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Abstract

In this paper, we discuss a manager's allocation problem. Two managers allocate their heterogeneous employees - each manager allocates two high types and two low types - in groups of two in order to compete for an exogenous contest prize in a two period model. There are three possibilities of groups' constellation depending on manager's allocation decision: Strong groups (two high types), balanced groups (one high and one low type) and weak groups (two low types). These allocations determine the managers' performance. We show that equilibria in a simultaneous as well as in a sequential game only depend on the difference of the heterogeneous groups' outputs. Furthermore, we show that there is no second mover advantage according this model. Therefore, firms' performances are independent of the model's timing. A typical application of the model fits to coaches' decisions in ice hockey concerning the optimal constellation of the first, second, third (and so forth) lines.

1 Introduction

In contest models, agents invest in order to possibly obtain a contest prize. Dixit (1987), for example, states:

> "Many economic and social games are contests where the players expend effort to increase their probability of winning a given prize. Examples include: (i) inter-firm or international R&D rivalry for a profitable innovation; (ii) bribery to secure a lucrative license or contract from a government; and (iii) the Wimbledon final or the Superbowl. Economists have studied such games in many contexts." (p. 891)

As we can see in Dixit's statement, contest theory has been applied to a wide range of economic problems. Loury (1979), for instance, formulates a model in which firms invest into R&D under technological and market uncertainty in order to produce a new good. In this economy the first firm succeeding in the innovation process will be rewarded by a patent protection. Nevertheless, firms' chances of success depend on each other. Loury shows that a higher rivalry, i.e. more firms in the market, reduces firms' investments in equilibrium. Therefore, it takes a longer time until the innovation has been arrived. Dasgupta and Stiglitz (1980) and Mortensen (1982) provide similar R&D rivalry models. All of these models have important implications for the regulatory policy aimed at the improvement of social welfare in the corresponding industries because the theoretical understanding of innovation and market processes is very critical and may help to avoid regulatory mistakes.

Lazear and Rosen (1981) examine optimal payment schemes based on the rank order in a tournament. Employees are not paid based on just their individual efforts. Own efforts and a random component together determine the individual output. This individual output is then used for a ranking of the employee's performance. Firms offer different prizes for the different ranking positions.[1] Lazear and Rosen are able to show that a payment according to the rank position can result in an efficient behaviour of employees in case of risk neutral employees. They extend their model and allow for heterogenous and risk-averse workers. Then, it is possible that the efficient solution collapses.[2]

El-Hodiri and Quirk (1971) provide a dynamic n-team sports model in order to analyze whether the structure of the professional sports industry legitimates

[1] The idea behind this modelling is that it might be interesting for firms to offer a payment scheme not based on individual effort since individual effort is often difficult or costly to observe. In some industries, workers' relative output might be easier to determine.

[2] In case of asymmetric information, low quality workers may enter into firms consisting of high quality workers such that the efficient solution collapses.

an exemption of some antitrust bills. Clubs invest into human capital in each period in order to maximize expected discounted profits. The time horizon is infinite. Moreover, the model incorporates certain fundamental features of the North American sports industry such as the reserve clause, player drafts, the sale of player contracts among teams and gate revenue sharing. Gate revenue sharing indicates the split of gate revenues between the home team and the away team. Furthermore, El-Hodiri and Quirk assume a fixed supply of talents since in their model the total number of talents is exogenously given. In order to ensure that the model possesses a solution, El-Hodiri and Quirk assume that net sales of contracts by each team are zero. Then, El-Hodiri and Quirk are able to conclude that profit maximization by each team is generally not consistent with a time path of equal playing strengths for all teams. However, a time path with equal playing strength is feasible in case of equal revenue functions as long as the gate revenue sharing parameter exceeds 0.5.[3] El-Hodiri and Quirk provide a simple rule guaranteeing the convergence of team's playing strength even if the revenue functions differ among teams:

> "Assume that the supply of new playing skills is constant over time. If sales of player contracts for cash are forbidden, then, given any distribution of initial stocks of playing skills among teams, time paths of stocks of playing skills converge to one of equal playing strengths for all teams." (p. 1314)

Under this rule they are able to show that the speed of convergence of teams' playing strengths is higher the larger the depreciation rate of playing strength is. Due to this result, El-Hodiri and Quirk expect that the speed of convergence is higher in football than in baseball because careers usually last longer in baseball than in football.

In contest models, agents decide how much to invest in each period in order to maximize expected profits. However, one could also ask: Given agents' specific investments, what is the agent's optimal allocation of the resulting resources? The main goal of this paper is to provide an answer to this question.

In many sports disciplines, there are periods in which it is prohibited to buy or sell players (e.g. soccer, icehockey). Then, coaches have to decide how to allocate optimally the available resources. They specifically determine how to form their teams against each competitor. We often observe that coaches substitute players, change the team constellation or even change their tactics during a single game.

One of the fundamental characteristics in team sports is the heterogeneity of player's strength. We rarely observe teams in which all players are equally strong.

[3] A gate revenue sharing parameter of 0.7, for instance, indicates that the home team receives 70 percent and the away team 30 percent of gate revenues.

However, there are often better players but also weaker players in a team. This heterogeneity may have an impact on the "optimal" tactics for coaches. For instance, a coach of an icehockey team considers whether to form a relatively strong first line or rather to balance the first, second, third, ..., lines. Furthermore, this decision may not be independent of the opponent's team constellation. In icehockey, there is another important feature which should be kept in mind: The home team has the possibility to react on the away team's decision because the away team has to enter the ice first after each break during a match.

Therefore, it is interesting to analyze the determinants of optimal team constellation in order to provide answers for coaches' optimal tactical behaviour. We provide a model in which there are two types of agents representing the heterogeneity of players. Typically, icehockey teams consist of more than two lines and there are five players in each line. However, we simplify matters by considering just two lines each consisting of two players. Although a match normally takes sixty minutes and lines substitute each other after approximately one minute, we simplify matters and just analyze a two period model.

The above-mentioned icehockey example may be generalized. We claim that there are analogous decision problems in the private industry: Suppose that managers have to form two teams each consisting of two employees. Each team produces one product such that each firm produces two different products.[4] Depending on the team formation, the employees together produce superior, moderate or poor outputs. Finally, firm's resulting outputs compete with the outputs of the other firm on markets. Therefore, we henceforth use a more general notation. We call the agents of the model "employees" and "manager" of firms.

The paper has the following structure: In section 2, we present the assumptions and possible strategies for managers. Then, we determine Nash equilibria in case of simultaneous decisions of managers in section 3.1. In section 3.2, we consider subgame perfect equilibria in case of a sequential game. Finally, we present the conclusions in section 4. In the appendix, we extend the model by relaxing the assumption of two different types. Instead, we allow for four different types in firms. Furthermore, we use a different measurement of the winning probabilities in order to detect the effects of each specification.

[4] Note that the two products in this example correspond with the two periods in the icehockey example.

2 Model

2.1 Assumptions

Two firms compete for an exogenous prize. Each firm consists of four employees and one manager. The employees differ with respect to their productivity. On the one hand there is a high productive type called A and on the other hand there is a low productive type called B. Each firm consists of two A types and two B types. Hence, four employees are engaged within each firm. There are two periods. We assume that a specific employee is working just in one period. Afterwards, he is exhausted. Furthermore, we suppose that employees are working in a two-party group. The firm's manager decides in which constellation the employees work. For instance, he is able to assort the two high productive types A to a group which we denote by AA and the two low productive types B to a group which we denote by BB. Otherwise, the manager has the possibility to mix the high and low types to a group which we denote by AB (or equivalently BA). There are no costs in order to assemble the groups. For reasons of simplicity, we assume that the output of group AA is X, the output of group AB is Y, and the output of group BB is $Z > 0$. We suppose that group AA is more productive than group AB (or BA) such that $X > Y$. Moreover, group AB is more productive than group BB such that $Y > Z$. The timing of the model is as follows: The managers simultaneously or sequentially decide how to allocate the employees over the two periods.[5] Firm one wins the exogenous prize with probability $p^1(\cdot)$ which consists of two components incorporating the two periods:

$$p^1(s_{11}, s_{12}; s_{21}, s_{22}) = \frac{1}{2}\frac{e^{s_{11}}}{e^{s_{11}} + e^{s_{21}}} + \frac{1}{2}\frac{e^{s_{12}}}{e^{s_{12}} + e^{s_{22}}}$$

where s_{ij} indicates the group output of firm i in period j for $i = 1, 2$ and $j = 1, 2$. For instance, $s_{12} = X$ means that firm one plays AA in period two. Analogously, firm two has the following winning probability $p_2(\cdot)$:

$$p^2(s_{21}, s_{22}; s_{11}, s_{12}) = \frac{1}{2}\frac{e^{s_{21}}}{e^{s_{21}} + e^{s_{11}}} + \frac{1}{2}\frac{e^{s_{22}}}{e^{s_{22}} + e^{s_{12}}}$$

Note that this logistic contest success function is standard in contest models (cf. Hirshleifer (1989)). The special characteristic of the contest success function is the splitting into two periods each weighted by 0.5.

We assume that each manager tries to maximize expected profits. Since the prize is exogenous and there are no costs, managers maximize their winning probabilities.

[5] Otherwise, we can modify the assumption as follows: Both managers simultaneously or sequentially choose the employees for the first period. Obviously, the remaining employees are deployed automatically in the second period.

2.2 Strategies

Manager one has the following three possible strategies:

- Strategy (i): If he plays AA in the first period and BB in the second, then his winning probability is

$$p^1(X, Z; s_{21}, s_{22}) = \frac{1}{2}\frac{e^X}{e^X + e^{s_{21}}} + \frac{1}{2}\frac{e^Z}{e^Z + e^{s_{22}}}.$$

- Strategy (ii): If he plays BB in the first period and AA in the second, then his winning probability is

$$p^1(Z, X; s_{21}, s_{22}) = \frac{1}{2}\frac{e^Z}{e^Z + e^{s_{21}}} + \frac{1}{2}\frac{e^X}{e^X + e^{s_{22}}}.$$

- Strategy (iii): If he plays AB (*or* BA) in the first period and AB (*or* BA) in the second, then his winning probability is

$$p^1(Y, Y; s_{21}, s_{22}) = \frac{1}{2}\frac{e^Y}{e^Y + e^{s_{21}}} + \frac{1}{2}\frac{e^Y}{e^Y + e^{s_{22}}}.$$

Manager two has the following three possible strategies:

- Strategy (iv): If he plays AA in the first period and BB in the second, then his winning probability is

$$p^2(X, Z; s_{11}, s_{12}) = \frac{1}{2}\frac{e^X}{e^X + e^{s_{11}}} + \frac{1}{2}\frac{e^Z}{e^Z + e^{s_{12}}}.$$

- Strategy (v): If he plays BB in the first period and AA in the second, then his winning probability is

$$p^2(Z, X; s_{11}, s_{12}) = \frac{1}{2}\frac{e^Z}{e^Z + e^{s_{11}}} + \frac{1}{2}\frac{e^X}{e^X + e^{s_{12}}}.$$

- Strategy (vi): If he plays AB (*or* BA) in the first period and AB (*or* BA) in the second, then his winning probability is

$$p^2(Y, Y; s_{11}, s_{12}) = \frac{1}{2}\frac{e^Y}{e^Y + e^{s_{11}}} + \frac{1}{2}\frac{e^Y}{e^Y + e^{s_{12}}}.$$

A strategy combination is defined according to the following example: If manager one plays strategy (ii) and manager two plays strategy (vi), then the strategy combination is defined by (ii, vi).

3 Equilibrium

3.1 Simultaneous Game

In this section, we consider simultaneous decisions of managers. Thus, each manager determines the team constellation without knowing the rival's behaviour. Even if there are two periods, we do not have to use a dynamic game approach since decisions in period one determine the behaviour in period two. Therefore, the game is static with respect to decisions and we are able to use Nash strategies to determine the manager's rational behaviour. Using this procedure, we get proposition 1:

Proposition 1 *If $X-Y>Y-Z$ then strategy combinations $(i,iv),(i,v),(ii,iv)$ and (ii,v) constitute Nash equilibria. If $X-Y<Y-Z$ then the strategy combination (iii,vi) constitutes a Nash equilibrium. In case of $X-Y=Y-Z$ all strategy combinations are Nash equilibria. Independent of the parameter conditions, the winning probabilities equal 0.5 for each firm.*

Proof. The first part of the proof for proposition 1 is as follows: Suppose that manager one plays strategy (i) and manager two plays strategy (iv). It follows that both firms win the contest with probability 0.5 because

$$p^1(X,Z;X,Z)=p^2(X,Z;X,Z)=\frac{1}{2}\frac{e^X}{e^X+e^X}+\frac{1}{2}\frac{e^Z}{e^Z+e^Z}=\frac{1}{2}.$$

Do they have incentives to deviate? Suppose that manager one assumes that manager two plays the Nash-strategy (iv). Manager one has the possibilities to deviate by playing either strategies (ii) or (iii). Manager one has no incentive to deviate to strategy (ii) if and only if

$$\begin{aligned}\frac{1}{2} &\geqslant \frac{1}{2}\frac{e^Z}{e^Z+e^X}+\frac{1}{2}\frac{e^X}{e^X+e^Z}\\ \iff & 1\geqslant\frac{e^Z}{e^Z+e^X}+\frac{e^X}{e^X+e^Z}\\ \iff & 1\geqslant 1\end{aligned}$$

This condition is always satisfied. Thus, there is no incentive for manager one to deviate to strategy (ii). But, does manager one has incentives to deviate to strategy (iii)? There is no incentive if and only if

$$\begin{aligned}
\frac{1}{2} &\geqslant \frac{1}{2}\frac{e^Y}{e^Y+e^X}+\frac{1}{2}\frac{e^Y}{e^Y+e^Z}\\
&\iff 1 \geqslant \frac{e^Y(e^Y+e^Z)+e^Y(e^Y+e^X)}{(e^Y+e^X)(e^Y+e^Z)}\\
&\iff 1 \geqslant \frac{e^{2Y}+e^{Y+Z}+e^{2Y}+e^{X+Y}}{e^{2Y}+e^{Y+Z}+e^{X+Y}+e^{X+Z}}\\
&\iff e^{X+Z} \geqslant e^{2Y}\\
&\iff X+Z \geqslant 2Y\\
&\iff X-Y \geqslant Y-Z
\end{aligned}$$

Note that manager two has similar incentives to deviate as manager one. Therefore, we have shown that the strategy combination (i, iv) constitutes a Nash equilibrium if $X - Y \geqslant Y - Z$. It is easy to see that the strategy combinations $(i, v), (ii, iv)$ and (ii, v) are Nash equilibria, too, since both managers are indifferent between choosing strategy AA followed by BB or BB followed by AA as long as $X - Y \geqslant Y - Z$.[6] Next, we proof the second part of proposition 1: Under which condition is the strategy combination (iii, vi) a Nash equilibrium? In this case, both managers have a winning probability of 0.5. Suppose that manager two plays strategy (vi). Does manager one has an incentive to deviate by playing strategy (i) or (ii)? There is no incentive to choose strategy (i) or (ii) if and only if

$$\begin{aligned}
\frac{1}{2} &\geqslant \frac{1}{2}\frac{e^X}{e^X+e^Y}+\frac{1}{2}\frac{e^Z}{e^Z+e^Y}\\
&\iff 1 \geqslant \frac{e^X}{e^X+e^Y}+\frac{e^Z}{e^Z+e^Y}\\
&\iff 1 \geqslant \frac{e^X(e^Z+e^Y)+e^Z(e^X+e^Y)}{(e^X+e^Y)(e^Z+e^Y)}\\
&\iff 1 \geqslant \frac{e^{X+Z}+e^{Y+X}+e^{X+Z}+e^{Y+Z}}{e^{X+Z}+e^{X+Y}+e^{Y+Z}+e^{2Y}}\\
&\iff e^{2Y} \geqslant e^{X+Z}\\
&\iff 2Y \geqslant X+Z\\
&\iff Y-Z \geqslant X-Y
\end{aligned}$$

Note that manager two has similar incentives for deviation as manager one. Thus, the strategy combination (iii, vi) constitutes a Nash equilibrium if $Y - Z \geqslant$

[6] It directly follows that the aforementioned strategy combinations determine Nash equilibria if $X - Y > Y - Z$.

$X-Y$.[7] In case of $Y-Z=X-Y$, it directly follows from the previous proof that strategy combinations $(i,iv),(i,v),(ii,iv)$ (ii,v) and (iii,vi) are Nash-equilibria. Furthermore, we next show that strategy combinations $(i,vi),(ii,vi),(iii,iv)$ and (iii,v) represent Nash equilibria if $Y-Z=X-Y$ such that all strategy combinations are possible. Obviously, we have to analyze just one combination of $(i,vi),(ii,vi),(iii,iv)$ and (iii,v) since incentives are symmetrical. The strategy combination (i,vi) is a Nash equilibrium in case of $Y-Z=X-Y$ since

$$\begin{aligned}
p^1(X,Z;Y,Y) &= \frac{1}{2}\frac{e^X}{e^X+e^Y}+\frac{1}{2}\frac{e^Z}{e^Z+e^Y} \\
&= \frac{1}{2}\frac{e^Z}{e^Z+e^Y}+\frac{1}{2}\frac{e^X}{e^X+e^Y} = \underbrace{p^1(Z,X;Y,Y)}_{(deviation\ of\ manager\ one)} \\
&= \frac{1}{2}\frac{e^Y}{e^Y+e^Y}+\frac{1}{2}\frac{e^Y}{e^Y+e^Y} = \underbrace{p^1(Y,Y;Y,Y)}_{(deviation\ of\ manager\ one)} \\
&= \frac{1}{2} \\
p^2(Y,Y;X,Z) &= \frac{1}{2}\frac{e^Y}{e^Y+e^X}+\frac{1}{2}\frac{e^Y}{e^Y+e^Z} \\
&= \frac{1}{2}\frac{e^Z}{e^Z+e^X}+\frac{1}{2}\frac{e^X}{e^X+e^Z} = \underbrace{p^2(Z,X;X,Z)}_{(deviation\ of\ manager\ two)} \\
&= \frac{1}{2}\frac{e^X}{e^X+e^X}+\frac{1}{2}\frac{e^Z}{e^Z+e^Z} = \underbrace{p^2(X,Z;X,Z)}_{(deviation\ of\ manager\ two)} \\
&= \frac{1}{2}.
\end{aligned}$$

According to the above arguments, we get proposition 1. ■

It is interesting to see that only the difference between $X-Y$ and $Y-Z$ is decisive for the equilibrium. The intuition for this result is as follows: Managers would deviate from equilibrium if the advantage of one period is higher than the disadvantage of the other period. Suppose that $X-Y \geqslant Y-Z$ and the equilibrium strategy combinations is (i,iv). Manager one assumes that manager two plays strategy (iv). Then, manager one has no incentive to deviate from strategy (i) to (ii) since marginal benefits in one period is exactly compensated by marginal losses in the other period. Furthermore, manager one does not

[7] It directly follows that the strategy combination (iii,vi) constitutes a Nash equilibrium if $Y-Z>X-Y$.

deviate to strategy (iii) because marginal losses (which depend on $X-Y$) would be at least as high as marginal benefits (which depend on $Y-Z$) due to the fact that $X-Y \geqslant Y-Z$. On the other hand, a strategy combination (iii, vi) constitutes a Nash equilibrium if $X-Y \leqslant Y-Z$. No manager has incentives to deviate to another strategy since marginal benefits (which depend on $X-Y$) are equal or smaller than marginal losses (which depend on $Y-Z$) due to the fact that $X-Y \leqslant Y-Z$.

3.2 Sequential Game

In the last section we have considered managers' optimal allocation in case of simultaneous choices. However, a sequential choice may be appropriate regarding icehockey since the home team has the possibility to react on the opponent's choice. Therefore, we solve the model taking into account that one manager moves first and the other manager reacts. Now, the game is dynamic and backward induction is adequate in order to solve the model. We assume that manager two moves first, and manager one reacts. Manager two anticipates manager one's optimal behaviour and adjusts its decision. We guess that manager one's situation is at least as good as in the simultaneous game since he can react on manager two's decision and could always imitate manager two's decision. We get the following proposition using backward induction:

Proposition 2 *If manager two moves first, then manager one has no second mover advantage. The winning probability equals 0.5 for both managers in a subgame perfect equilibrium. If* $Y-Z > X-Y$*, then manager two optimally plays strategy* (vi)*, manager one's optimal answer is strategy* (iii)*. Thus, the strategy combination* (iii, vi) *constitutes a subgame perfect Nash equilibrium. If* $Y-Z < X-Y$*, then manager two optimally plays strategy* (iv) *or* (v)*, manager one's optimal answer is strategy* (i) *or* (ii)*. Thus, strategy combinations* $(i, iv), (i, v), (ii, iv)$ *and* (ii, v) *constitute subgame perfect Nash equilibria. If* $Y-Z = X-Y$*, then all strategy combinations constitute a subgame perfect Nash equilibrium.*

Proof. What is manager one's optimal reaction to a given action of manager two? Suppose manager two plays (AA, BB) or (BB, AA). If manager one's answer is (AA, BB) or (BB, AA), then both managers' winning probability is equal to 0.5. If manager one's answer is (AB, AB), then manager one's winning probability is

$$\begin{aligned} p^1(Y, Y; X, Z) &= \frac{1}{2}\frac{e^Y}{e^Y + e^X} + \frac{1}{2}\frac{e^Y}{e^Y + e^Z} \\ &= \frac{1}{2}\frac{e^{2Y} + e^{Y+Z} + e^{2Y} + e^{Y+X}}{e^{2Y} + e^{Y+Z} + e^{X+Y} + e^{X+Z}} \end{aligned}$$

Thus, manager one plays (AB, AB) if

$$\begin{aligned} e^{2Y} &> e^{X+Z} \\ &\iff 2Y > X + Z \\ &\iff Y - Z > X - Y \end{aligned}$$

such that $p^1(Y, Y; X, Z) > 0.5$ and therefore $p^2(X, Z; Y, Y) < 0.5$. If $Y - Z < X - Y$, then manager one's optimal answer is (AA, BB) or (BB, AA) such that $p^1(\cdot) = p^2(\cdot)$. In case of $Y - Z = X - Y$, manager one is indifferent between his strategies. Otherwise, suppose that manager two plays (AB, AB). If manager one's answer is (AB, AB), then both managers have an equal winning probability of 0.5. If manager one plays (AA, BB) or (BB, AA), then manager one's winning probability is

$$\begin{aligned} p^1(X, Z; Y, Y) &= \frac{1}{2}\frac{e^X}{e^X + e^Y} + \frac{1}{2}\frac{e^Z}{e^Z + e^Y} \\ &= \frac{1}{2}\frac{e^{X+Z} + e^{X+Y} + e^{Z+X} + e^{Z+Y}}{e^{X+Z} + e^{X+Y} + e^{Y+Z} + e^{2Y}} \end{aligned}$$

Thus, manager one plays (AA, BB) or (BB, AA) if

$$\begin{aligned} e^{Z+X} &> e^{2Y} \\ Z + X &> 2Y \\ X - Y &> Y - Z \end{aligned}$$

such that $p^1(X, Z; Y, Y) > 0.5$ and $p^2(Y, Y; X, Z) < 0.5$. If $X - Y < Y - Z$, then manager one's optimal answer is (AB, AB) such that $p^1(\cdot) = p^2(\cdot)$. In case of $Y - Z = X - Y$, manager one is indifferent between his strategies. It is obvious that manager one achieves at least a winning probability of 0.5 by simulating manager two's action in both periods. However, manager one can possibly do better by choosing a different strategy. Note that manager two anticipates manager one's optimal behaviour. Thus, manager two optimally plays (AA, BB) or (BB, AA) if $Y - Z < X - Y$. In this case, manager one's optimal answer is (AA, BB) or (BB, AA) such that $p^1(\cdot) = p^2(\cdot)$. Moreover, manager two optimally plays (AB, AB) if $Y - Z > X - Y$. In this case, manager one's optimal answer is (AB, AB) such that $p^1(\cdot) = p^2(\cdot)$. If $Y - Z = X - Y$, then all strategy combinations constitute a subgame perfect Nash equilibrium. According to these arguments, we get proposition 2. ■

Proposition 2 shows that there is no second mover advantage for manager one since both managers have identical winning probabilities in equilibrium. Manager two anticipates manager one's optimal reaction and adequately adjusts his first move.

4 Conclusion

In this paper, we discussed a manager's allocation problem. Two managers allocate their heterogeneous employees - each manager allocates two high types and two low types - in groups of two in order to compete for an exogenous contest prize in a two period model. There are three possible group constellations within each firm depending on the manager's allocation decision: Strong groups (two high types), balanced groups (one high and one low type) and weak groups (two low types). The allocation determines the manager's winning probability. We show that equilibria in a simultaneous as well as in a sequential game depend on the difference of the groups' outputs. If the difference between the strong group's and balanced group's output is higher than the difference between the balanced group's and weak group's output, then each group is composed of identical types independent of the model's timing (i.e. simultaneous or sequential game). Otherwise, groups consist of different types, if the difference between the strong group's and balanced group's output is smaller than the difference between the balanced group's and weak group's output independent of the model's timing. Furthermore, we show that there is no second mover advantage according to this model. Therefore, the firms' winning probability is independent of the model's timing. Applying this result on coaches' optimal line constellation in icehockey, there is theoretically no home game advantage under the assumption that the heterogeneity is identical in both competing teams. Even if the home team can react on the opponent's decision, there is no advantage in equilibrium. Of course, a team may reap the benefits of a home match because the fans typically cheer more for the home team.

5 Appendix

In this appendix, we relax the assumption of two different types. We allow for four different types in order to generalize the model. Furthermore, we use a different measurement of the winning probability in order to detect the effects of each specification.

5.1 Notation

Manager one has two (high) A-types and two (low) B-types. Manager two has two (high) C-types and two (low) D-types. AA means that manager one combines the two types A together as an input. The resulting output is denoted by a. Table 1 summarizes the notation.

Input	$\longrightarrow$	*Output*
AA	$\longrightarrow$	a
BB	$\longrightarrow$	b
AB	$\longrightarrow$	ϕ
CC	$\longrightarrow$	c
DD	$\longrightarrow$	d
CD	$\longrightarrow$	γ

Table 1: Notation

Henceforth, (AA, BB) means that manager one plays AA in $t = 1$ and BB in $t = 2$. (CD, CD) means that manager two plays CD in $t = 1$ and CD in $t = 2$, and so forth.

5.2 Assumptions

We use the following assumptions regarding output:

$$\begin{array}{ccccccc} a & > & \phi & > & b & > & 0 \\ c & > & \gamma & > & d & > & 0 \end{array}$$

Thus, a combination of the high types AA (CC) results in higher output than a composition of the different types AB (CD). Therefore, $a > \phi$ ($c > \gamma$). Furthermore, a composition AB (CD) yields a higher output than a combination of two low types BB (DD) such that $\phi > b$ ($\gamma > d$). Moreover, we simply assume that $a \gtreqless c$, $\phi \gtreqless \gamma$, $b \gtreqless d$ comparing manager one's with manager two's output. Different to the last section, we use the logit contest success function in order to model the competition.[8] Hence, firm one wins the exogenous prize with probability $p^1(\cdot)$ which consists of two components incorporating the two periods:

$$p^1(s_{11}, s_{12}; s_{21}, s_{22}) = \frac{1}{2}\frac{s_{11}}{s_{11}+s_{21}} + \frac{1}{2}\frac{s_{12}}{s_{12}+s_{22}}$$

where s_{ij} indicates the group output of firm i in period j for $i = 1, 2$ and $j = 1, 2$. For instance, $s_{12} = b$ means that firm one plays BB in period two. Analogously, firm two has the following winning probability $p_2(\cdot)$:

$$p^2(s_{21}, s_{22}; s_{11}, s_{12}) = \frac{1}{2}\frac{s_{21}}{s_{21}+s_{11}} + \frac{1}{2}\frac{s_{22}}{s_{22}+s_{12}}$$

We will identify and point out the similarities to the logistic contest success function later.

5.3 Strategies

Manager one has the following three possible strategies:

- Strategy (i): If he plays AA in the first period and BB in the second, then his winning probability is

$$p^1(a, b; s_{21}, s_{22}) = \frac{1}{2}\frac{a}{a+s_{21}} + \frac{1}{2}\frac{b}{b+s_{22}}.$$

- Strategy (ii): If he plays BB in the first period and AA in the second, then his winning probability is

$$p^1(b, a; s_{21}, s_{22}) = \frac{1}{2}\frac{b}{b+s_{21}} + \frac{1}{2}\frac{a}{a+s_{22}}.$$

- Strategy (iii): If he plays AB (*or* BA) in the first period and AB (*or* BA) in the second, then his winning probability is

$$p^1(\phi, \phi; s_{21}, s_{22}) = \frac{1}{2}\frac{\phi}{\phi+s_{21}} + \frac{1}{2}\frac{\phi}{\phi+s_{22}}.$$

  Manager two has the following three possible strategies:

- Strategy (iv): If he plays CC in the first period and DD in the second, then his winning probability is

$$p^2(c, d; s_{11}, s_{12}) = \frac{1}{2}\frac{c}{c+s_{11}} + \frac{1}{2}\frac{d}{d+s_{12}}.$$

[8] Note that this function was introduced by Tullock (1980).

- Strategy (v): If he plays DD in the first period and CC in the second, then his winning probability is
$$p^2(d, c; s_{11}, s_{12}) = \frac{1}{2}\frac{d}{d + s_{11}} + \frac{1}{2}\frac{c}{c + s_{12}}.$$
- Strategy (vi): If he plays CD $(or\ DC)$ in the first period and CD $(or\ DC)$ in the second, then his winning probability is
$$p^2(\gamma, \gamma; s_{11}, s_{12}) = \frac{1}{2}\frac{\gamma}{\gamma + s_{11}} + \frac{1}{2}\frac{\gamma}{\gamma + s_{12}}.$$

A strategy combination is defined as follows: If manager one plays strategy (ii) and manager two plays strategy (vi), then the strategy combination is defined by (ii, vi).

A strategy combination is called homogeneous if the different types are not mixed within a group. Therefore, strategy combinations consisting of AA, BB, CC and DD are called homogeneous strategy combinations. However, the strategy combination (iii, vi) is heterogeneous. We call a strategy combination semi-heterogeneous if one manager plays a homogeneous and the other a heterogeneous strategy.

5.4 Simultaneous Equilibrium

In the next sections, we consider simultaneous equilibria. Thus, both manager simultaneously determine the allocation of their types. First, we analyze homogeneous strategy combinations. Afterwards, we consider heterogeneous and then semi-heterogeneous strategy combinations.

5.4.1 Homogeneous Strategy Combinations

Under which conditions constitute (AA, BB) and (CC, DD) a Nash equilibrium? Manager one has no incentives to deviate if the following two inequalities are fulfilled:

$$\begin{aligned} \frac{1}{2}\frac{a}{a+c} + \frac{1}{2}\frac{b}{b+d} &\geqslant \frac{1}{2}\frac{\phi}{\phi+c} + \frac{1}{2}\frac{\phi}{\phi+d} \\ &\Longleftrightarrow \\ \frac{a}{a+c} + \frac{b}{b+d} &\geqslant \frac{\phi}{\phi+c} + \frac{\phi}{\phi+d} \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\frac{a}{a+c} + \frac{1}{2}\frac{b}{b+d} &\geqslant \frac{1}{2}\frac{b}{b+c} + \frac{1}{2}\frac{a}{a+d} \\ &\Longleftrightarrow \\ ab &\geqslant cd \end{aligned}$$

Manager two has no incentives to deviate if the following two inequalities are fulfilled:

$$\begin{aligned} \frac{1}{2}\frac{c}{c+a}+\frac{1}{2}\frac{d}{d+b} &\geqslant \frac{1}{2}\frac{\gamma}{\gamma+a}+\frac{1}{2}\frac{\gamma}{\gamma+b} \\ &\Longleftrightarrow \\ \frac{c}{c+a}+\frac{d}{d+b} &\geqslant \frac{\gamma}{\gamma+a}+\frac{\gamma}{\gamma+b} \end{aligned}$$

$$\begin{aligned} \frac{1}{2}\frac{c}{c+a}+\frac{1}{2}\frac{d}{d+b} &\geqslant \frac{1}{2}\frac{d}{d+a}+\frac{1}{2}\frac{c}{c+b} \\ &\Longleftrightarrow \\ cd &\geqslant ab \end{aligned}$$

These four inequalities can be summarized by the following three inequalities:

$$\frac{a}{a+c}+\frac{b}{b+d} \geqslant \frac{\phi}{\phi+c}+\frac{\phi}{\phi+d} \tag{1}$$

$$\frac{c}{c+a}+\frac{d}{d+b} \geqslant \frac{\gamma}{\gamma+a}+\frac{\gamma}{\gamma+b} \tag{2}$$

$$ab = cd \tag{3}$$

Combining equation (1) with (3) we get

$$cd \geqslant \phi^2 \tag{4}$$

Combining equation (2) with (3) we get

$$ab \geqslant \gamma^2 \tag{5}$$

Combining equations (3), (4) and (5) we need

$$ab = cd \geqslant \max\{\phi^2, \gamma^2\} \tag{6}$$

for (AA, BB) and (CC, DD) to define Nash strategies. Note that the condition $ab = cd$ has to be fulfilled in order to satisfy that no manager has incentives to play the inverse strategy (BB, AA) or (DD, CC), respectively. The opportunity to play strategies (AB, AB) or (CD, CD) are not attractive as long as ϕ^2 and γ^2 are smaller than ab. It is easy to see that under condition (6) there are other possible equilibria: (AA, BB) and (DD, CC), (BB, AA) and (CC, DD),

(BB, AA) and (DD, CC). We briefly show that (AA, BB) and (DD, CC) constitute a Nash equilibrium under condition (6).[9] Manager one does not deviate from this equilibrium strategy to either (AB, AB) or (BB, AA) if

$$\begin{aligned} \frac{1}{2}\frac{a}{a+d}+\frac{1}{2}\frac{b}{b+c} \quad &\geqslant \quad \frac{1}{2}\frac{\phi}{\phi+d}+\frac{1}{2}\frac{\phi}{\phi+c} \\ &\iff \quad \frac{a}{a+d}+\frac{b}{b+c}\geqslant\frac{\phi}{\phi+d}+\frac{\phi}{\phi+c} \end{aligned} \tag{7}$$

and

$$\begin{aligned} \frac{1}{2}\frac{a}{a+d}+\frac{1}{2}\frac{b}{b+c} \quad &\geqslant \quad \frac{1}{2}\frac{b}{b+d}+\frac{1}{2}\frac{a}{a+c} \\ &\iff \quad cd\geqslant ab. \end{aligned} \tag{8}$$

Manager two does not deviate from equilibrium strategy to either (CD, CD) or (CC, DD) iff

$$\begin{aligned} \frac{1}{2}\frac{d}{d+a}+\frac{1}{2}\frac{c}{c+b} \quad &\geqslant \quad \frac{1}{2}\frac{\gamma}{\gamma+a}+\frac{1}{2}\frac{\gamma}{\gamma+b} \\ &\iff \quad \frac{d}{d+a}+\frac{c}{c+b}\geqslant\frac{\gamma}{\gamma+a}+\frac{\gamma}{\gamma+b} \end{aligned} \tag{9}$$

and

$$\begin{aligned} \frac{1}{2}\frac{d}{d+a}+\frac{1}{2}\frac{c}{c+b} \quad &\geqslant \quad \frac{1}{2}\frac{c}{c+a}+\frac{1}{2}\frac{d}{d+b} \\ &\iff \quad ab\geqslant cd. \end{aligned} \tag{10}$$

Combining equations (7),(8),(9) and (10) we are able to reduce the four conditions to the following condition, once again:

$$ab = cd \geqslant \max\ \{\phi^2, \gamma^2\}$$

Thus, we conclude that these equilibria with homogeneous strategies balance on the knife's edge condition $ab = cd$. If $ab \neq cd$, at least one manager has incentives to deviate from the corresponding strategy. Table 2 sums up the four homogeneous equilibria.[10]

[9] However, we omit the proofs for the other equilibria since it directly follows by reason of symmetry.

[10] Note that the different lines in the table constitute Nash Equilibria. The condition in the first column must hold in each equilibrium.

Condition	Manager one's Nash Strategy	Manager two's Nash Strategy
$ab = cd \geqslant \max\{\phi^2, \gamma^2\}$	AA, BB	CC, DD
	AA, BB	DD, CC
	BB, AA	CC, DD
	BB, AA	DD, CC

Table 2: Homogeneous Equilibria

In the last section, we used the logistic model and assumed that there exist just two types. The above solutions with four types coincide to the two type results, if we assume that $ab = cd$ and $\phi = \gamma$. In case of the logistic model, we showed that differences were crucial. However, the logit model implicates that ratios are decisive for equilibria. In order to see this, use $ab = cd$ and $\phi = \gamma$ in the homogeneous equilibrium condition $ab = cd \geqslant \max\{\phi^2, \gamma^2\}$. Then, it is obvious that this condition reduces to

$$\frac{a}{\phi} \geqslant \frac{\phi}{b}.$$

The ratio of the two high type's output (a) to the output of one low and one high type (ϕ) has to be larger than the ratio of the output of one low and one high type (ϕ) to the output of two low types (b). If ϕ is relatively low, then no manager has incentives to deviate to heterogeneous strategies.

5.4.2 Heterogeneous Strategy Combinations

Now, we analyze heterogeneous strategies. Under which conditions constitute (AB, AB) and (CD, CD) a Nash equilibrium? Manager one has no incentives to deviate if the following inequality is fulfilled:

$$\begin{aligned} \frac{1}{2}\frac{\phi}{\phi+\gamma} + \frac{1}{2}\frac{\phi}{\phi+\gamma} &\geqslant \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma} \\ &\Longleftrightarrow \\ \frac{\phi}{\phi+\gamma} &\geqslant \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma} \end{aligned}$$

Manager two has no incentives to deviate if the following inequality is fulfilled:

$$\begin{aligned} \frac{1}{2}\frac{\gamma}{\phi+\gamma} + \frac{1}{2}\frac{\gamma}{\phi+\gamma} &\geqslant \frac{1}{2}\frac{c}{c+\phi} + \frac{1}{2}\frac{d}{d+\phi} \\ &\Longleftrightarrow \\ \frac{\gamma}{\phi+\gamma} &\geqslant \frac{1}{2}\frac{c}{c+\phi} + \frac{1}{2}\frac{d}{d+\phi} \end{aligned}$$

We sum up the results in case of a heterogeneous strategy combination in table 3.[11]

Conditions	Manager one's Nash Strategy	Manager two's Nash Strategy
$\frac{\phi}{\phi+\gamma} \geqslant \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma}$ $\frac{\gamma}{\gamma+\phi} \geqslant \frac{1}{2}\frac{c}{c+\phi} + \frac{1}{2}\frac{d}{d+\phi}$	AB, AB	CD, CD

Table 3: Heterogeneous Equilibrium

5.4.3 Semi-Heterogeneous Strategy Combinations

In case of semi-heterogeneous strategy combinations, one manager plays homogeneous and the other heterogeneous strategies. The results mainly depend on the condition $ab \gtreqless cd$. We just explain the following example: Equilibrium (AA, BB) and (CD, CD) with $ab = cd$:

Manager one does not deviate from equilibrium strategy to either (AB, AB) or (BB, AA) if

$$\begin{aligned} \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma} \quad &\geqslant \quad \frac{1}{2}\frac{\phi}{\phi+\gamma} + \frac{1}{2}\frac{\phi}{\phi+\gamma} \\ &\Longleftrightarrow \quad \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma} \geqslant \frac{\phi}{\phi+\gamma} \end{aligned} \tag{11}$$

and

$$\begin{aligned} \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma} \quad &\geqslant \quad \frac{1}{2}\frac{b}{b+\gamma} + \frac{1}{2}\frac{a}{a+\gamma} \\ &\Longleftrightarrow \quad 1 \geqslant 1 \end{aligned} \tag{12}$$

Manager two does not deviate from equilibrium strategy to either (CC, DD) or (DD, CC) if

$$\begin{aligned} \frac{1}{2}\frac{\gamma}{\gamma+a} + \frac{1}{2}\frac{\gamma}{\gamma+b} \quad &\geqslant \quad \frac{1}{2}\frac{c}{c+a} + \frac{1}{2}\frac{d}{d+b} \\ &\Longleftrightarrow \quad \frac{\gamma}{\gamma+a} + \frac{\gamma}{\gamma+b} \geqslant \frac{c}{c+a} + \frac{d}{d+b} \end{aligned} \tag{13}$$

[11] Note that the indicated heterogeneous equilibrium exists if both conditions in the first column hold.

and

$$\begin{aligned} \frac{1}{2}\frac{\gamma}{\gamma+a}+\frac{1}{2}\frac{\gamma}{\gamma+b} &\geqslant \frac{1}{2}\frac{d}{d+a}+\frac{1}{2}\frac{c}{c+b} \\ \iff & \frac{\gamma}{\gamma+a}+\frac{\gamma}{\gamma+b} \geqslant \frac{d}{d+a}+\frac{c}{c+b} \end{aligned} \tag{14}$$

Note that inequality (11) must hold, inequality (12) always holds and inequalities (13) and (14) together imply that $\gamma^2 \geqslant ab = cd$. Therefore, (AA, BB) and (CD, CD) is an equilibrium if $\gamma^2 \geqslant ab = cd$ and $\frac{1}{2}\frac{a}{a+\gamma}+\frac{1}{2}\frac{b}{b+\gamma} \geqslant \frac{\phi}{\phi+\gamma}$.

Tables 4-6 sum up all possible results in case of semi-heterogeneous strategy combinations:[12]

Conditions ($ab = cd$)	Manager one	Manager two
$\gamma^2 \geqslant ab = cd$	AA, BB	CD, CD
$\frac{1}{2}\frac{a}{a+\gamma}+\frac{1}{2}\frac{b}{b+\gamma} \geqslant \frac{\phi}{\phi+\gamma}$	BB, AA	CD, CD
$\phi^2 \geqslant cd = ab$	AB, AB	CC, DD
$\frac{1}{2}\frac{c}{c+\phi}+\frac{1}{2}\frac{d}{d+\phi} \geqslant \frac{\gamma}{\gamma+\phi}$	AB, AB	DD, CC

Table 4: Semi-Heterogeneous Equilibria with ab=cd

Conditions ($ab > cd$)	Manager one	Manager two
$\frac{1}{2}\frac{a}{a+\gamma}+\frac{1}{2}\frac{b}{b+\gamma} \geqslant \frac{\phi}{\phi+\gamma}$	AA, BB	CD, CD
$\frac{\gamma}{\gamma+a}+\frac{\gamma}{\gamma+b} \geqslant \frac{c}{c+b}+\frac{d}{d+a}$	BB, AA	CD, CD
$\frac{1}{2}\frac{c}{c+\phi}+\frac{1}{2}\frac{d}{d+\phi} \geqslant \frac{\gamma}{\gamma+\phi}$	AB, AB	CC, DD
$\frac{\phi}{\phi+c}+\frac{\phi}{\phi+d} \geqslant \frac{a}{a+c}+\frac{b}{b+d}$	AB, AB	DD, CC

Table 5: Semi-Heterogeneous Equilibria with ab>cd

Conditions ($ab < cd$)	Manager one	Manager two
$\frac{1}{2}\frac{a}{a+\gamma}+\frac{1}{2}\frac{b}{b+\gamma} \geqslant \frac{\phi}{\phi+\gamma}$	AA, BB	CD, CD
$\frac{\gamma}{\gamma+a}+\frac{\gamma}{\gamma+b} \geqslant \frac{c}{c+a}+\frac{d}{d+b}$	BB, AA	CD, CD
$\frac{1}{2}\frac{c}{c+\phi}+\frac{1}{2}\frac{d}{d+\phi} \geqslant \frac{\gamma}{\gamma+\phi}$	AB, AB	CC, DD
$\frac{\phi}{\phi+c}+\frac{\phi}{\phi+d} \geqslant \frac{b}{b+c}+\frac{a}{a+d}$	AB, AB	DD, CC

Table 6: Semi-Heterogeneous Equilibria with ab<cd

[12] We differentiate between the following three cases: $ab = cd, ab > cd$ and $ab < cd$. Note that the previous example is represented in the first line of table 4.

5.4.4 Summary of Equilibria

In table 7-9, we present all equilibria in another version, once again. We simply distinct the following three cases (i) $ab = cd$, (ii) $ab > cd$ and (iii) $ab < cd$ and we do not differentiate between homogeneous, heterogeneous and semi-heterogeneous equilibria.

Conditions	Manager one	Manager two
$ab = cd \geqslant \max\{\phi^2, \gamma^2\}$	AA, BB	CC, DD
	AA, BB	DD, CC
	BB, AA	CC, DD
	BB, AA	DD, CC
$\gamma^2 \geqslant ab = cd$	AA, BB	CD, CD
$\frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma} \geqslant \frac{\phi}{\phi+\gamma}$	BB, AA	CD, CD
$\phi^2 \geqslant cd = ab$	AB, AB	CC, DD
$\frac{1}{2}\frac{c}{c+\phi} + \frac{1}{2}\frac{d}{d+\phi} \geqslant \frac{\gamma}{\gamma+\phi}$	AB, AB	DD, CC
$\frac{\phi}{\phi+\gamma} \geqslant \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma}$	AB, AB	CD, CD
$\frac{\gamma}{\gamma+\phi} \geqslant \frac{1}{2}\frac{c}{c+\phi} + \frac{1}{2}\frac{d}{d+\phi}$		

Table 7: Equilibria with ab=cd

Conditions	Manager one	Manager two
$\frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma} \geqslant \frac{\phi}{\phi+\gamma}$	AA, BB	CD, CD
$\frac{\gamma}{\gamma+a} + \frac{\gamma}{\gamma+b} \geqslant \frac{c}{c+b} + \frac{d}{d+a}$	BB, AA	CD, CD
$\frac{1}{2}\frac{c}{c+\phi} + \frac{1}{2}\frac{d}{d+\phi} \geqslant \frac{\gamma}{\gamma+\phi}$	AB, AB	CC, DD
$\frac{\phi}{\phi+c} + \frac{\phi}{\phi+d} \geqslant \frac{a}{a+c} + \frac{b}{b+d}$	AB, AB	DD, CC
$\frac{\phi}{\phi+\gamma} \geqslant \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma}$	AB, AB	CD, CD
$\frac{\gamma}{\gamma+\phi} \geqslant \frac{1}{2}\frac{c}{c+\phi} + \frac{1}{2}\frac{d}{d+\phi}$		

Table 8: Equilibria with ab>cd

Conditions	Manager one	Manager two
$\frac{1}{2}\frac{a}{a+\gamma}+\frac{1}{2}\frac{b}{b+\gamma} \geqslant \frac{\phi}{\phi+\gamma}$	AA, BB	CD, CD
$\frac{\gamma}{\gamma+a}+\frac{\gamma}{\gamma+b} \geqslant \frac{c}{c+a}+\frac{d}{d+b}$	BB, AA	CD, CD
$\frac{1}{2}\frac{c}{c+\phi}+\frac{1}{2}\frac{d}{d+\phi} \geqslant \frac{\gamma}{\gamma+\phi}$	AB, AB	CC, DD
$\frac{\phi}{\phi+c}+\frac{\phi}{\phi+d} \geqslant \frac{b}{b+c}+\frac{a}{a+d}$	AB, AB	DD, CC
$\frac{\phi}{\phi+\gamma} \geqslant \frac{1}{2}\frac{a}{a+\gamma}+\frac{1}{2}\frac{b}{b+\gamma}$	AB, AB	CD, CD
$\frac{\gamma}{\gamma+\phi} \geqslant \frac{1}{2}\frac{c}{c+\phi}+\frac{1}{2}\frac{d}{d+\phi}$		

Table 9: Equilibria with ab<cd

5.5 Sequential Equilibrium

Comparing the results of the simultaneous with the sequential game, we do not get a general solution. We cannot generally show that both the simultaneous and sequential game coincide. However, we consider some examples of sequential equilibria in this section. In the sequential game, manager two moves first and manager one reacts.

Example 3 *Suppose that the parameter conditions are as follows:* $a = 10, \phi = 6, b = 5, c = 12.5, \gamma = 8, d = 4$ *such that* $\gamma^2 > ab = cd > \phi^2$*. In a simultaneous game, we get the following strategy combinations determining Nash equilibria according to the last section:* (i, vi) *or* (ii, vi)*. The winning probabilities in a simultaneous game are as follows:*

$$p^1(\cdot) = \frac{1}{2}\frac{a}{a+\gamma}+\frac{1}{2}\frac{b}{b+\gamma} = 0.47$$
$$p^2(\cdot) = \frac{1}{2}\frac{\gamma}{\gamma+a}+\frac{1}{2}\frac{\gamma}{\gamma+b} = 0.53$$

However, the subgame perfect Nash equilibria in a corresponding sequential game is determined by backwards induction. Manager two anticipates manager one's optimal reaction. It is easy to show that manager two optimally plays (CD, CD) *and then manager one plays* (AA, BB) *or* (BB, AA) *in a subgame perfect equilibrium. The results of the sequential game coincides with the results of the simultaneous game. However, this equilibrium has a special property because manager two's winning probability is higher than manager one's winning probability even if we have the parameter condition* $ab = cd$*. The reason is that manager two has a relatively high heterogeneous output* γ *such that his winning probability is higher than* 0.5 *in equilibrium. Thus, a good harmony of heterogeneous employees may be valuable for firms. Investing into the harmony of different types is profitable according to this example. For instance, if - ceteris paribus -* $\gamma = 9$ $(\gamma = 10)$, *then manager two's winning probability is* 0.54 (0.58) *which is even higher than* 0.53.

Example 4 *Suppose that the parameter conditions are as follows:* $a = 12, \phi = 6, b = 5, c = 12.5, \gamma = 8, d = 4$ *such that* $\gamma^2 > ab > cd > \phi^2$*. In a simultaneous game, we get the following two strategy combinations determining Nash equilibria according to the last section:* (i, vi) *or* (ii, vi)*. The winning probabilities in a simultaneous game are as follows:*

$$\begin{aligned} p^1(\cdot) &= \frac{1}{2}\frac{a}{a+\gamma} + \frac{1}{2}\frac{b}{b+\gamma} = 0.49 \\ p^2(\cdot) &= \frac{1}{2}\frac{\gamma}{\gamma+a} + \frac{1}{2}\frac{\gamma}{\gamma+b} = 0.51 \end{aligned}$$

However, the subgame perfect Nash equilibria in a corresponding sequential game are determined by backwards induction. Manager two anticipates manager one's optimal reaction. It is easy to show that manager two optimally plays (CD, CD) *and manager one reacts with* (AA, BB) *or* (BB, AA) *in a subgame perfect equilibrium. Once again, the results of the sequential game coincides with the results of the simultaneous game even if we have the condition* $ab > cd$ *(in contrast to the previous example).*

Example 5 *Suppose that the parameter conditions are as follows:* $a = 10, \phi = 9, b = 5, c = 12.5, \gamma = 8, d = 4$ *such that* $\phi^2 > \gamma^2 > ab = cd$*. In a simultaneous game, we get the following two strategy combinations determining the Nash equilibrium according to the last section:* $(iiii, vi)$*. The winning probabilities in a simultaneous game are as follows:*

$$\begin{aligned} p^1(\cdot) &= \frac{1}{2}\frac{\phi}{\phi+\gamma} + \frac{1}{2}\frac{\phi}{\phi+\gamma} = 0.53 \\ p^2(\cdot) &= \frac{1}{2}\frac{\gamma}{\gamma+\phi} + \frac{1}{2}\frac{\gamma}{\gamma+\phi} = 0.47 \end{aligned}$$

However, the subgame perfect Nash equilibria in a corresponding sequential game is determined by backwards induction. Manager two anticipates manager one's optimal reaction. Manager two optimally plays (CD, CD) *and manager one reacts with* (AB, AB) *in a subgame perfect equilibrium. It is easy to see that the results of the sequential game coincides with the results of the simultaneous game.*

5.6 Comparison

The generalized model in this appendix shows that firms can have different winning probabilities in equilibrium in contrast to the basic model. Furthermore, we show that the application with a logit contest function does not qualitatively differ from the application with a logistic contest function. Equilibria in the logistic setting depend on differences. Similarly, equilibria in the logit setting depend on ratios. In the generalized model, however, we cannot generally conclude that there is no second mover advantage in a sequential game in contrast to the basic model.

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