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Helmut Dietl, Egon Franck and Markus Lang

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Helmut Dietl, Egon Franck, Markus Lang**

University of Zurich

Abstract

This paper applies contest theory to provide an integrated framework of a team sports league and analyses the competitive interaction between clubs. We show that dissipation of the league revenue arises from "overinvestment" in playing talent as a direct consequence of the ruinous competitive interaction between clubs. This overinvestment problem increases if the discriminatory power of the contest function increases, revenue-sharing decreases, and the size of an additional exogenous prize increases. We further show that clubs invest more when they play in an open compared to a closed league. Moreover, the overinvestment problem within open leagues increases with the revenue differential between leagues.

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** All authors from the University of Zurich, Institute for Strategy and Business Economics, Plattenstrasse 14, 8032 Zurich, Switzerland. Phone: +41 44 634 53 11, Fax: +41 44 634 53 29. Emails: helmut.dietl@isu.uzh.ch, egon.franck@isu.uzh.ch, markus.lang@isu.uzh.ch. Corresponding author: Markus Lang

1 Introduction

One of the economic peculiarities of professional team sports is the associative character of competition. No club (team) can improve its position in the standings without worsening the position of other teams. If sportive and economic success are correlated, which they usually are, this rank order contest may result in destructive competition. In this paper we show that the dissipation of the league revenue in a team sports league arises from 'overinvestment' in playing talent as a direct consequence of the ruinous competitive interaction between clubs. This 'overinvestment' problem aggravates if the discriminatory power of the contest function increases, revenue-sharing decreases, and the size of an additional exogenous prize increases. We further show that clubs invest more when they play in an open compared to a closed league. Moreover, the 'overinvestment' problem within open leagues increases with the revenue differential between leagues.

Before proceeding with the model, we will give a short overview of the related literature: The first academic analyses of the economics of sports were presented by Rottenberg (1956), Neale (1964) and Canes (1974). In his seminal article Rottenberg studied the structural characteristics of the markets in which professional sports teams operate. Neale described the peculiarities of team sports leagues which are characterized by their mutual interdependence. The participants need each other because it is impossible to produce any output without the assistance of the other producers. Canes showed that improvements in team quality have important negative external effects which may induce clubs to over-employ athletic talent. He suggested the need for institutional mechanisms such as revenue-sharing, reserve clauses and player drafts in order to 'counteract the incentive to overinvest in team quality.' However, the precise rationale for this tendency to 'overinvest' into team quality remains obscure, since the specific nature of competitive interaction between the clubs in a league was not explicitly addressed. El-Hodiri & Quirk (1971) formalized the insights developed in the early literature in the first general economic model of a sports league, based on a dynamic decision-making mathematical framework. Fort & Quirk (1995), Vrooman (1995) and Vrooman (2000) updated this framework, however, without explicitly modelling competition and interaction among the clubs. Whitney (1993) was the first to formalize ruinous competitions within sports leagues using a labour market model. He suggested that the market for star athletes could be subject to 'destructive competition' which drives some participants out of the market even though it is inefficient for them to leave. The recent sports economics literature has suggested modelling a team sports league by making use of contest theory which reflects the noncooperative nature of such leagues.[1] Szymanski (2003)Szymanski (2003) applied Tullock's (1980) rent-seeking contest[2] to find the optimal design of sports leagues. However, he did not explicitly address the problem of 'overinvestment' in his model. Dietl and Franck (2000) and Dietl et al. (2003) were the first to model the 'overinvestment' problem based on contest theory. Our paper substantially extends their analysis by providing an integrated framework based on contest theory which allows studying the strategic interaction in a league with clubs competing for an endogenously-determined league prize. In this respect we are able to explain the tendency to 'overinvest' in playing talent as a direct consequence of the ruinous competitive interaction between clubs. We also model the effects that typical features of team sports leagues such as exogenous prizes and promotion and relegation have on talent investments.

The remainder of the paper is organized as follows: In Section 2 we present our basic model of a league with profit-maximizing clubs competing for an endogenously-determined league prize. In Section 3 we consider a league in which an exogenously-given prize is offered to the winner of the championship in addition to the endogenous league prize. Section 4 provides a two-period dynamic two-league model incorporating a system of promotion and relegation. Finally, section 5 concludes.

2 A league with an endogenous league prize

In this section we attempt to model the investment behavior of profit maximizing clubs which are organized as public limited companies in a team sports league. We follow the sports economic literature by portraying the league as a non-cooperative venture in order to analyze club's strategic behaviour. The teams are affiliated in a league with nothing but a common schedule and rules and there are no mechanisms that can put a damper on their externality-generating behavior. This is, of course, an extreme assumption and

[1] The first approaches in contest theory were made by Lazear & Rosen (1981), Green & Stokey (1983) and Nalebuff & Stiglitz (1983).

[2] The simple Tullock model has been extended in various ways (for a collection of relevant articles see e.g. Lockard and Tullock, 2001): inter alia different valuations of the prize, asymmetric players, sequential play, cooperative behaviour and dynamic games have been considered.

it is arguable if it provides a perfect description of real-world leagues. If, on the other hand, teams were affiliated in a league that had binding power to enforce a certain talent distribution, then the league should, of course, be modelled in a cooperative way. However, there is a broad consensus in sports economics that the non-cooperative approach is more adequate (see e.g. Atkinson et al., 1988; Fort and Quirk, 1995; Szymanski, 2003; Szymanski, 2004 and Szymanski and Késenne, 2004). For example Szymanski (2003), p.1167 argues that 'it seems more natural, however, to examine revenue-sharing rules in the context of a noncooperative game.' Moreover, Szymanski (2004), p.112 states: 'It is natural to think of the choice of playing talent by teams in a professional sports league as a noncooperative game. Teams choose independently how many players to hire and how much to pay them, subject to the rules and bylaws of the league.' Also, Szymanski and Késenne (2004), p.166 adopt a contest model in order to analyze the influence of revenue-sharing on competitive balance in team sports: 'Analysis of this problem requires a contest model. A sporting contest is a type of all-pay auction in which the players or teams make bids in the form of effort or investment in talent.'

In our opinion real-world sports leagues must retain basic characteristics of non-cooperative ventures in order to maintain the integrity of the game, which is sold as 'genuine competition' and not as a 'show' to spectators. This is in line with the argument of Atkinson et al. (1988), p.28: 'In professional team sports the league cannot directly enforce talent distribution without jeopardizing the integrity of the league and likely violating antitrust laws: noncooperative behavior among teams is essential to maintain fan interest.'[3]

Since the non-cooperative approach captures this basic insight we have decided to follow the mainstream and model the league as a non-cooperative venture, bearing in mind, of course, that this is not a perfect description of real-world leagues.

Let the league consist of n clubs where each club $i \in I = \{1, .., n\}$ invests independently a certain amount x_i in playing talent. This amount includes transfer fees, player

[3]Note that this article is written in the context of North American Leagues which are generally regarded as much closer to the cooperative venture model since they operate as 'closed shops' without promotion and relegation. European clubs pursueing their strategies in a pyramid of partially overlapping competitions (e.g. national championship and Champions League) might be even less inclined to cease power over talent distribution to a central league authority.

and coach salaries, winning bonuses, training expenses and medical attendance.[4] The investments x_i generate costs for each club, which are given by $C_i(x_i)$. We follow the literature and assume that the investment costs in playing talent are linear, i.e. $C_i(x_i) = cx_i$, resulting in constant marginal costs $C_i'(x_i) = c_i$ and are equal for the n clubs, i.e. $c_i = c$ $\forall i \in I$.

The league's total revenue $R(x_1, x_2, ..., x_n)$ is assumed to be a concave function of aggregate investments in playing talent, given by

$$R(x_1, x_2, ..., x_n) := \left(\sum_{i=1}^{n} x_i\right)^{\frac{1}{2}}.$$

This function reflects the fact that with raising investments in playing talent, e.g. better players, the league becomes more attractive for fans or TV-broadcasters and therefore the league income increases. In addition to this, we assume that investments in playing talent have decreasing returns to scale. Furthermore, we are considering a league with a revenue sharing arrangement, i.e. also the defeated clubs receive a certain amount of the league revenue. In our model the share of the endogenously-determined league prize $R(x_1, .., x_n)$ which is awarded to the winner of the championship is given by the parameter $\alpha \in [\frac{1}{2}, 1]$, while $\frac{1-\alpha}{n-1}$ is assumed to be the share of the endogenous league prize received by each of the defeated clubs.[5] The limiting case $\alpha = 1$ describes a 'winner-takes-all' league, whereas $\alpha = \frac{1}{2}$ describes a league with full revenue-sharing in which all clubs get the same share of the league revenue, independent of on-field success.

The probability of success is a function, called 'contest success function' (CSF) and, in our model, equals the ratio of each club's talent investment to total talent investment. Formally, the CSF maps the vector $(x_1, ., x_n)$ of talent investment into probabilities for each club. We apply the logit approach which is the most widely used functional form of a CSF in sporting contests.[6] The probability of success for club $i \in I$ in this imperfectly discriminating contest is defined as[7]

[4]For reasons of simplicity we sum up these different investments under the notion 'investment in playing talent.'

[5]For reasons of simplicity, we have assumed that each of the defeated $(n-1)$ clubs receives the same share of the remaining league's revenue.

[6]The logit CSF was generally introduced by Tullock (1980) and subsequently was axiomatized by Skaperdas (1996). An alternative functional form would be the probit CSF (e.g. Dixit, 1987) and the difference-form CSF (e.g. Hirshleifer, 1989).

[7]Note that the probabilities must sum up to unity, i.e. $\sum_{j=1}^{n} P_j^{\gamma}(x_1, .., x_n) = 1$.

$$P_i^\gamma(x_1,..,x_n) := \frac{x_i^\gamma}{\sum_{j=1}^n x_j^\gamma},$$

with the corresponding derivative given as

$$\frac{\partial P_i^\gamma}{\partial x_i} = \frac{\gamma x_i^{\gamma-1}\left(\sum_{j=1,j\neq i}^n x_j^\gamma\right)}{\left(\sum_{j=1}^n x_j^\gamma\right)^2}.$$

The parameter $\gamma > 0$, the so-called 'discriminatory power' of the CSF, measures how easily money buys on-field success. With other words, γ determines the ease of affecting the probability of winning the championship by a certain level of talent investment and specifies how much impact the club's own investments in playing talent have on its winning probability. The parameter γ also reflects the importance of luck or coincidence in a game. Luck plays a less important role in sports with high scores or a high frequency of matches. As γ increases,[8] the marginal costs of influencing the probability of success decreases, i.e. the probability of winning the championship increases for the club i with the highest level of talent investment x_i and differences in talent investments affect the winning probability in a stronger way. In the limiting case where γ goes to infinity, we would have a so-called 'all-pay auction,' i.e. a perfectly discriminating contest, where the club with the highest talent investment wins the prize with probability one. However, for a sports league this is not a realistic assumption since the club with the highest investment in playing talent cannot be certain of winning the championship race. If each club invests the same amount in playing talent, the probability of winning equals $\frac{1}{n}$ for each club. In case that no club is willing to invest a positive amount in talents, i.e. $x_i = 0$ $\forall i \in I$, the corresponding probability is then defined as $P_i^\gamma(0,..,0) := \frac{1}{n}$. Furthermore, it is straightforward to verify that the CSF of club i is an increasing function in the club's own investments x_i and a decreasing function in the other clubs' investments x_j $(j \neq i \in I)$.[9]

We start our analysis by considering the league's optimum which serves as a benchmark case. The league's optimal level $(\bar{x}_1,..,\bar{x}_n)$ of talent investments maximizes the

[8] Note that $\frac{\partial P_i^\gamma}{\partial \gamma} = \frac{x_i^\gamma\left(\sum_{j=1,j\neq i}^n x_j^\gamma \log\left(\frac{x_i}{x_j}\right)\right)}{\left(\sum_{j=1}^n x_j^\gamma\right)^2} > 0 \Leftrightarrow x_i > x_j$ for all $j \neq i \in I$.

[9] Formally, $\frac{\partial P_i^\gamma}{\partial x_i} = \frac{\gamma x_i^{\gamma-1}\left(\sum_{j=1,j\neq i}^n x_j^\gamma\right)}{\left(\sum_{j=1}^n x_j^\gamma\right)^2} > 0$ and $\frac{\partial P_i^\gamma}{\partial x_j} = -\frac{\gamma x_i^\gamma x_j^\gamma}{x_j\left(\sum_{j=1}^n x_j^\gamma\right)^2} < 0$.

social surplus of the clubs and is defined as

$$(\bar{x}_1, .., \bar{x}_n) = \arg \max_{(x_1,.,x_n)} \left(R(x_1, ., x_n) - c \sum_{i=1}^{n} x_i \right).$$

Solving this maximization problem yields[10]

$$\bar{x}_i = \frac{1}{4c^2 n}, \quad i \in I. \tag{1}$$

The terminologies 'overinvest' and 'underinvest' are defined as situations in which a club invests in equilibrium more and less, respectively, than in the league optimum.

The expected payoff for club $i \in I$ is determined by the following (expected) profit function:

$$\begin{aligned} E(\Pi_i) &= P_i^\gamma(x_1, .., x_n)\alpha R(x_1, .., x_n) \\ &\quad +(1 - P_i^\gamma(x_1, .., x_n))\frac{1-\alpha}{n-1} R(x_1, .., x_n) - C_i(x_i) \\ &= \left(\alpha \frac{x_i^\gamma}{\sum_{j=1}^{n} x_j^\gamma} + \frac{1-\alpha}{n-1} \frac{\sum_{j=1, j\neq i}^{n} x_j^\gamma}{\sum_{j=1}^{n} x_j^\gamma} \right) \left(\sum_{j=1}^{n} x_j \right)^{\frac{1}{2}} - c x_i \end{aligned} \tag{2}$$

The expected payoff of club i depends on the probability of winning P_i^γ multiplied by the share α of the endogenous league prize $R(x_1, ., x_n)$ awarded to the winner, plus the probability of losing $(1 - P_i^\gamma)$ multiplied by the share $\frac{1-\alpha}{n-1}$ of the endogenous league prize $R(x_1, ., x_n)$ awarded to each of the defeated clubs, minus the investment costs in playing talent $C_i(x_i)$.

The club-owners choose an investment level of playing talent such that expected profits are maximized, i.e. club i solves $\max_{x_i} E(\Pi_i)$, where $E(\Pi_i)$ is given by equation (2). Hence, the FOC for club $i \in I$ is derived as[11]

$$\frac{\partial E(\Pi_i)}{\partial x_i} = \frac{\alpha n - 1}{n-1} \left(\frac{\partial P_i^\gamma}{\partial x_i} R + P_i^\gamma \frac{\partial R}{\partial x_i} \right) + \frac{1-\alpha}{n-1} \frac{\partial R}{\partial x_i} - c = 0.$$

By solving this system of (implicitly defined) reaction functions, we obtain the following equilibrium expected investment level for club i:[12]

$$x_i^* = \frac{(1 + 2\gamma(\alpha n - 1))^2}{4c^2 n^3}, \quad i \in I. \tag{3}$$

In the symmetric equilibrium (3) the clubs realize identical strictly positive investment levels and therefore obtain an equal probability of $\frac{1}{n}$ to receive the share α of the

[10] Since clubs are symmetric we only consider the symmetric optimum.

[11] It is straightforward to verify that the second-order conditions for a maximum are satisfied.

[12] Note that the league size n is fixed since it is assumed to be exogenously-given.

endogenously-determined league prize $R(x_1^*, .., x_n^*) = \frac{1+2\gamma(\alpha n-1)}{2cn}$. Furthermore, the equilibrium investments $(x_1^*, .., x_n^*)$ in playing talent generate costs for each club i given by $C_i(x_i^*) = \frac{(1+2\gamma(\alpha n-1))^2}{4cn^3}$.

Plugging these investment levels into the (expected) profit function (2) yields the equilibrium expected payoff $E(\Pi_i^*)$ for club i as

$$E(\Pi_i^*) = \frac{(1+2\gamma(\alpha n-1))(2n-1-2\gamma(\alpha n-1))}{4cn^3}, \quad i \in I.$$

The existence of an equilibrium in pure strategies depends on the discriminatory power γ of the CSF and the parameter α of the revenue-sharing arrangement:

Lemma 1

The existence of a Nash equilibrium in pure strategies is guaranteed if (i) the discriminatory power γ is restricted to $0 < \gamma \leq \bar{\gamma}(\alpha) := \frac{2n-1}{2(\alpha n-1)}$ or (ii) the revenue-sharing parameter α is restricted to $\frac{1}{2} \leq \alpha \leq \bar{\alpha}(\gamma) := \frac{2\gamma+2n-1}{2\gamma n}$.

Proof. See Appendix A.1. ■

If $\gamma > \bar{\gamma}(\alpha)$ or $\alpha > \bar{\alpha}(\gamma)$, then the FOCs and SOCs fail to characterize the global maximum. Nevertheless, there exists a symmetric mixed-strategy equilibrium.[13] Moreover, note that $\gamma \leq \bar{\gamma}(\alpha)$ is equivalent to $\alpha \leq \bar{\alpha}(\gamma)$.

The 'ratio of revenue dissipation,' denoted D, measures the degree of dissipation of the league revenue and is defined in our model as[14]

$$D(\alpha, \gamma) := \frac{\bar{T} - T^*}{\bar{T}} = \frac{(1-n+2\gamma(\alpha n-1))^2}{n^2}.$$

The terms $\bar{T} := R(\bar{x}_1, .., \bar{x}_n) - c\sum_{j=1}^{n} \bar{x}_j$ and $T^* := R(x_1^*, .., x_n^*) - c\sum_{j=1}^{n} x_j^*$ characterize the net surplus at the league optimum and the Nash-equilibrium, respectively. The higher the ratio $D(\alpha, \gamma)$, the higher the degree of dissipation in the league. Note that if α or γ are bigger than the threshold values $\alpha^*(\gamma) := \frac{2\gamma+n-1}{2\gamma n}$ and $\gamma^*(\alpha) := \frac{n-1}{2(\alpha n-1)}$, then D is an

[13] The existence of Nash equilibria in the Tullock contest is discussed in the rent-seeking literature e.g. in Lockard and Tullock (2001). The case of mixed-strategies in a discrete choice set is analyzed by Baye et al. (1994).

[14] Note that in the rent-seeking literature the ratio D is called 'ratio of rent dissipation.' See for instance Chung (1996).

increasing function in α and γ. Moreover, the ratio D is within the interval $[0, 1]$ since we have assumed that $\gamma \leq \bar{\gamma}(\alpha)$ and $\alpha \leq \bar{\alpha}(\gamma)$.[15]

The next proposition provides comparative statics for the equilibrium investments $(x_1^*, .., x_n^*)$:

Proposition 1

The equilibrium investments x_i^* of club $i \in I$ increase if (i) the discriminatory power γ of the CSF increases, i.e. money buys on-field success more easily, (ii) the share α of the league prize awarded to the winner increases, i.e. the league's revenue is distributed more unequally, or (iii) marginal costs c for talent investments decrease.

Proof. Straightforward. ■

From this proposition we derive the following results:

ad (i) If γ is bigger than the threshold value $\gamma^*(\alpha) = \frac{n-1}{2(\alpha n-1)}$, then each club 'overinvests' in playing talent, i.e.$x_i^* > \bar{x}_i$, and the degree of revenue dissipation in the league increases with γ.[16] Intuitively, this is clear: If smaller differences in playing talent have a stronger impact on the probability of success, then the clubs have a stronger incentive for higher talent investments. If the discriminatory power γ equals the other threshold value $\bar{\gamma}(\alpha) = \frac{2n-1}{2(\alpha n-1)}$, then the net surplus T^* at the Nash equilibrium amounts to zero and the ratio of dissipation $D(\alpha, \gamma)$ reaches its maximum of one. In this case the clubs dissipate the whole league revenue through their investment behavior.

ad (ii) Similarly, if α is bigger than the threshold value $\alpha^*(\gamma) = \frac{2\gamma+n-1}{2\gamma n}$, then each club 'overinvests' in playing talent and the degree of dissipation of the league revenue increases with α. Moreover, revenue dissipation is maximal, i.e. the ratio $D(\alpha, \gamma)$ amounts to one, if the parameter α equals the other threshold value $\bar{\alpha}(\gamma) = \frac{2\gamma+2n-1}{2\gamma n}$. In this case the net surplus T^* at the Nash equilibrium is zero. We conclude that less revenue-sharing induces the clubs to increase their investments in playing talent and therefore contributes

[15] Formally, $\frac{\partial D(\alpha,\gamma)}{\partial \alpha} = \frac{4\gamma(1-n+2\gamma(\alpha n-1))}{n} > 0 \Leftrightarrow \alpha > \frac{2\gamma+n-1}{2\gamma n}$. In this case it is also guaranteed that $\frac{\partial D(\alpha,\gamma)}{\partial \gamma} = \frac{4(\alpha n-1)(1-n+2\gamma(\alpha n-1)}{n^2} > 0$. Moreover, $\lim_{\alpha \to \alpha^*} D(\alpha, \gamma) = \lim_{\gamma \to \gamma^*} D(\alpha, \gamma) = 0$ and $\lim_{\alpha \to \bar{\alpha}} D(\alpha, \gamma) = \lim_{\gamma \to \bar{\gamma}} D(\alpha, \gamma) = 1$.

[16] However, in a league with full revenue-sharing, i.e. $\alpha = 0.5$, each club will invest less in equilibrium than in the league optimum, independent of the discriminatory power γ. Clearly, if each club gets the same share of the league's revenue, irrespective of field success, incentives to invest in playing talents are low.

to aggravate the 'overinvestment' problem. The result that a bigger spread between first and second prize leads to higher equilibrium efforts is well-known in contest theory and follows from the stronger incentives to win.

ad (iii) Even though marginal costs influence the equilibrium investments, altering marginal costs does not affect the dissipation of the league revenue since the ratio of dissipation $D(\alpha, \gamma)$ is independent of c. Hence, marginal costs have no influence on the 'overinvestment' problem.

Summarizing the results derived above yields that if (a) the discriminatory power γ of the CSF is within the interval $(\gamma^*, \bar{\gamma}] = (\frac{n-1}{2(\alpha n-1)}, \frac{2n-1}{2(\alpha n-1)}]$ or (b) the parameter α of the revenue-sharing arrangement is within the interval $(\alpha^*, \bar{\alpha}] = (\frac{2\gamma+n-1}{2\gamma n}, \frac{2\gamma+2n-1}{2\gamma n}]$ existence of a Nash-equilibrium is guaranteed in which each club 'overinvests' in playing talent and therefore dissipates parts of the league's revenue. However, the increase of the investment level in playing talent does not affect the winning-probability in equilibrium, since the clubs simultaneously increase their investments and will end up with identical equilibrium investments. The same relative performance among the clubs could be obtained at the league optimum, i.e. $P_i^\gamma(x_1^*, .., x_n^*) = P_i^\gamma(\bar{x}_1, .., \bar{x}_n) = \frac{1}{2}$. Even though the clubs would be better off if they agreed upon the investment level in the league optimum, this solution does not characterize a feasible equilibrium strategy due to strategic interaction, i.e. cannot be sustained without cooperation. Starting at the league optimum $\bar{x}_i$, club i has an incentive to increase its investments in talents, since this behavior raises the probability of winning the share of the endogenous league prize awarded to the winner. However, the other clubs have the same incentives and therefore the clubs are caught in a typical prisoners' dilemma type of equilibrium. As a result, each club will enter in a ruinous competition leading to the symmetric Nash-equilibrium where the clubs 'overinvest' in playing talent, with no relative gain in performance compared to the league optimum.

3 A league with an additional exogenous league prize

We assume that our league now offers an exogenously-given prize besides the endogenously-determined league prize. This exogenous league prize is a proxy for all sorts of performance-related revenues like sponsorship contracts and the secure monetary value of qualifying for international competition.[17] The exogenous league prize, denoted Q, is solely awarded to the winner of the championship while the endogenous league prize $R(x_1,..,x_n) = (\sum_{i=1}^{n} x_i)^{\frac{1}{2}}$ is again distributed among the clubs according to the revenue-sharing arrangement from Section 2. For the sake of simplicity, we henceforth assume that the discriminatory power γ of the CSF equals unity, i.e. the probability of success for club $i \in I$ is given by $P_i(x_1,..,x_n) = \frac{x_i}{\sum_{j=1}^{n} x_j}$. The expected profit of club $i \in I$ is now given by

$$\begin{aligned} E(\Pi_i) &= P_i(x_1,..,x_n)(\alpha R(x_1,..,x_n) + Q) \\ &\quad + (1 - P_i(x_1,..,x_n))\frac{1-\alpha}{n-1} R(x_1,..,x_n) - C_i(x_i) \\ &= \frac{x_i}{\sum_{j=1}^{n} x_j}\left(\alpha\left(\sum_{i=1}^{n} x_i\right)^{\frac{1}{2}} + Q\right) + \frac{1-\alpha}{n-1}\frac{\sum_{j\neq i}^{n} x_j^{\gamma}}{\sum_{j=1}^{n} x_j^{\gamma}}\left(\sum_{i=1}^{n} x_i\right)^{\frac{1}{2}} - cx_i, \end{aligned}$$

yielding the following FOC of profit-maximization:[18]

$$\frac{\partial E(\Pi_i)}{\partial x_i} = \frac{1}{n-1}\left((1-\alpha)\frac{\partial R}{\partial x_i} + (2n-1)\left(\frac{\partial R}{\partial x_i}P_i + \frac{\partial P_i}{\partial x_i}R\right)\right) + \frac{\partial P_i}{\partial x_i}Q - c = 0.$$

By solving this system of reaction functions, we determine the following Nash-equilibrium for club $i \in I$ as[19]

$$x_i^*(Q) = \frac{(n-1)}{cn^2}Q + \frac{(2\alpha n - 1)}{8c^2n^3}\left((2\alpha n - 1) + \left((2\alpha n - 1)^2 + 16cn(n-1)Q\right)^{\frac{1}{2}}\right) \tag{4}$$

and derive the following results:

Proposition 2

(i) The equilibrium investments $x_i^*(Q)$ of club $i \in I$ increase in the exogenous prize Q.

(ii) The equilibrium investments and the ratio of dissipation are higher in a league that offers an exogenous prize besides an endogenous prize compared with a league that offers an endogenous prize only, i.e. $x_i^*(Q) > x_i^*$ and $D(Q) > D$ $\forall Q > 0$.

[17] For example, in the European football leagues the clubs compete against each other also for the right to participate in international competition like the UEFA Champions League. The participation in the Champions League guarantees participants a minimum number of matches at the group stage and therefore secure revenue.

[18] Note that the second-order conditions for a maximum are satisfied.

[19] Formally, we obtain two equilibria. However, the negative one can be ruled out, since it does not constitute an interior solution.

Proof. See Appendix A.2 ■

The proposition shows that in a league that offers an additional exogenous prize awarded to the winner of the championship the clubs are induced to spend more on playing talent and the 'overinvestment' problem is aggravated compared with a league that offers an endogenous prize only. The potential extra prize Q generates additional financial incentives that encourages clubs to gamble on success by 'overinvesting' in playing talent in the hope of gaining admission to lucrative international competition and therefore to compensate their expenditures. Even though expected profits are non-negative, such a strategy is risky since the clubs cannot be sure of receiving the prize.

4 A league with promotion and relegation

In this section we provide a model in which the leagues are organized hierarchically in ascending divisions, offering a system of promotion and relegation. At the end of each season the worst performing clubs in each division are relegated to the next lower division and are replaced by the best performing clubs from that division. In order to analyze how a system of promotion and relegation affects the investment behavior of clubs, we will incorporate such a system in our league model by considering an open winner-takes-all league, i.e. a league without a revenue-sharing arrangement but which is open to promotion and relegation. Our dynamic model covers two periods and consists of two divisions denoted division A and division B, with each division containing two clubs. The time dimension becomes relevant now, because current investment behavior depends on the expected future profits as well as current profits. In other words, the prospect of promotion and relegation affects the first-period investments in playing talent. For the sake of simplicity, we assume that the revenue of each division is exogenously-given with R_A and R_B denoting the prize of division A and B, respectively. Division A is considered as the top-flight division which offers a higher prize than the second division B, i.e. $R_A > R_B$.

We assume that club 1 and club 2 start in period one in division A competing for the first-division prize R_A. The first-period champion receives the prize R_A, remains in division A and competes in period two against the promoted club from division B. The defeated club from division A receives nothing, is relegated to the second division and competes in the second period against the defeated club from division B. Club 3 and club 4 start in the first period in division B and compete for the second-division prize R_B. The first-period champion receives the prize R_B, is promoted to division A and competes in period two against the first-period champion of division A. The defeated club from division B receives nothing, remains in the division and competes in the second period against the relegated club from division A.

The investments in playing talent of club $\mu \in I = \{1, 2, 3, 4\}$ in period $t \in \{1, 2\}$ are denoted $x_{\mu,t}$ generating costs $C_\mu(x_{\mu,t}) = x_{\mu,t}$ $\forall \mu \in I$, i.e. marginal costs are normalized to one. Expected profits of club μ, if this club competes in division k against club ν in period t, are denoted $E(\Pi^{t,k}_{\mu,\nu})$, with $\mu, \nu \in I$. Again, we assume that the discriminatory power γ of the CSF in our dynamic model amounts to one. Hence, the probability that club $\mu \in I$ wins against club $\nu \in I$ in period $t \in \{1, 2\}$ is given by

$$P^t_{\mu,\nu}(x_{\mu,t}, x_{\nu,t}) = \frac{x_{\mu,t}}{x_{\mu,t} + x_{\nu,t}}.$$

Since it is assumed that the division prize R_k is won by one of the two clubs in the corresponding division $k \in \{A, B\}$ with certainty, it must be the case that $P^t_{\mu,\nu} = (1 - P^t_{\nu,\mu})$. For notational clarity, we exclusively use the subscripts $i, j \in \{1, 2\}$ to characterize the division A clubs 1 and 2, while the subscripts $r, s \in \{3, 4\}$ stand for the division B clubs 3 and 4. The superscript k denotes the division, with $k \in \{A, B\}$ and t stands for the period, with $t \in \{1, 2\}$.

In the top-flight division A, expected first-period profits $E(\Pi^{1,A}_{i,j})$ of club i and j are given by

$$E(\Pi^{1,A}_{i,j}) = P^1_{i,j}(R_A + E(\Pi^{2,A}_{i,r})) + (1 - P^1_{i,j})E(\Pi^{2,B}_{i,s}) - x_{i,1}. \tag{5}$$

With probability $P^1_{i,j}$ club i wins against club j in period one and obtains the first division prize R_A. Club i then remains in division A, competes in period two against the promoted club r from division B and receives an expected second-period payoff of $E(\Pi^{2,A}_{i,r})$. With probability $(1 - P^1_{i,j})$ club i loses against club j and is relegated to division B without receiving a prize in period one. Then, club i competes in the second period against the defeated club s of division B, obtaining an expected second-period payoff of $E(\Pi^{2,B}_{i,s})$.

In the second division expected first-period profits $E(\Pi^{1,B}_{r,s})$ of club r and s are given by

$$E(\Pi^{1,B}_{r,s}) = P^1_{r,s}(R_B + E(\Pi^{2,A}_{r,i})) + (1 - P^1_{r,s})E(\Pi^{2,B}_{r,j}) - x_{r,1}. \tag{6}$$

With probability $P^1_{r,s}$ club r is successful against club s in period one and receives the division B prize R_B. Club r is then promoted to division A, obtaining an expected payoff of $E(\Pi^{2,A}_{r,i})$ in period two. With probability $(1-P^1_{r,s})$ club r loses against club s and stays in division B, receiving in period two an expected payoff of $E(\Pi^{2,B}_{r,j})$.

Following the logic of backward induction, we first determine expected profits $E(\Pi^{2,k}_{i,s})$ for club i and expected profits $E(\Pi^{2,k}_{r,j})$ for club r in division k of the subgame beginning in period two. Since clubs are assumed to be symmetric, it is irrelevant for the division A club i against which division B club r it will compete in the second period in division k and vice versa, i.e. it must be the case that $E(\Pi^{2,k}_{i,3}) = E(\Pi^{2,k}_{i,4})$ and $E(\Pi^{2,k}_{r,1}) = E(\Pi^{2,k}_{r,2})$, respectively. Therefore, expected payoffs in period two are given by

$$E(\Pi^{2,k}_{i,s}) = P^2_{i,s}R_k - x_{i,2} \quad \text{and} \quad E(\Pi^{2,k}_{r,j}) = P^2_{r,j}R_k - x_{r,2}.$$

By deriving the respective FOCs and solving the system of reaction functions, we determine the equilibrium investment levels $x^o_{i,2}$ and $x^o_{r,2}$ besides the equilibrium payoffs $E^o(\Pi^{2,k}_{i,s})$ and $E^o(\Pi^{2,k}_{r,j})$ in the second period for club i and club r, respectively, as[20]

$$x^o_{i,2} = x^o_{r,2} = \frac{R_k}{4} \quad \text{and} \quad E^o(\Pi^{2,k}_{i,s}) = E^o(\Pi^{2,k}_{r,j}) = \frac{R_k}{4}.$$

In an open league, each of the four clubs invests in period two $\frac{R_k}{4}$ in playing talent and receives an expected payoff of $\frac{R_k}{4}$, dependent in which division k it competes. Plugging the second-period expected payoffs $E^o(\Pi^{2,k}_{i,s})$ and $E^o(\Pi^{2,k}_{r,j})$ into the first-period profit functions (5) and (6), respectively, yields

$$\begin{aligned} E(\Pi^{1,A}_{i,j}) &= P^1_{i,j}(R_A + \frac{R_A}{4}) + (1-P^1_{i,j})\frac{R_B}{4} - x_{i,1}, \\ E(\Pi^{1,B}_{r,s}) &= P^1_{r,s}(R_B + \frac{R_A}{4}) + (1-P^1_{r,s})\frac{R_B}{4} - x_{r,1}. \end{aligned}$$

By deriving the corresponding FOCs and solving the system of reaction functions, we determine the first-period equilibrium investments $x^o_{i,1}$ and $x^o_{r,1}$ besides the expected profits $E^o(\Pi^{1,A}_{i,j})$ and $E^o(\Pi^{1,B}_{r,s})$ for club i and club r, respectively, as

$$x^o_{i,1} = \frac{1}{16}(5R_A - R_B) \quad \text{and} \quad E^o(\Pi^{1,A}_{i,j}) = \frac{1}{16}(5R_A + 3R_B), \tag{7}$$

$$x^o_{r,1} = \frac{1}{16}(R_A + 3R_B) \quad \text{and} \quad E^o(\Pi^{1,B}_{r,s}) = \frac{1}{16}(R_A + 7R_B). \tag{8}$$

[20]Note that the superscript o stands for 'open' league whereas c stands for 'closed' league.

The division A clubs 1 and 2 spend more on playing talent in the first period than the division B clubs 3 and 4. But, they also receive a higher expected payoff.[21]

As a reference point, we now calculate the respective investment levels and payoffs in a closed league, i.e. in a league where it is not possible to be promoted or relegated from one division to another. In a closed league the first-period expected profits of the division k clubs μ and ν are given by

$$E(\Pi^{1,k}_{\mu,\nu}) = P^1_{\mu,\nu}(R_k + E(\Pi^{2,k}_{\mu,\nu})) + (1 - P^1_{\mu,\nu})E(\Pi^{2,k}_{\mu,\nu}) - x_{\mu,1}, \tag{9}$$

with $k = A$ if $\mu, \nu \in \{1, 2\}$ and $k = B$ if $\mu, \nu \in \{3, 4\}$. With probability $P^1_{\mu,\nu}$ the division k club μ wins against club ν in period one, obtains the division k prize R_k and competes in period two again with club ν for the prize R_k, receiving an expected payoff of $E(\Pi^{2,k}_{\mu,\nu})$. With probability $(1 - P^1_{\mu,\nu})$ club μ is defeated by club ν in period one, receives nothing and plays in the second period again against club ν in division k, obtaining an expected payoff of $E(\Pi^{2,k}_{\mu,\nu})$.

For the subgame beginning in period two, expected profits $E(\Pi^{2,A}_{i,j})$ for the division A clubs 1 and 2 and expected profits $E(\Pi^{2,B}_{r,s})$ for the division B clubs 3 and 4, respectively, are given by

$$E(\Pi^{2,A}_{i,j}) = P^2_{i,j}R_A - x_{i,2} \quad \text{and} \quad E(\Pi^{2,B}_{r,s}) = P^2_{r,s}R_B - x_{r,2},$$

yielding the following second-period equilibrium investments and payoffs in division A and B respectively:

$$x^c_{i,2} = \frac{R_A}{4},\ E^c(\Pi^{2,A}_{i,j}) = \frac{R_A}{4} \quad \text{and} \quad x^c_{r,2} = \frac{R_B}{4},\ E^c(\Pi^{2,B}_{r,s}) = \frac{R_B}{4}.$$

By plugging the equilibrium payoffs $E^c(\Pi^{2,A}_{i,j})$ and $E^c(\Pi^{2,B}_{r,s})$ into (9) and computing the corresponding FOCs we derive the first-period equilibrium investments and payoffs in division A and B as

$$x^c_{i,1} = \frac{R_A}{4} \quad \text{and} \quad E^c(\Pi^{1,A}_{i,j}) = \frac{R_A}{2}, \tag{10}$$

$$x^c_{r,1} = \frac{R_B}{4} \quad \text{and} \quad E^c\left(\Pi^{1,B}_{r,s}\right) = \frac{R_B}{2}. \tag{11}$$

Comparison of the first-period investment levels (7) with (10) in division A and (8) with (11) in division B, respectively, yields the following results:

[21] $x^O_{i,1} > x^O_{r,1} \Leftrightarrow R_A > R_B$ and $E^O(\Pi^{1,A}_{i,j}) > E^O(\Pi^{1,B}_{r,s}) \Leftrightarrow R_A > R_B$.

Proposition 3

In an open league, the aggregate first-period investments in both divisions are higher than the respective investments in a closed league. This difference increases if the spread between the division A prize and the division B prize augments.

Proof. See Appendix A.3. ■

This proposition shows that clubs compete more intensively in an open league than in a closed league in the first period. In the second period, however, the investment levels in an open league and in a closed league are equal in both divisions, i.e. $x^o_{\mu,2} = x^c_{\mu,2}$ $\forall \mu \in I$. In an open league the prospect of promotion as an additional reward for clubs in the second division and the threat of relegation for clubs in the top division both induce an increase of talent investments in the first period, compared with a closed league. We conclude that under a system of promotion and relegation the incentives to improve team quality by investing a higher amount in playing talent are enhanced since clubs obtain financial benefits from promotion and suffer financial penalties from relegation.

Moreover, the larger the difference between division A and division B in terms of revenues, the bigger the difference between the first-period investments in an open and a closed league. Hence, each club will spend more on playing talent in an open league, if the promotion from division B to division A becomes more lucrative and the relegation from A to B more 'costly' (in terms of reduced revenues).

5 Conclusions

The precise rationale why clubs tend to 'overinvest' in playing talent has been astonishingly neglected in the sports economics literature. In this paper we have tried to fill the gap based on the analysis of a theoretical league model with profit maximizing clubs competing for a league prize.

The analysis shows that the tendency to 'overinvest' in playing talent leading to the dissipation of the league's revenue is a direct consequence of the ruinous competition between the clubs. The following factors enhance the incentives to 'overinvest' and therefore to dissipate the league's revenue:

- a stronger correlation between talent investments and league performance.

- a more unequal distribution of the league's revenue.
- an additional exogenous prize (e.g. participation to international competition) awarded to the winner of the domestic championship.
- a system of promotion and relegation.
- an increased inequality between first and second division of a domestic league.

A Appendix

A.1 Proof of Lemma 1

The existence of a Nash equilibrium in pure strategies is guaranteed if each club receives non-negative equilibrium-payoffs, i.e. $E(\Pi_i^*) \geq 0 \quad \forall \gamma \in (0, \bar{\gamma}]$ and $i \in I$.

ad (i) We derive $E(\Pi_i^*) \geq 0 \ \forall \gamma \in [\bar{\gamma}_1, \bar{\gamma}_2]$ with $\bar{\gamma}_1 = -\frac{1}{2(\alpha n-1)}$ and $\bar{\gamma}_2 = \frac{2n-1}{2(\alpha n-1)}$. Since γ is assumed to be strictly positive we conclude $\bar{\gamma}_1 < 0$ and thus we can concentrate on the interval $(0, \bar{\gamma}_2]$. Hence by restricting the discriminatory power γ to $0 < \gamma \leq \bar{\gamma}(\alpha) := \frac{2n-1}{2(\alpha n-1)}$, we obtain non-negative equilibrium-payoffs and therefore the existence of the Nash equilibrium.

ad (ii) Similarly,[22] we derive $E(\Pi_i^*) \geq 0 \ \forall \alpha \in [\bar{\alpha}_1, \bar{\alpha}_2]$ with $\bar{\alpha}_1 = \frac{1}{n} - \frac{1}{2\gamma n}$ and $\bar{\alpha}_2 = \frac{1}{n} + \frac{2n-1}{2\gamma n}$. Since α is assumed to be bigger or equal $\frac{1}{2}$ we conclude $\bar{\alpha}_1 < \frac{1}{2}$ and thus we can concentrate on the interval $[\frac{1}{2}, \bar{\alpha}_2]$. Hence, by restricting α to $\frac{1}{2} \leq \alpha \leq \bar{\alpha}(\gamma) := \frac{2\gamma+2n-1}{2\gamma n}$ proves the claim.

A.2 Proof of Proposition 2

ad (i) We compute $\frac{\partial x_i^*(Q)}{\partial Q} = \frac{n-1}{n}\left(1 + \frac{2\alpha n-1}{\sqrt{16cn(n-1)Q+(2\alpha n-1)^2}}\right) > 0$. Thus, by increasing the exogenous prize Q each club is induced to spend more on playing talent.

ad (ii) The equilibrium investments in a league that offers an endogenous prize only are given by equation (3) as $x_i^* = \frac{(2\alpha n-1)^2}{4c^2n^3}$, $i \in I$. Note that we have set γ equal unity. By comparing these investments with the corresponding equilibrium investments $x_i^*(Q)$ given by (4) in a league that offers an exogenous prize besides an endogenous prize proves the claim that $x_i^*(Q) > x_i^*$.

The additional exogenous prize, however, has no influence on the league optimum which is still given, as in the basic model in Section 2, by $\bar{x}_i = \frac{1}{4nc^2}$. Hence, the net surplus at the league optimum is given by $\bar{T}(Q) = \frac{1}{4c} + Q$. We derive that due to the additional exogenous prize, the corresponding ratio of dissipation $D(Q) = \frac{\bar{T}(Q)-T^*(Q)}{\bar{T}(Q)}$ is higher than the ratio $D = \frac{\bar{T}-T^*}{\bar{T}}$ of a league with an endogenous prize only. This claim can be straightforward proved by noting that $T^*(Q)$ is a decreasing function in Q.

[22] Note that $\gamma \leq \bar{\gamma}(\alpha)$ is equivalent to $\alpha \leq \bar{\alpha}(\gamma)$.

A.3 Proof of Proposition 3

In an open league the division A clubs 1 and 2 realize an investment level of $x^o_{1,1} = x^o_{2,1} = \frac{1}{16}(5R_A - R_B)$ in the first period. This investment level lays above the first-period investment level $x^c_{1,1} = x^c_{2,1} = \frac{R_A}{4}$ of the respective clubs in a closed league since we have assumed that $R_A > R_B$. The same holds true for the division B clubs 3 and 4: the first-period talent investments $x^o_{3,1} = x^o_{4,1} = \frac{1}{16}(R_A + 3R_B)$ in an open league are higher than the respective investment levels $x^c_{3,1} = x^c_{4,1} = \frac{R_B}{4}$ in a closed league. Hence, the first-period aggregate investment level in both divisions is higher in an open league than the respective level in a closed league.

Moreover, the difference between the first-period investments in playing talent in an open and a closed league is given for both divisions by $x^o_{\mu,1} - x^c_{\mu,1} = \frac{1}{16}(R_A - R_B) \forall \mu \in I$. The difference $x^o_{\mu,1} - x^c_{\mu,1}$ becomes larger if the spread between the division prize R_A and R_B increases.

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