = 2(1-w_1) - (1-w_1)^2 = 1 - w_1^2$. By plugging in, we compute $W = \frac{3}{8}(m(2w_1 - w_1^2) + (1 - w_1^2))$. Maximizing W with respect to w_1 yields the welfare optimal win percentage $w_1^* = \frac{m}{m+1}$. Due to the adding-up constraint, we derive $w_2^* = 1 - w_1^* = \frac{1}{m+1}$. This proves the proposition.

Proof of Lemma 1

Aggregate consumer surplus in a PM league is given by $CS^{PM} = \frac{1}{8}\left(mq_1^{PM} + q_2^{PM}\right)$, with $q_1^{PM} = 1 - \frac{1}{(1+\sqrt{m})^2}$ and $q_2^{PM} = \frac{1+2\sqrt{m}}{(1+\sqrt{m})^2}$. Addition of aggregate club profits and aggregate salary payments yields aggregate club revenues, given by $R_1^{PM} + R_2^{PM} = \frac{1}{4}\left(mq_1^{PM} + q_2^{PM}\right)$. Recall that salary payments do not directly influence social welfare because salaries merely represent a transfer from clubs to players. By summing up aggregate club revenues and aggregate consumer surplus, we derive $W^{PM} = \frac{3}{8}\left(mq_1^{PM} + q_2^{PM}\right) = \frac{3\left(1+\sqrt{m}(2+2m+m^{3/2})\right)}{8(1+\sqrt{m})^2}$.

Proof of Lemma 2

Similar to the PM league, we derive aggregate consumer surplus in a WM league as $CS^{WM} = \frac{1}{8}\left(mq_1^{WM} + q_2^{WM}\right)$, with $q_1^{WM} = \frac{3(2m-1)}{(1+m)^2}$ and $q_2^{WM} = \frac{3(2-m)m}{(1+m)^2}$. Aggregate club profits are zero because each club spends all its revenue on playing talent, i.e. $cx_i^{WM} = R_i^{WM}$. As a consequence, aggregate salary payments are derived as $cx_1^{WM} + cx_2^{WM} = R_1^{WM} + R_2^{WM} = \frac{1}{4}\left(mq_1^{WM} + q_2^{WM}\right)$. By summing up aggregate salary payments and aggregate consumer surplus, we derive $W^{WM} = \frac{3}{8}\left(mq_1^{WM} + q_2^{WM}\right) = \frac{9m}{8(1+m)}$.

Proof of Lemma 3

Similar to the PM and WM leagues, aggregate consumer surplus in a type-I mixed league is given by $CS^{TypeI} = \frac{1}{8}\left(mq_1^{TypeI} + q_2^{TypeI}\right)$, with $q_1^{TypeI} = \frac{\sqrt{m(16+m)}(4+m) - m(12+m)}{8}$ and $q_2^{TypeI} = \frac{8 - m(8+m-\sqrt{m(16+m)})}{8}$. Note that club 1 spends all its revenue on playing talent, i.e. $cx_1^{TypeI} = R_1^{TypeI} = \frac{m}{4}q_1^{TypeI}$, such that it makes zero profits. By summing up club 2's profit and salary payments, we derive club 2's revenue as $R_2^{TypeI} =$

$\frac{1}{4}q_2^{TypeI}$. Note again that salaries represent a transfer from club 2 to its players. Addition of aggregate consumer surplus CS^{TypeI}, club 1's salary payments $\frac{m}{4}q_1^{TypeI}$, and club 2's revenue $\frac{1}{4}q_2^{TypeI}$ produces social welfare as $W^{TypeI} = \frac{3}{8}\left(mq_1^{TypeI} + q_2^{TypeI}\right) = \frac{3\left(8-m(8+m(13+m)-\sqrt{m(16+m)}(5+m)\right)}{64}$.

Proof of Lemma 4

Similar to the proof for a type-I mixed league, we derive social welfare in a type-II mixed league as $W^{TypeII} = \frac{3}{8}\left(mq_1^{TypeII} + q_2^{TypeII}\right) = \frac{3\left(\sqrt{16m+1}-1+m(8m(m-1)+5\sqrt{16m+1}-13)\right)}{64m^2}$, with $q_1^{TypeII} = \frac{8m(m-1)-1+\sqrt{16m+1}}{8m^2}$ and $q_2^{TypeII} = \frac{\sqrt{16m+1}(4m+1)-(1+12m)}{8m^2}$.

Proof of Proposition 2

Competitive balance $CB^{\mu}(m)$ with $\mu \in \{PM, WM, TypeI, TypeII\}$, as a function of the market size parameter m, has the following properties:

(1) $CB^{\mu}(m)$ with $\mu \in \{PM, WM, TypeI, TypeII\}$ is a well-defined and continuous function in $(\underline{m}, \overline{m})$.

(2) $CB^{\mu}(m)$ is a strictly increasing function for $\mu = TypeII$ and a strictly decreasing function for $\mu \in \{PM, WM, TypeI\}$ in $(\underline{m}, \overline{m})$.

(3) $CB^{PM}(1) = CB^{WM}(1) = \frac{1}{4} > CB^{TypeI}(1) = CB^{TypeII}(1) = \frac{1}{8}(3\sqrt{17} - 11) \approx 0.17$ and $CB^{PM}(2) = 3\sqrt{2}-4 \approx 0.242 > CB^{TypeII}(2) = \frac{1}{32}(5\sqrt{33}-21) \approx 0.241 > CB^{WM}(2) = CB^{TypeI}(2) = 0$.

(4) $\nexists\ m \in (\underline{m}, \overline{m})$ such that $CB^{PM}(m) = CB^{\mu}(m)$ with $\mu \in \{WM, TypeI, TypeII\}$.

(5) $\nexists\ m \in (\underline{m}, \overline{m})$ such that $CB^{WM}(m) = CB^{\mu}(m)$ with $\mu \in \{PM, TypeI, TypeII\}$.

(6) $\exists!\ m \in (\underline{m}, \overline{m})$ such that $CB^{TypeII}(m^{CB}) = CB^{WM}(m^{CB})$ with $m^{CB} \approx 1.3128$.

From Properties 1-4, we derive that $CB^{PM}(m) > CB^{\mu}(m)\ \forall m \in (\underline{m}, \overline{m})$ with $\mu \in \{WM, TypeI, TypeII\}$. This proves claim (i).

From Properties 1-3 and 5, we derive that $CB^{TypeI}(m) < CB^{\mu}(m)\ \forall m \in (\underline{m}, \overline{m})$ with $\mu \in \{PM, WM, TypeII\}$. This proves claim (ii).

From Properties 1-3 and 6, we derive that $CB^{TypeII}(m) > CB^{WM}(m) \Leftrightarrow m > m^{CB} \approx 1.3128$. This proves claim (iii).

Proof of Proposition 3

(i) By comparing the welfare-maximizing win percentages $(w_1^*, w_2^*) = \left(\frac{m}{m+1}, \frac{1}{m+1}\right)$ with the equilibrium win percentages in the PM league $\left(w_1^{PM}, w_2^{PM}\right) = \left(\frac{\sqrt{m}}{\sqrt{m}+1}, \frac{1}{\sqrt{m}+1}\right)$ and the type-II mixed league $\left(w_1^{TypeII}, w_2^{TypeII}\right) = \left(\frac{1+4m-\sqrt{16m+1}}{4m}, \frac{\sqrt{16m+1}-1}{4m}\right)$, we derive

$$w_1^* > w_1^\mu \text{ and } w_2^* < w_2^\mu \ \forall m \in (\underline{m}, \overline{m}) \text{ with } \mu \in \{PM, TypeII\}.$$

Thus, from a welfare point of view the small club wins too often and the large club does not win often enough in both the PM league and the type-II mixed league.

(ii) By comparing the welfare-maximizing win percentages $(w_1^*, w_2^*) = \left(\frac{m}{m+1}, \frac{1}{m+1}\right)$ with the equilibrium win percentages in the WM league $\left(w_1^{WM}, w_2^{WM}\right) = \left(\frac{2m-1}{m+1}, \frac{2-m}{m+1}\right)$ and the type-I mixed league $\left(w_1^{TypeI}, w_2^{TypeI}\right) = \left(\frac{\sqrt{m(16+m)}-m}{4}, \frac{4+m-\sqrt{m(16+m)}}{4}\right)$, we derive

$$w_1^* < w_1^\mu \text{ and } w_2^* > w_2^\mu \ \forall m \in (\underline{m}, \overline{m}) \text{ with } \mu \in \{WM, TypeI\}.$$

Thus, from a welfare point of view the large club wins too often and the small club does not win often enough in both the WM league and the type-I mixed league.

Proof of Proposition 4

Analogous to the proof of Proposition 2, we derive that $W^\mu(m)$ with $\mu \in \{PM, WM, TypeI, TypeII\}$, as a function of the market size parameter m, has the following properties:

(1) $W^\mu(m)$ with $\mu \in \{PM, WM, TypeI, TypeII\}$ is a well-defined and continuous function $\forall(\underline{m}, \overline{m})$.

(2) $W^\mu(m)$ with $\mu \in \{PM, WM, TypeI, TypeII\}$ is a strictly increasing function $\forall(\underline{m}, \overline{m})$.

(3) $W^{PM}(1) = W^{WM}(1) = \frac{9}{16} > W^{TypeI}(1) = W^{TypeII}(1) = \frac{3}{32}(3\sqrt{17}-7) \approx 0.503$ and $W^{PM}(2) = 3\sqrt{2} - \frac{27}{8} \approx 0.868 > W^{TypeII}(2) = \frac{3}{256}(11\sqrt{33}+5) \approx 0.799 > CB^{WM}(2) = CB^{TypeI}(2) = \frac{3}{4}$.

(4) $\nexists\ m \in (\underline{m}, \overline{m})$ such that $W^{PM}(m) = W^\mu(m)$ with $\mu \in \{WM, TypeI, TypeII\}$.

(5) $\nexists\ m \in (\underline{m}, \overline{m})$ such that $W^{WM}(m) = W^\mu(m)$ with $\mu \in \{PM, TypeI, TypeII\}$.

(6) $\exists!\ m \in (\underline{m}, \overline{m})$ such that $W^{TypeII}(m^W) = W^{WM}(m^W)$ with $m^W \approx 1.72$.

From Properties 1-4, we derive that $W^{PM}(m) > W^\mu(m)\ \forall m \in (\underline{m}, \overline{m})$ with $\mu \in$

$\{WM, TypeI, TypeII\}$. This proves claim (i).

From Properties 1-3 and 5, we derive that $W^{TypeI}(m) < W^{\mu}(m)\ \forall m \in (\underline{m}, \overline{m})$ with $\mu \in \{PM, WM, TypeII\}$. This proves claim (ii).

From Properties 1-3 and 6, we derive that $W^{TypeII}(m) > W^{WM}(m) \Leftrightarrow m > m^W \approx 1.72$. This proves claim (iii).

Proof of Proposition 5

ad (i) The effect of revenue sharing on competitive balance in homogeneous leagues has been extensively analyzed in the literature: see e.g. Szymanski and Késenne (2004), Késenne (2005), Késenne (2006), Késenne (2007), and Dietl and Lang (2008). We omit the proof and refer to the articles quoted.

ad (ii) In a first step (a), we will show that revenue sharing decreases competitive balance in mixed leagues where clubs differ with respect to their objective function but are symmetric with respect to their market size, i.e. $m = 1$. In a second step (b), we will show numerically that this claim is true more generally for clubs with asymmetric market size.

(a) By normalizing the market size parameter m of club 1 to unity, the revenues of club i are given by $R_i = \frac{1}{4}q_i = \frac{1}{4}(2w_i - w_i^2)$ for $i = 1, 2$. The after-sharing revenues of club i, denoted by R_i^*, are then given by $R_i^* = \alpha R_i + (1-\alpha)R_j$ with $\alpha \in [\frac{1}{2}, 1], i, j = 1, 2$, $i \neq j$. Without loss of generality, we assume that club 1 is a win-maximizer and club 2 a profit-maximizer.

The maximization problem for the win-maximizing club 1 is given by $\max_{x_1 \geq 0} x_1$ subject to $cx_1 \leq R_1^*(x_1, x_2)$. Similar to section 3, the corresponding optimality condition yields

$$\frac{1}{x_1}\left(\alpha R_1 + (1-\alpha)R_2\right) = \frac{1}{x_1}\left(\alpha \frac{1}{4}\left(2w_1 - w_1^2\right) + (1-\alpha)\frac{1}{4}\left(1 - w_1^2\right)\right) = c. \tag{A1}$$

Note that $R_2 = \frac{1}{4}\left(2w_2 - w_2^2\right) = \frac{1}{4}\left(1 - w_1^2\right)$ due to the adding-up constraint $w_2 = 1 - w_1$.

The maximization problem for the profit-maximizing club 2 is given by $\max_{x_2 \geq 0} \pi_2 = R_2^*(x_1, x_2) - cx_2$. Similar to section 3, the corresponding optimality condition yields

$$\frac{\partial \pi_2}{\partial x_2} = \alpha \frac{\partial R_2}{\partial w_2}\frac{\partial w_2}{\partial x_2} + (1-\alpha)\frac{\partial R_1}{\partial w_1}\frac{\partial w_1}{\partial x_2} = \left(\alpha \frac{\partial R_2}{\partial w_2} - (1-\alpha)\frac{\partial R_1}{\partial w_1}\right)\frac{\partial w_2}{\partial x_2} = c, \tag{A2}$$

with $\frac{\partial w_1}{\partial x_2} = -\frac{\partial w_2}{\partial x_2}$ due to the adding-up constraint. With $\frac{\partial R_2}{\partial w_2} = \frac{1}{2}(1-w_2) = \frac{1}{2}w_1$, $\frac{\partial R_1}{\partial w_1} = \frac{1}{2}(1-w_1)$, and $\frac{\partial w_2}{\partial x_2} = \frac{x_1}{(x_1+x_2)^2} = w_1\frac{1}{x_1+x_2}$, Equation A2 can be simplified as follows

$$\frac{1}{2}\left(\alpha w_1 - (1-\alpha)(1-w_1)\right) w_1 \frac{1}{x_1+x_2} = c. \tag{A3}$$

Combining Equations A1 and A3, we compute:

$$\alpha\left(2w_1 - w_1^2\right) + (1-\alpha)\left(1-w_1^2\right) = 2\left(\alpha w_1 - (1-\alpha)(1-w_1)\right) w_1^2.$$

Rearranging yields $\frac{1}{4}\left(2w_1^3 + (2\alpha-1)w_1^2 - 2\alpha w_1 + (\alpha-1)\right) = 0$. This equation implicitly characterizes the equilibrium win percentage w_1^* of club 1 as a function of the revenue-sharing parameter α. Applying the implicit function theorem yields

$$\frac{\partial w_1^*}{\partial \alpha} = -\frac{2w_1(w_1-1)+1}{2(3w_1^2 + w_1(2\alpha-1) - \alpha)} < 0.$$

The numerator and the denominator are both positive since $\alpha \in [\frac{1}{2}, 1]$ and $w_1 > \frac{1}{2}$. Remember that club 1 is the win-maximizing club that invests more in equilibrium than the profit-maximizing club 2. As a consequence, a higher degree of revenue sharing (i.e. a lower value of α) increases the difference between the clubs' win percentages in equilibrium and thus produces a more unbalanced league.

(b) We provide a numerical simulation to show that revenue sharing decreases competitive balance in type-I and type-II mixed leagues. Table (1a) gives a numerical simulation for a type-I mixed league, where club 1 is the large, win-maximizing club and club 2 is the small, profit-maximizing club. Table (1b) provides a type-II mixed league where club 1 is the large, profit-maximizing club and club 2 is the small, win-maximizing club. We differentiate the case where the difference between both clubs in terms of market size is low ($m = 1.1$) and the case where the difference in terms of market size is high ($m = 1.9$). The first column shows the degree of revenue sharing where a higher value of α characterizes a league with a lower degree of revenue sharing. Note that $\alpha = 1$ denotes a league with no revenue sharing, whereas $\alpha = 1/2$ characterizes a league with full revenue sharing. The next columns characterize the investment level x_i and win percentage w_i of club $i = 1, 2$, as well as the level of competitive balance CB in equilibrium. Without loss of generality, we can normalize the marginal unit cost of talent c to unity.

Table 1: The effect of revenue sharing in mixed leagues

a) Type-I mixed league

	$m = 1.1$					$m = 1.9$				
	x_1	x_2	w_1	w_2	CB	x_1	x_2	w_1	w_2	CB
$\alpha = 1$	.265	.062	.809	.191	.154	.475	.008	.983	.017	.017
$\alpha = 0.8$	.229	.042	.845	.155	.131	.381	.005	.986	.014	.013
$\alpha = 0.6$	.185	.025	.881	.119	.104	.287	.003	.990	.010	.010
$\alpha = 0.5$	.160	.018	.900	.100	.090	.240	.002	.991	.009	.009

b) Type-II mixed league

	$m = 1.1$					$m = 1.9$				
	x_1	x_2	w_1	w_2	CB	x_1	x_2	w_1	w_2	CB
$\alpha = 1$	.077	.235	.247	.753	.186	.137	.211	.394	.606	.239
$\alpha = 0.8$	.053	.212	.201	.799	.160	.106	.231	.314	.686	.215
$\alpha = 0.6$	.032	.178	.154	.846	.130	.070	.222	.241	.759	.183
$\alpha = 0.5$	.023	.156	.130	.870	.113	.053	.207	.204	.796	.163

Notes: α revenue-sharing parameter, m market size parameter of club 1,
x_i investment level and w_i win percentages of club $i = 1, 2$ in equilibrium,
CB level of competitive balance in equilibrium

Table 1 shows that both types of club reduce their investment level in equilibrium through a higher degree of revenue sharing. The profit-maximizing club, however, reduces its investment level in a stronger way than the win-maximizing club in both types of mixed league. As a consequence, the win percentage of the profit-maximizing club decreases and the win percentage of the win-maximizing club increases. Since the profit-maximizing club always invests less than the win-maximizing club independent of the market size, competitive balance decreases and the (type-I and type-II) mixed league becomes more unbalanced through a higher degree of revenue sharing.

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