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Advertising Pricing Models in Media Markets: Lump-Sum versus Per-Consumer Charges*

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Abstract

This paper develops a model of asymmetric competition between a pay and a free media platform. The pay media platform generates revenues from media consumers through subscription fees, while the free media platform generates revenues from charging advertisers either on a lump-sum basis (regime A) or on a per-consumer basis (regime B). We show that the free platform produces a higher advertising level and attracts more consumers in regime A than B although advertisers must pay more for ads and consumers dislike ads. Moreover, the pay media platform faces higher subscription fees and lower consumer demand in regime A than B. Compared to regime B, the profit of the free (pay) media platform is higher (lower) in regime A, while aggregate profits are higher only if the consumers' disutility from ads is sufficiently low. In addition, advertisers are better off in regime A than B, while the opposite is true for the media consumers. Finally, in small media markets, social welfare is lower in regime A than B, while this is true in large media markets only if the media consumers' disutility from advertising is sufficiently high.

Keywords: Advertising, media platform, two-sided market, lump-sum charge, per-consumer charge, asymmetric competition

JEL Classification: D40, L10

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1 Introduction

Two generic business models coexist and compete in the various media markets: either media platforms provide their content to the media consumers for free and generate revenues from advertising (free media platform), or media platforms do not place ads but charge their consumers a subscription fee for access to their contents (pay media platform).[1] One justification for the coexistence between pay and free media platforms is that media consumers usually dislike the presence of ads because they decrease the entertainment value of consuming the media content.[2] As a result, some media consumers are willing to pay for media content and switching to ad-free pay platforms to avoid ads (Tag, 2009).

In general, free media platforms possess two basic ways to charge advertisers. Advertisers are charged a lump-sum fee for placing an ad or they are charged on a per-consumer basis such that the advertising charges are a positive function of the consumer size. For example, the online version of The New York Times can ask advertisers a certain fixed amount for placing an ad (lump-sum charges) or it can charge advertisers via the concept of Pay-per-Click or Cost-per-Click where advertisers must pay for each click on the ad link (per-consumer charges).[3]

Given these two distinct advertising pricing models, several research questions arise: What are the economic effects of both pricing models? Which pricing model generates higher profit for the free media platform and for the pay media platform, respectively? What are the market reactions in both pricing models? From which pricing model can media consumers and advertisers benefit more? This paper tries to answer these and related questions by developing a simple theoretical model of a media market that is served by one pay media and one free media platform. In accordance with the existing literature, media competition is modeled in the Hotelling fashion. That is, the media consumers consume ad-free media content on the pay platform and pay a positive subscription fee or they consume the media content for free and accept the presence of advertising. The free media platform can charge its advertisers either a lump-sum charge (regime A) or on a per-consumer basis (regime B). In regime A, the advertisers pay a fixed amount for placing an ad on the free media platform, which does not explicitly depend on the consumer size. In regime B, the price that advertisers must pay for placing an ad is an increasing function of the consumer size. To analyze these two pricing models, we model

[1] A third hybrid business model exists where media platforms place ads and charge consumers (e.g., daily newspapers and magazines). However, in this paper we focus on the two generic models: pay vs free platforms.

[2] See Depken II and Wilson (2004), Anderson and Coate (2005), Wilbur (2008), and Casadesus-Masanell and Zhu (2010).

[3] Other examples for per-consumer advertising charges include pricing models such as CPM (cost per thousand impressions/views), CPA (cost per action, where the required action is defined by the advertisers, e.g., signing up for a service or ordering products etc.), and CPV (cost per view/visitor).

the advertising market explicitly and assume that advertiser demand positively depends on the consumer size.

Our model shows that a dominant pricing strategy for the free media platform is to apply lump-sum charges for the advertisers because it realizes higher profit compared to a per-consumer advertising charge. Moreover, the advertising level on the free platform is higher and attracts more consumers under lump-sum charges although advertisers must pay more per ad and consumers dislike ads. We find that the competing pay media platform's profit is lower if the free platform imposes a lump-sum charge on advertisers because the lower consumer demand overcompensates for the higher subscription fee yielding a lower profit. As a result, the strength of media consumers' disutility from ads determines whether aggregate profits are higher in regime A or B. Moreover, the advertisers are always better off and the media consumers are worse off if the advertiser charge is levied on a lump-sum basis. Overall, in small media markets, applying lump-sum advertiser charges always yields lower social welfare; in large media markets, this finding is true only if the media consumers' disutility from ads is sufficiently high.

In the remainder of the paper we proceed as follows. In the next section, we review the related literature. Section 3 introduces the basic model setup and Section 4 provides the equilibrium analysis. In Section 5, we compare the relevant outcomes of both regimes and derive our main results. Section 6 discusses our results and concludes the paper.

2 Related Literature

Our examination of asymmetric competition between a pay media platform and a free media platform that charges advertisers contributes to the literature on the economics of media markets in two dimensions.[4] First, we add to this literature by comparing lump-sum and per-consumer advertiser charges in an integrated framework. Second, we contribute to the literature because prior research focuses on symmetric competition between either free media platforms or pay media platforms and then compares the two independent scenarios separately.

In the area of media economics, most papers that explicitly model the advertising market explore one of the two advertising pricing models (lump-sum or per-consumer charges). Papers that assume a lump-sum advertising charge include, e.g., Gabszewicz et al. (2001), Crampes et al. (2009), Kind et al. (2009) and Reisinger (2011).

Gabszewicz et al. (2001) develop a model in which two symmetric competing newspapers play a three-stage game and sequentially set the political opinion, the prices of newspapers, and the advertising prices. They show that newspaper editors often tend to sell tasteless political messages to their readers in order to augment the audience size

[4]For a summary of the literature, see Anderson and Gabszewicz (2006).

and therefore to become more attractive to advertisers. Crampes et al. (2009) present a model of media competition with free entry by considering the number of active media platforms as endogenous.[5] In their model of symmetric competition, the media platforms are either financed with advertising and subscription revenues or they are solely funded by advertising receipts. The authors find that under constant or increasing returns to scale in the audience size, there are an excessive number of firms and underprovision of advertising in the markets. Kind et al. (2009) investigate how the number of the media platforms and the level of horizontal differentiation between media platforms could affect the way media firms raise their revenues. They demonstrate that symmetric media platforms generate less revenue from consumers when the horizontal differentiation is low. Media firms also generate less revenue from advertisers when there are more firms in the markets. Reisinger (2011) presents a two-sided market model of symmetric free media platforms that compete for advertisers and consumers. He also extends his model to a setting in which platforms charge consumers for the consumption of the platform's content. He shows that media platforms' profits can increase with users' nuisance cost of ads.

Models with a per-consumer advertising charge can be found, for example, in Anderson and Coate (2005) and Peitz and Valletti (2008). Anderson and Coate (2005) develop a general theory of market provision of broadcasting and analyze the nature of market failure in this industry. They show that symmetric commercial broadcasters provide advertising levels and programming amounts that can be above or below socially optimal levels, depending on how strongly viewers dislike advertising (among other factors). Peitz and Valletti (2008) focus on the endogenous provision of program diversity by symmetric television broadcasters. They analyze how the program diversity and advertising level (among others) may be affected under two different revenue regimes adopted by the TV broadcasters, pay TV with income from both viewers and advertisers and free TV with only advertising receipts. Broadcasters tend to vertically differentiate their channel programs more when they adopt pay TV than free TV. Moreover, the advertising level is higher under the free TV regime when viewers strongly dislike advertising.

The only paper that explicitly compares lump-sum and per-consumer advertising charges is Armstrong (2006). In his framework of a so-called competitive bottleneck, two media platforms (newspapers) generate revenues from two sources, readers and advertisers. There is competition for readers, but not for advertisers. Under the assumption that readers like (dislike) ads, the equilibrium reader price and platform profit is lower (higher) if platforms charge advertisers on a lump-sum rather than per-reader basis. In contrast to Armstrong (2006), who analyzes the symmetric competition between two pay media platforms (with subscription fees and advertising charges), we consider a scenario of asymmetric competition between one pay media platform (only with subscription rev-

[5]See also Choi (2006) for a model of broadcast competition with free entry.

enues) and one free media platform (only with advertising revenues).

In sum, neither of the above-mentioned papers on competition in media markets compares the two advertising pricing models in a framework of asymmetric competition between pay and free media platforms. Thus, our paper can offer insights about a scenario, in which pay and free media platforms coexist and compete for the same consumers. To the best of our knowledge, only a few papers model the direct competition between pay media and free media in a integrated framework.

Casadesus-Masanell and Zhu (2010) develop a model of duopoly competition between a high-quality incumbent and a low-quality ad-sponsored entrant. They investigate what the optimal reaction regarding own business model for the incumbent could be when it faces a new ad-sponsored entrant. They consider four different business models for the incumbent: a subscription-based model; an ad-sponsored model; a mixed model with both subscription and ads; and a dual model with two products (one based on the ad-sponsored model and the other based on the mixed model). The case in which the incumbent chooses a subscription-based model is similar to the asymmetric competition in our setting. Lin (2011) studies the endogenous provision of program quality by one pay TV broadcaster and one free TV broadcaster competing directly against each other. He shows that depending on the viewer's nuisance cost of ads and on the degree of horizontal differentiation, pay TV does not always offer higher quality programming than free TV. Dietl et al. (2012) also model asymmetric competition between a pay TV broadcaster and a free TV broadcaster to analyze the economic effects of introducing advertising on the pay TV channel. They show that under certain circumstances there is scope for the pay TV broadcaster to place ads on its channel. By doing so, viewers will always benefit from it while aggregate broadcaster profits may increase if the viewer's disutility from ads is sufficiently high. However, neither of the three aforementioned papers explicitly models the advertising market nor compares the two different advertising pricing models (lump-sum versus per-consumer charges).

Our paper is related also to the literature on two-sided markets.[6] Media markets are canonical examples of two-sided markets in which media platforms serve two distinct groups of agents, media consumers and advertisers. It is essential for the media platforms to take into account the existence of indirect network effects between media consumers and advertisers. There are positive network effects that operate from media consumers to advertisers such that the value of the media platform for the advertisers increases with the number of media consumers.[7] In contrast, the network effects that operate from advertisers to consumers are considered to be mainly negative in media industries.[8]

[6]Classical works on two-sided markets include Caillaud and Jullien (2003), Rochet and Tirole (2003), Rysman (2004), Armstrong (2006), Hagiu (2006), Kaiser and Wright (2006), Belleflamme and Toulemonde (2009), and Weyl (2010).

[7]See Gabszewicz et al. (2004) and Kind et al. (2007).

[8]An exception is Kaiser and Song (2009) who find that in the German magazine market, readers

In our model only the free media platform operates a two-sided market strategy while the pay media platform operates in a traditional one-sided market. Although the free media platform does not price both market sides, due to the existence of indirect network effects between advertisers and media consumers, such a platform can also be considered two-sided.

3 Model

We consider a media market with three types of agents: consumers (users), platforms, and advertisers. The media market is served by one pay media platform and one free media platform. The pay media platform charges its consumers a subscription fee for access to the media content whereas the free media platform gives free access to its consumers with no further monetary charges. There are no ads on the pay media platform while the consumers are exposed to ads on the free media platform. We differentiate two pricing regimes for advertisers, lump-sum and per-consumer charges.

3.1 Consumers

Suppose there are $\theta \in \mathbb{R}^+$ media consumers uniformly distributed along the unit interval. The two media platforms are located at the extremes of the unit interval where the pay media platform (denoted by subscript p) is situated at 0 and the free media platform (denoted by subscript f) is situated at 1. We consider the Hotelling model with linear transport cost of $t \in \mathbb{R}^+$ per unit of length. Hence, the two media platforms are horizontally differentiated from the perspective of consumers and the parameter t can be interpreted as the differentiation parameter. A lower value of t means that the media platforms (or rather their media content) are perceived as closer substitutes by the consumers.

The indirect utility function of a consumer located at point $x \in [0, 1]$ when consuming media on the pay media platform or on the free media platform is given by

$$u_p = v - s_p - tx, \tag{1}$$

$$u_f = v - \gamma a_f - t(1 - x), \tag{2}$$

where s_p is the subscription fee and a_f is the level of advertising on the free media platform. The parameter $v \in \mathbb{R}^+$ denotes the consumers' intrinsic value from consuming media and the parameter $\gamma \in \mathbb{R}^+$ measures the level of consumers' disutility from ads.[9]

actually appreciate informative ads, such as car ads in car magazines. However, in this case, we believe advertising is rather part of media content than a separate by-product.

[9] As mentioned in Section 2, recent works in media industries assume that consumers do not like ads and derive a disutility from it. We follow this approach, which allows us to focus on the trade-off for

We assume full consumer market coverage, i.e., the consumers' intrinsic value from consuming media is sufficiently large such that all consumers will join one media platform. We also assume that no media platform can corner the consumer market such that each media platform gains a positive market share. The marginal consumer, who is indifferent between consuming pay media and free media, is located at $\overline{x} = \frac{1}{2} + \frac{1}{2t}(\gamma a_f - s_p)$. All consumers to the left of $\overline{x}$ consume the content of the pay media platform and all consumers to the right of $\overline{x}$ consume the content of the free media platform. As a result, the demand function of the pay media and free media consumers, respectively, are given by

$$n_p = \frac{\theta}{2}\left[1 + \frac{1}{t}(\gamma a_f - s_p)\right], \tag{3}$$

$$n_f = \theta - n_p = \frac{\theta}{2}\left[1 + \frac{1}{t}(s_p - \gamma a_f)\right]. \tag{4}$$

The consumers of the pay media and free media derive the following surpluses:

$$CS_p = \theta \int_0^{\overline{x}} (v - s_p - tz)\, dz \text{ and } CS_f = \theta \int_{\overline{x}}^{1} (v - \gamma a_f - t(1-z))\, dz. \tag{5}$$

Because the consumer market is fully covered, the aggregate consumer surplus is the sum of all consumers' net benefits from consuming media, $CS = CS_p + CS_f$.

3.2 Advertisers

Advertisers are producers of goods or services who want to attract potential buyers through ads on the free media platform. As in previous studies (see, e.g., Crampes et al., 2009), we assume that each advertiser can place only one ad on the free media platform such that the number of advertisers also represents the number of ads. We assume that the advertisers incur the cost η for designing and producing one ad. Advertisers are heterogeneous with respect to η where η is assumed to be uniformly distributed in the unit interval, $\eta \sim U[0;1]$. We assume that the net utility of advertisers is given by

$$u_a = \beta n_f - p_f - \eta,$$

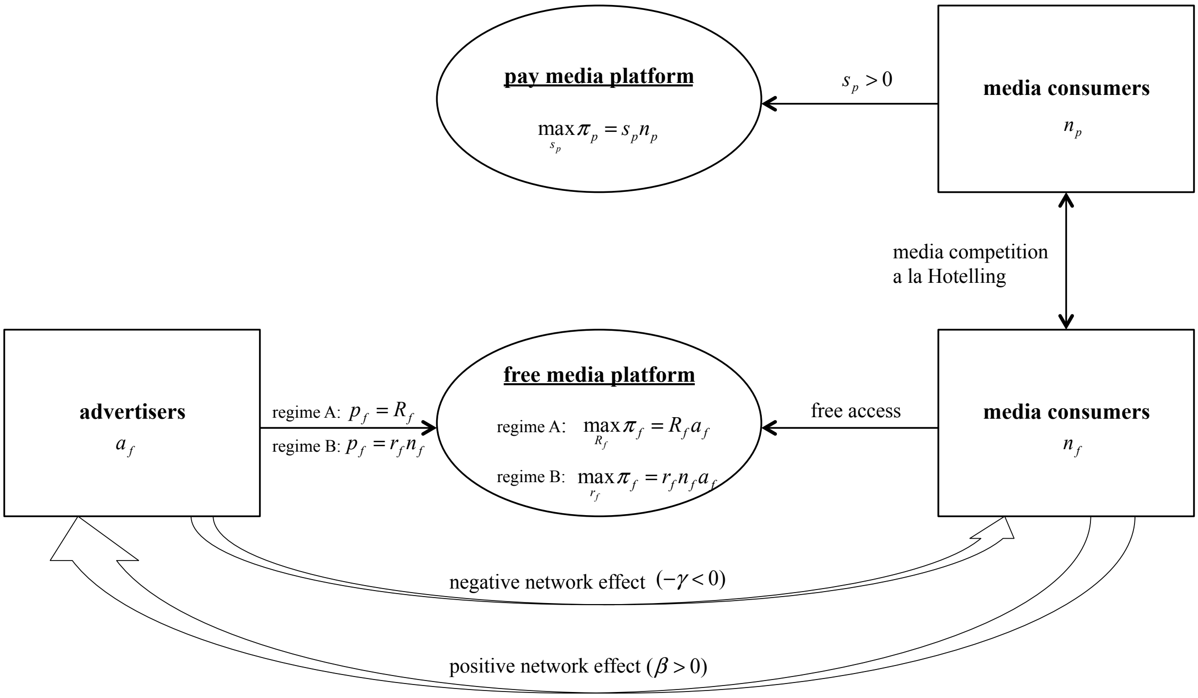
where $\beta \in \mathbb{R}^+$ measures the marginal gross benefit of an advertiser derived from each media consumer and p_f is the price an advertiser has to pay per ad. An advertiser decides to place an ad if her net utility is non-negative, $u_a \geq 0$. By normalizing the mass of advertisers to unity, we derive the advertiser demand as[10]

$$a_f = \beta n_f - p_f.$$

consumers between ads-free media with a subscription fee and free media that includes ads.

[10]For a similar derivation of advertiser demand, see e.g., Li (2009).

Figure 1: Model Illustration



Advertiser surplus is then given by

$$AS = \int_0^{a_f} (\beta n_f - y - p_f)\, dy. \tag{6}$$

Advertiser surplus is the positive difference between the amount that advertisers are willing and able to pay for placing an ad and the amount that they actually pay.

3.3 Media Platforms

The media platforms provide the content for the consumers. The pay media platform generates its revenues purely from the media consumers through subscription fees, while the free media platform generates its revenues from advertising receipts. For simplicity, all incurring costs for the media platforms are assumed to be 0. The profit functions of the pay and free media platforms, respectively, are then given by

$$\pi_p = s_p n_p \text{ and } \pi_f = p_f a_f.$$

We consider two advertising pricing models on the free media platform: In regime A, the advertising charge is levied on a lump-sum basis where R_f denotes the fixed price that each advertiser has to pay per ad, i.e., $p_f = R_f$. In regime B, advertisers are charged on a per-consumer basis where r_f denotes the charge that each advertiser has to pay per consumer for placing one ad. Hence, in regime B the price per ad now positively depends on the number of attracted media consumers, i.e., $p_f = r_f n_f$. Figure 1 graphically illustrates our model.

4 Analysis

In this section, we derive the equilibrium outcomes by assuming that both media platforms simultaneously maximize their profits. The pay media platform sets the subscription fee s_p, while the free media platform either sets a fixed price for an ad or sets a per-consumer price for an ad. First, we analyze regime A in which advertisers pay a lump-sum charge. Second, we investigate regime B where advertisers are charged on a per-consumer basis.

4.1 Regime A: lump-sum advertising charges

In regime A, the advertising charge R_f is levied on a lump-sum basis and hence advertisers derive the net utility $u_a = \beta n_f - R_f - \eta$. The advertiser demand function is therefore given by $a_f = \beta n_f - R_f$. By substituting this demand function into (3) and (4), we obtain free media and pay media consumer demand in regime A as

$$n_f^A = \frac{\theta}{2t + \theta\beta\gamma} (t + s_p + \gamma R_f) \text{ and } n_p^A = \theta - n_f^A. \tag{7}$$

Consequently, the advertising level is given by

$$a_f^A = \beta n_f^A - R_f = \frac{\theta\beta}{2t + \theta\beta\gamma} \left(t + s_p - \frac{2t}{\theta\beta} R_f \right). \tag{8}$$

It is intuitively clear that a higher subscription fee s_p on the pay media platform decreases consumer demand n_p^A on this platform and increases consumer demand n_f^A on the free media platform. As a result, the advertising level a_f^A on the free media platform also increases with a higher subscription fee. On the other hand, a higher advertising charge R_f on the free media platform induces a lower advertising level a_f^A on this platform. Because media consumers dislike ads, this reduction in the advertising level leads to a higher free media consumer demand n_f^A and consequently to a lower pay media consumer demand n_p^A.

The pay and free media platforms simultaneously solve $\max_{s_p > 0} \left\{ \pi_p^A = s_p n_p^A \right\}$ and $\max_{R_f > 0} \{ \pi_f^A = R_f a_f^A \}$, respectively. The equilibrium (s_p^{A*}, R_f^{A*}) is then characterized by[11]

$$\frac{\partial \pi_p^A}{\partial s_p} = n_p^A + s_p^A \frac{\partial n_p^A}{\partial s_p} = 0 \text{ and } \frac{\partial \pi_f^A}{\partial R_f} = a_f^A + R_f^A \frac{\partial a_f^A}{\partial R_f} = 0. \tag{9}$$

The two first-order conditions have an intuitive interpretation. For the pay media platform, a marginally higher subscription fee s_p induces a direct positive revenue effect n_p^A and an indirect negative consumer-mediated effect $s_p^A \frac{\partial n_p^A}{\partial s_p}$ through a reduction in consumer demand. The optimal subscription fee s_p^{A*} is chosen such that the revenue effect and consumer effect are balanced.

For the free media platform, marginally increasing the lump-sum advertising charge R_f triggers a direct positive revenue effect $a_f^A = \beta n_f^A - R_f$ and an indirect negative advertiser-mediated effect $R_f^A \frac{\partial a_f^A}{\partial R_f} = R_f^A (\beta \frac{\partial n_f^A}{\partial R_f} - 1)$ through a lower advertising level. The advertiser effect $\frac{\partial a_f^A}{\partial R_f}$ is composed of two effects: the first term, $\beta \frac{\partial n_f^A}{\partial R_f}$, represents the positive effect on the advertisers through higher consumer demand and the second term, -1, is the negative effect of a higher advertising price. The second effect dominates the first such that $\frac{\partial a_f^A}{\partial R_f} < 0$. Again, the platform chooses the optimal advertising price in a way that balances both countervailing effects (i.e., revenue effect and advertiser-mediated effect).

To make the notation simpler, we henceforth write $\lambda \equiv \theta\beta\gamma$. By solving the above system of first-order conditions, we compute the subscription fee and the lump-sum advertiser charge in equilibrium as

$$\left(s_p^{A*}, R_f^{A*}\right) = \left(\frac{t\,(4t + 3\lambda)}{8t + \lambda}, \frac{\theta\beta\,(3t + \lambda)}{8t + \lambda}\right).$$

Hence, each advertiser has to pay $p_f^{A*} = R_f^{A*}$ per ad on the free media platform. Substituting s_p^{A*}, and p_f^{A*} into the demand functions yields equilibrium demands of the pay media consumers and free media consumers (n_p^{A*}, n_f^{A*}) as well as the advertising level a_f^{A*} on the free media platform. Similarly, we obtain equilibrium platform profits (π_p^{A*}, π_f^{A*}), aggregate consumer surplus CS^{A*}, and advertiser surplus AS^{A*}. See the appendix for a detailed derivation of these outcomes.

4.2 Regime B: per-consumer advertising charges

In regime B, the advertising charge r_f is levied on a per-consumer basis and the price per ad is $p_f = r_f n_f$. Advertisers therefore enjoy a net utility of $u_a = \beta n_f - r_f n_f - \eta$ and their demand function is given by $a_f = \beta n_f - r_f n_f$. With a similar approach as in regime

[11]The second-order conditions are satisfied because $\frac{\partial^2 \pi_p}{\partial s_p^2} = -\frac{2\theta}{2t+\theta\beta\gamma} < 0$ and $\frac{\partial^2 \pi_f}{\partial r_f^2} = -\frac{4t}{2t+\theta\beta\gamma} < 0$.

A, we obtain free media and pay media consumer demand in regime B as

$$n_f^B = \frac{\theta}{2t + \theta\left(\beta - r_f\right)\gamma}\left(t + s_p\right) \text{ and } n_p^B = \theta - n_f^B. \tag{10}$$

Advertiser demand is then given by

$$a_f^B = n_f^B(\beta - r_f) = \frac{\theta}{2t + \theta\left(\beta - r_f\right)\gamma}\left(\beta - r_f\right)\left(t + s_p\right). \tag{11}$$

The same qualitative comparative statics for the demand functions hold as in regime A.

The pay and free media platforms solve $\max_{s_p>0}\left\{\pi_p^B = s_p n_p^B\right\}$ and $\max_{r_f>0}\{\pi_f^B = r_f n_f^B a_f^B\}$, respectively. The equilibrium (s_p^{B*}, r_f^{B*}) is then characterized by[12]

$$\frac{\partial \pi_p^B}{\partial s_p} = n_p^B + s_p^B \frac{\partial n_p^B}{\partial s_p} = 0, \tag{12}$$

$$\frac{\partial \pi_f^B}{\partial r_f} = n_f^B a_f^B + r_f^B\left(\frac{\partial n_f^B}{\partial r_f} a_f^B + n_f^B \frac{\partial a_f^B}{\partial r_f}\right) = 0. \tag{13}$$

A marginal higher subscription fee induces a direct positive revenue effect n_p^B and an indirect negative consumer-mediated effect $s_p\frac{\partial n_p^B}{\partial s_p}$. For the free media platform, marginally increasing the advertising charge triggers a direct positive revenue effect $n_f^B a_f^B$ and an indirect effect through changes in $n_f^B a_f^B$. This indirect effect is composed of two effects: first, marginally increasing r_f causes a positive consumer effect $\frac{\partial n_f^B}{\partial r_f} a_f^B > 0$ through a higher consumer demand level. Second, marginally increasing r_f induces a negative advertiser effect $n_f^B \frac{\partial a_f^B}{\partial r_f} < 0$ through a lower advertiser demand level. Which of the two effects dominates depends on the level of γ. We derive $\frac{\partial n_f^B a_f^B}{\partial r_f} < 0 \Leftrightarrow \gamma < \widehat{\gamma} = \frac{2t}{\left(\beta - r_f\right)\theta}$. Hence, if the media consumers' disutility from ads is sufficiently low, the positive consumer effect will be dominated by the negative advertiser effect because consumer demand increases to such an extent that it cannot overcompensate for the lower advertising level.

By solving the above system of first-order conditions, we compute the subscription fee and the per-consumer advertising charge in equilibrium as

$$\left(s_p^{B*}, r_f^{B*}\right) = \left(\frac{t\left(4t + 3\lambda\right)}{2\left(4t + \lambda\right)}, \frac{\beta\left(2t + \lambda\right)}{4t + \lambda}\right).$$

Substituting s_p^{B*} and r_f^{B*} into the demand functions yields equilibrium demands of the pay and free media consumers (n_p^{B*}, n_f^{B*}) as well as the advertising level a_f^{B*} on the free

[12] The second-order conditions are satisfied: $\frac{\partial^2 \pi_p}{\partial s_p^2} = \theta\left(-\frac{1}{2t} - \frac{1}{2t+\lambda}\right) < 0$ and $\frac{\partial^2 \pi_f}{\partial r_f^2} = -\frac{\theta^2(4t+\lambda)^2(12t+5\lambda)^2}{256t(2t+\lambda)^3} < 0$.

media platform. We further derive that each advertiser has to pay

$$p_f^{B*} = r_f^{B*} n_f^{B*} = \frac{\beta\theta\,(12t + 5\lambda)}{8(4t + \lambda)}$$

per ad. Analogous to above, we obtain equilibrium platform profits (π_p^{B*}, π_f^{B*}), aggregate consumer surplus CS^{B*}, and advertiser surplus AS^{B*}. See the appendix for a detailed derivation of these outcomes.

5 Comparison of Equilibrium Outcomes

In this section, we compare the outcomes of the two regimes. First, we compare the subscription fees on the pay media platform and the prices advertisers must pay for an ad placed on the free media platform.

Proposition 1 *(i) The subscription fee that a consumer must pay on the pay media platform is higher in regime A than in regime B, i.e.,* $s_p^{A*} > s_p^{B*}$*.*

(ii) The price that an advertiser must pay per ad on the free media platform is higher in regime A than in regime B, i.e., $p_f^{A*} > p_f^{B*}$*.*

Proof. See Appendix B.1. ■

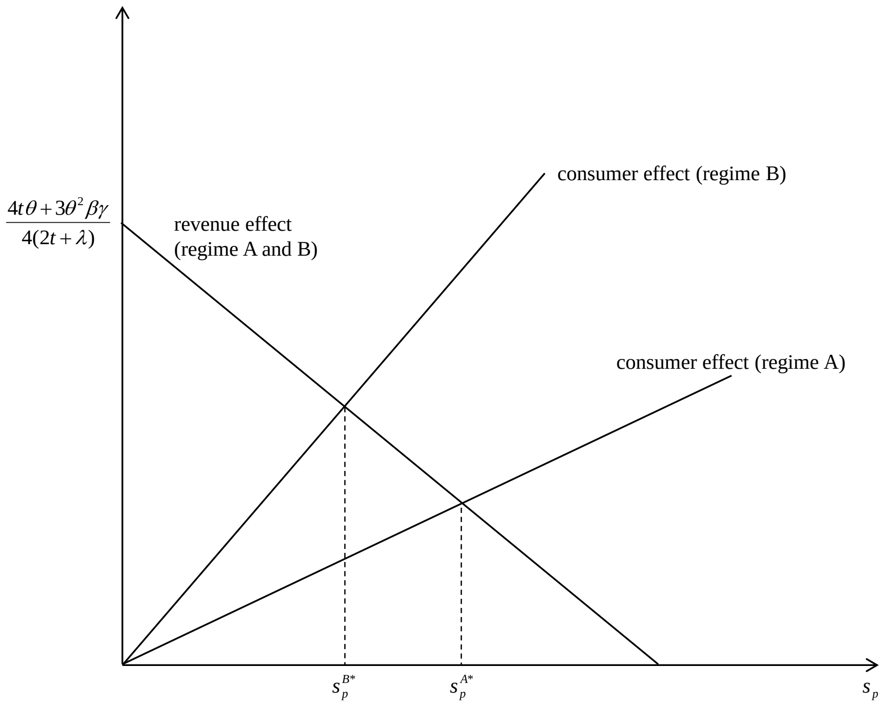
Part (i) posits that the pay media platform charges consumers a higher subscription fee if the free media platform chooses a lump-sum advertiser charge as compared to a per-consumer charge. To understand the intuition behind this result, we rearrange the first-order conditions (9) and (12) to obtain

$$\frac{\theta}{2t + \lambda}\left(t + \lambda - \gamma R_f^A - s_p^A\right) = s_p^A \left(\frac{\theta}{2t + \lambda}\right), \tag{14}$$

$$\frac{\theta}{2t + \theta\gamma\left(\beta - r_f^B\right)}\left(t + \lambda - \gamma\theta r_f^B - s_p^B\right) = s_p^B \left(\frac{\theta}{2t + \theta\gamma\left(\beta - r_f^B\right)}\right). \tag{15}$$

The left-hand side (lhs) of both equations represent the revenue effect n_p, while the right-hand sides (rhs) characterize the consumer-mediated effects $s_p(-\frac{\partial n_p}{\partial s_p})$. Substituting the best-response functions $R_f^A(s_p^A) = \frac{\theta\beta(t+s_p^A)}{4t}$ and $r_f^B = \frac{\beta(2t+\theta\beta\gamma)}{4t+\theta\beta\gamma}$ into (14) and (15), we obtain the following results. For a given subscription fee s_p, the revenue effects (lhs) are equal in both regimes, while the consumer-mediated effects (rhs) are stronger in regime B than in regime A. Hence, increasing the subscription fee decreases the revenue effects with equal strength in both regimes, while the consumer effect increases more strongly in regime B than in A. As a result, the equilibrium subscription fee is larger in regime A than in B, i.e., $s_p^{A*} > s_p^{B*}$. Figure 2 depicts these effects as a function of the subscription fee s_p.

Figure 2: Intuition for Proposition 1



According to part (ii), the per-ad price $p_f^{A*} = R_f^{A*}$ that each advertiser must pay if it is charged on a lump-sum basis is higher than the per-ad price $p_f^{B*} = r_f^{B*} n_f^{B*}$ if it is charged on a per-consumer basis. The intuition behind this result is as follows. In both regimes, marginally increasing the advertising charge induces a positive revenue effect and a negative advertiser effect. However, an additional effect is present in regime B: the platform takes into account that a higher advertising charge induces a higher consumer demand (positive consumer effect), which enters the first-order condition with a positive sign. This effect is not present in regime A because advertisers are charged on a lump-sum basis. As a result, we derive

$$p_f^A(s_p^A) = \frac{\theta\beta(t + s_p^A)}{4t} \text{ and } p_f^B(s_p^B) = \frac{\theta\beta(t + s_p^B)}{4t}.$$

Because we know that the subscription fee is higher in regime A than in B, i.e., $s_p^{A*} > s_p^{B*}$, it must be the case that $p_f^{A*} > p_f^{B*}$.

In the next proposition, we compare both regimes with respect to consumer demands and advertising level.

Proposition 2 *(i) The free (pay) media platform attracts more (fewer) consumers in regime A than in regime B, i.e.,* $n_f^{A*} > n_f^{B*}$ *and* $n_p^{A*} < n_p^{B*}$.

(ii) The advertising level on the free media platform is higher in regime A than in

regime B, i.e., $a_f^{A*} > a_f^{B*}$.

Proof. See Appendix B.2. ■

Even though the advertising level on the free platform is higher in regime A than in B, part (i) of the proposition posits that this platform attracts more consumers in regime A than in B. To understand the intuition behind this result, recall that the subscription fee on the pay media platform is higher in regime A than in B and consumer demand on the free media platform is given by $n_f = \frac{\theta}{2}\left[1 + \frac{1}{t}(s_p - \gamma a_f)\right]$. Increasing the subscription fee on the pay platform decreases consumers on this platform, and in turn, increases consumer demand on the free platform. Hence, the higher subscription fee on the pay platform in regime A compared to B, overcompensates for the higher advertising level on the free platform such that the free media consumer demand is higher in regime A than in B. Due to our Hotelling specification, it follows that the pay media consumer demand is higher in regime B than in A.

Part (ii) of the proposition shows that the free platform attracts more advertisers in regime A than in B. This result is true despite higher price per ad in regime A than in B. However, the free platform attracts more consumers in regime A than in B, which makes it more attractive for advertisers to place ads. We conclude that the higher consumer demand overcompensates for the higher price in regime A compared to B.

Next, we compare the profits of the media platforms in both regimes and establish the following proposition:

Proposition 3 *(i) The profits of the free (pay) media platform is higher (lower) in regime A than in regime B, i.e.,* $\pi_f^{A*} > \pi_f^{B*}$ *and* $\pi_p^{A*} < \pi_p^{B*}$.

(ii) Aggregate platform profits are higher in regime A than in regime B if and only if the consumers' disutility from ads is sufficiently low, i.e., $\Pi^{A*} > \Pi^{B*} \Leftrightarrow \gamma < \gamma^{\pi}$.

Proof. See Appendix B.3. ■

Part (i) of the proposition states that the free media platform generates higher profits if it prices advertisers via a lump-sum charge as compared to a per-consumer charge. Recall that profits of the free platform are given by $\pi_f = p_f a_f$. Because each advertiser must pay a higher price p_f per ad in regime A than in B, and in addition, the advertising level a_f is also higher in regime A than in B, the claim follows immediately. Regarding the pay media platform, we find that this platform generates higher profits $\pi_p = s_p n_p$ if the free platform charges advertisers on a per-consumer basis as opposed to a lump-sum charge. Recall that the subscription fee is lower but consumer demand on the pay platform is higher in regime B compared to A. Hence, the higher consumer demand overcompensates for the lower subscription fee such that profits of the pay platforms are higher in regime B than in A.

Part (ii) shows that whether aggregate platform profits is higher in regime A or B crucially depends on the consumer preferences towards ads. If the consumers sufficiently dislike ads then aggregate profits are higher in the case that the free platform charges advertisers on a per-consumers basis. If, however, the consumers' disutility from ads is sufficiently low then aggregate profits are higher in the case that the free platform charges advertisers on a lump-sum basis.

To understand the intuition behind this result, we analyze how the components of the profit functions react to changes in the disutility parameter of ads γ. For the pay media platform, the subscription fee increases with γ in both regimes and the increase is stronger in regime A than in B. Hence, the spread between the subscription fees augments in γ. The effect of γ on consumer demand n_p^{A*} in regime A is ambiguous, while the effect is positive on consumer demand n_p^{B*} in regime B. Overall, the spread between n_p^{B*} and n_p^{A*} increases in γ. It follows that the difference in pay media profits $\Delta_p = \pi_p^{B*} - \pi_p^{A*}$ between regime B and A also augments in γ. Regarding the free media platform, it is intuitive that the advertising level a_f^* on the free media platform in both regimes decreases in γ. However, it depends on the level of γ whether the decrease is stronger in regime A or B. Furthermore, the price p_f^* that advertisers have to pay for placing an ad increases in γ because a higher level of γ leads to a lower advertising level which, ceteris paribus, increases advertisers' willingness to pay. The increase in the price for the advertisers is more pronounced in regime A than B such that the price spread between regime A and B augments for an increasing γ. Finally, we find that profits π_f^* of the free media platform decreases with γ in both regimes but it depends on the level of γ in which regime the decrease in profits is more pronounced. Particularly, the difference in free media profits $\Delta_f = \pi_f^{A*} - \pi_f^{B*}$ between regime A and B reaches its maximum for low values of γ and then diminishes for higher values of γ.

Overall, we conclude that for low values of γ the difference in free media profits Δ_f compensates for the difference in pay media profits Δ_p such that aggregate profits are higher in regime A than in B. Because Δ_f diminishes and Δ_p augments for higher values of γ, a critical value of $\gamma = \gamma^\pi$ exist above which aggregate profits are higher in regime B than in A.

The next proposition compares aggregate consumer surplus and advertiser surplus in both regimes.

Proposition 4 *(i) Aggregate consumer surplus is higher in regime B than in regime A, i.e.,* $CS^{B*} > CS^{A*}$.

(ii) The advertiser surplus is higher in regime A than in regime B, i.e., $AS^{A*} > AS^{B*}$.

Proof. See Appendix B.4. ■

Part (i) states that the media consumers are better off in regime B than in regime A. This result is intuitive because the consumers benefit from a lower subscription fee and a lower advertising level. In contrast, advertisers enjoy a higher surplus in regime A than in regime B, as stated in part (ii) of the proposition. On one hand, advertisers benefit from higher consumer demand in A compared to B, but on the other hand, they face higher prices. As we know from Proposition 2, the higher consumer demand outweighs the higher price such that the advertising level in A is higher than in B. Overall, the higher advertising level together with the higher consumer demand compensate for the higher price such that the advertiser surplus is higher in A compared to B.

In a final step, we compare social welfare in both regimes. We define social welfare W, as the sum of aggregate platform profits, aggregate consumers surplus and advertiser surplus

$$W = \Pi + CS + AS.$$

We establish the following proposition:

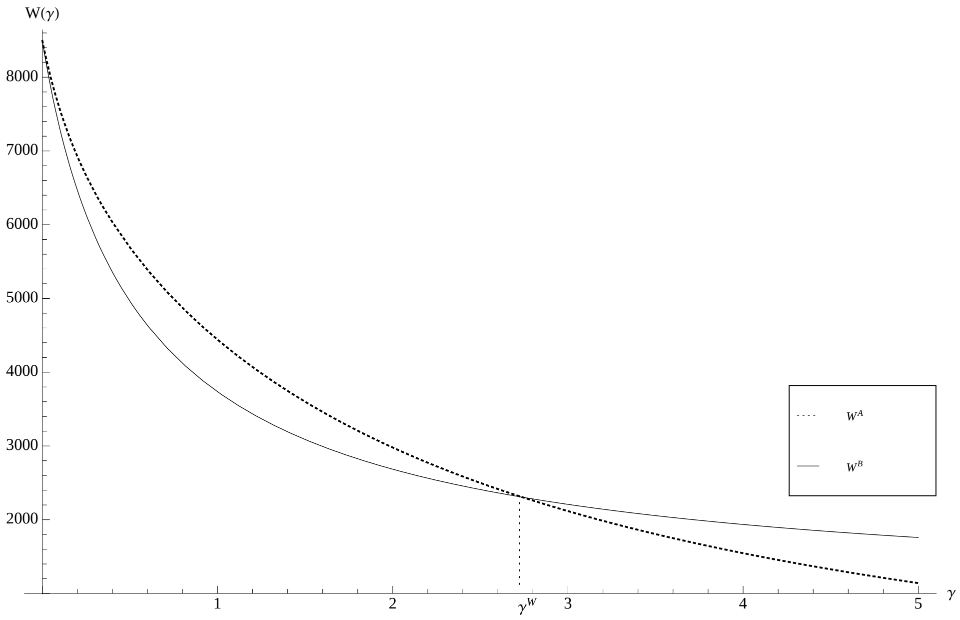
Proposition 5 *In large media markets (i.e., $\theta > \theta'$), social welfare is higher in regime B than in regime A if and only if the consumers' disutility from ads is sufficiently high, i.e., $W^{B*} > W^{A*} \Leftrightarrow \gamma > \gamma^W$. However, in small media markets (i.e., $\theta \leq \theta'$), social welfare is always higher in regime B than in regime A regardless of the consumers' disutility from ads.*

Proof. See Appendix B.5. ■

The proposition shows that the welfare effect of charging advertisers on a lump-sum basis or a per-consumer basis depends on the market size of consumers and the consumers' disutility from ads. In large media markets (i.e., $\theta > \theta' = \frac{8t}{9\beta^2}$) the disutility must be sufficiently low to ensure that social welfare is higher in regime A than in B. If, however, consumers sufficiently dislike ads then the opposite holds true. In small media markets, in contrast, social welfare is always higher if advertisers are charged on a per-consumer basis than if they are charged on a lump-sum basis. Figure 3 illustrates the result for large media markets by depicting social welfare in regime A and B as a function of the disutility γ. For the figure, we set the parameters as follows: $N = 50$, $t = 30$, $v = 50$ and $\beta = 3.5$.

The figure shows that in both regimes social welfare is a concave function in γ and that for $\gamma = 0$, social welfare would be equal in both regimes. For low values of γ, social welfare is higher in regime A than in B because welfare decreases with γ stronger in regime B than in A. However, for intermediate values of γ, social welfare decreases less strong in regime B than in A such that there exists a critical value $\gamma = \gamma^W$ for which welfare is equal in both regimes. Above this critical value γ^W, welfare is higher in regime B than in A. To understand the intuition behind this result, recall that the aggregate consumer surplus is always lower in A than in B, while the opposite is true for the advertiser surplus. In addition, aggregate platform profits are higher in regime B than in A if the consumers' disutility from ads is sufficiently high. Hence, if γ is low, then the higher advertiser surplus together with the higher aggregate platform profits

Figure 3: Social Welfare



outweigh the higher aggregate consumer surplus in regime A compared to B such that $W^{A*} > W^{B*}$, which is is true as long as $\gamma < \gamma^W$. If γ increases above this critical value γ^W, the difference in the aggregate consumer surpluses between regime B and A is so large that it compensates for the lower advertiser surplus and eventually lower platform profits, yielding a higher level of social welfare in regime B compared to A. In the case of small media markets, this overcompensation of aggregate consumer surpluses between regime B and A is true regardless of the consumers' disutility from ads such that social welfare is always higher in regime B compared to A.[13]

6 Conclusion

This paper is motivated by the observation that in reality pay and free media platforms often coexist and directly compete with each other. The existing literature, however, widely neglects to model asymmetric competition in media markets. To begin filling this research gap in this paper, we develop a simple model of asymmetric competition between a pay media platform and a free media platform. Specifically, we question how different advertising pricing models (lump-sum versus per-consumer charges) affect relevant equilibrium outcomes.

Our paper shows that profit-maximizing free media platforms should charge their

[13]Formally, the differences of aggregate platform profits as well as advertiser surpluses between regimes B and A decrease more strongly than the respective difference of consumer surplus between regimes A and B for low parameters of θ.

advertisers on a lump-sum basis rather than a per-consumer basis because in doing so they can realize higher profits. However, from the perspective of a social planner this is not always desirable because, at the same time, the profit of the pay media platform will be lower such that the effect on aggregate platform profits depends on the media consumers' disutility from ads. Particularly, if the disutility is sufficiently low, aggregate platform profits are higher under lump-sum charges. Furthermore, the advertisers are always better off if they are charged on a lump-sum basis, while the media consumers are better off if advertisers are charged on a per-consumer basis. Hence, choosing one of the two advertising pricing models benefits either the consumers or the advertisers. Finally, our model shows that social welfare is higher if the advertisers are charged on a per-consumer basis when the media market is small, while this is true in large media markets only if the media consumers' disutility from ads is sufficiently high.

Our analysis has implications for policy makers and regulators in the media industries. One common form of advertising regulation is to directly limit the number or length of ads on media platforms through so-called "advertising caps." Our results suggest another indirect instrument could achieve the aim of low advertising levels. According to our model, the prohibition of lump-sum advertising charges and the enforcement of per-consumer charges lead to a lower advertising level on free media platforms. Such an enforced per-consumer advertising charge could be particularly relevant for the television broadcasting industry where lump-sum advertising charges are predominant due to the difficulty of measuring the exact number of free TV viewers. However, policy makers and regulators should be aware of the resulting welfare effects of such a regulation because in large media markets social welfare could decrease through per-consumer charges, especially if the consumers' disutility from ads is low.

Our model could be extended in several directions. For example, a promising avenue for further research would be the integration of endogenous quality provision by the media platforms. In our model, we focused on the role of consumers' disutility from ads as a reason for the existence of pay media. However, this is obviously not the only reason. In addition to ad-free content, pay media platforms often provide high-quality content such as Hollywood Blockbusters or exclusive premium sports rights. Another possible extension of our model is to consider a situation in which the pay media platform also generates advertising revenues. An example for such platforms can be found in the newspaper industry where traditional paid newspapers (revenue from readers and advertisers) compete against free newspapers (revenue from advertisers). In such a setting, it could be interesting to investigate whether an asymmetric equilibrium arises in which the pay platform chooses lump-sum advertising charges and the free platform decides for per-consumer charges, or vice versa.

A Appendix: Equilibrium Outcomes

A.1 Regime A

Plugging (s_p^{A*}, R_f^{A*}) into the demand function yields equilibrium demands of the pay media consumers as

$$n_p^{A*} = \frac{\theta t\left(4t + 3\lambda\right)}{\left(2t + \lambda\right)\left(8t + \lambda\right)}.$$

On the free media platform, consumer and advertiser demands, respectively, are

$$\left(n_f^{A*}, a_f^{A*}\right) = \left(\frac{\theta\left(3t + \lambda\right)\left(4t + \lambda\right)}{\left(2t + \lambda\right)\left(8t + \lambda\right)}, \frac{2\theta t\beta\left(3t + \lambda\right)}{\left(2t + \lambda\right)\left(8t + \lambda\right)}\right).$$

By noting that the marginal consumer in equilibrium is given by $\overline{x} = \frac{t(4t+3\lambda)}{(2t+\lambda)(8t+\lambda)}$, consumer surplus is given by

$$CS^{A*} = \left(v - \frac{5t}{2} + \frac{19t^3 + 8\lambda t^2}{27\left(2t + \lambda\right)^2} + \frac{10\left(326t^3 + 37\lambda t^2\right)}{27\left(8t + \lambda\right)^2}\right)\theta > 0, \tag{16}$$

and advertiser surplus is

$$AS^{A*} = \frac{2\theta^2 t^2 \beta^2 \left(3t + \lambda\right)^2}{\left(2t + \lambda\right)^2 \left(8t + \lambda\right)^2}. \tag{17}$$

Equilibrium profits are then given by

$$\pi_p^{A*} = s_p^{A*} n_p^{A*} = \frac{\theta t^2 \left(4t + 3\lambda\right)^2}{\left(2t + \lambda\right)\left(8t + \lambda\right)^2} \text{ and } \pi_f^{A*} = p_f^{A*} a_f^{A*} = \frac{2t\theta^2 \beta^2 \left(3t + \lambda\right)^2}{\left(2t + \lambda\right)\left(8t + \lambda\right)^2}. \tag{18}$$

A.2 Regime B

Plugging (s_p^{B*}, r_f^{B*}) into the demand function yields equilibrium demands of the pay media consumers as

$$n_p^{B*} = \frac{\theta\left(4t + 3\lambda\right)}{8\left(2t + \lambda\right)}.$$

On the free media platform, consumer and advertiser demands, respectively, are

$$\left(n_f^{B*}, a_f^{B*}\right) = \left(\frac{\theta\left(12t + 5\lambda\right)}{8\left(2t + \lambda\right)}, \frac{\theta t\beta\left(12t + 5\lambda\right)}{4\left(2t + \lambda\right)\left(4t + \lambda\right)}\right).$$

By noting that the marginal consumer in equilibrium is given by $\overline{x} = \frac{4t+3\lambda}{8(2t+\lambda)}$, consumer surplus and advertiser surplus are respectively given by

$$CS^{B*} = \left(v - \frac{103t}{64} + \frac{11t^3 + 5\lambda t^2}{16\left(2t + \lambda\right)^2} + \frac{4t^2}{4t + \lambda}\right)\theta > 0, \tag{19}$$

$$AS^{B*} = \frac{\theta^2 t^2 \beta^2 \left(12t + 5\lambda\right)^2}{32\left(2t + \lambda\right)^2 \left(4t + \lambda\right)^2}. \tag{20}$$

Equilibrium profits can be derived as

$$\pi_p^{B*} = s_p^{B*} n_p^{B*} = \frac{\theta t \left(4t + 3\lambda\right)^2}{16 \left(2t + \lambda\right) \left(4t + \lambda\right)} \text{ and } \pi_f^{B*} = p_f^{B*} a_f^{B*} = \frac{t\theta^2 \beta^2 \left(12t + 5\lambda\right)^2}{32 \left(2t + \lambda\right) \left(4t + \lambda\right)^2}. \tag{21}$$

B Appendix: Proofs

B.1 Proof of Proposition 1

Part (i). This part posits that the subscription fee is higher in regime A than in regime B ($s_p^{A*} > s_p^{B*}$). This proof is straightforward and thus omitted.

Part (ii). This part claims that the advertiser's price per ad is higher in regime A than in regime B, i.e., $p_f^{A*} > p_f^{B*} = r_f^{B*} n_f^{B*}$. We derive

$$p_f^{A*} > p_f^{B*} \Leftrightarrow \frac{\theta\beta \left(3t + \lambda\right)}{8t + \lambda} > \frac{\beta \left(2t + \lambda\right)}{4t + \lambda} \frac{\theta \left(12t + 5\lambda\right)}{8 \left(2t + \lambda\right)} = \frac{\theta\beta \left(12t + 5\lambda\right)}{32t + 8\lambda}.$$

We rearrange the inequality in the following way and obtain

$$\theta\beta \left(3t + \lambda\right) \left(32t + 8\lambda\right) - \theta\beta \left(12t + 5\lambda\right) \left(8t + \lambda\right) > 0.$$

After simplifying, we have

$$4\theta^2 \beta^2 t\gamma + 3\theta^3 \beta^3 \gamma^2 > 0,$$

which proves part (ii) of proposition 1. This completes the proof of proposition 1.

B.2 Proof of Proposition 2

Part (i). This part of the proposition 2 claims that the consumer demand for the pay (free) media platform is lower (higher) in regime A than in regime B, i.e., $n_p^{A*} < n_p^{B*}$ and $n_f^{A*} > n_f^{B*}$. First, we prove the claim for the pay media platform. Hence,

$$n_p^{A*} < n_p^{B*} \Leftrightarrow \frac{\theta t \left(4t + 3\lambda\right)}{\left(2t + \lambda\right) \left(8t + \lambda\right)} < \frac{\theta \left(4t + 3\lambda\right)}{8 \left(2t + \lambda\right)}.$$

We rearrange the inequality in the following way and obtain

$$\theta t \left(4t + 3\lambda\right) 8 \left(2t + \lambda\right) - \left(2t + \lambda\right) \left(8t + \lambda\right) \theta \left(4t + 3\lambda\right) < 0.$$

After simplifying we get

$$-\lambda\theta \left(4t + 3\lambda\right) \left(2t + \lambda\right) < 0$$

and conclude $n_p^{A*} < n_p^{B*}$. By noting that $n_f^{k*} = \theta - n_f^{k*}$, $k \in \{A, B\}$, it immediately follows that the consumer demand for the free media platform is higher in regime A than in regime B, i.e., $n_f^{A*} > n_f^{B*}$.

Part (ii). This part of proposition 2 claims that the advertising level in regime A exceeds that in regime B, i.e., $a_f^{A*} > a_f^{B*}$. Hence,

$$a_f^{A*} > a_f^{B*} \Leftrightarrow \frac{2\theta t\beta\left(3t+\lambda\right)}{\left(2t+\lambda\right)\left(8t+\lambda\right)} > \frac{\theta t\beta\left(12t+5\lambda\right)}{4\left(2t+\lambda\right)\left(4t+\lambda\right)}.$$

We rearrange the inequality in the following way and obtain

$$2\theta t\beta\left(3t+\lambda\right)4\left(2t+\lambda\right)\left(4t+\lambda\right)-\left(2t+\lambda\right)\left(8t+\lambda\right)\theta t\beta\left(12t+5\lambda\right)>0.$$

After simplifying we get

$$3t\left(2t+\lambda\right)\theta^3\beta^3\gamma^2>0.$$

and it follows that part (ii) of proposition 2 holds. This completes the proof of proposition 2.

B.3 Proof of Proposition 3

Part (i). To prove part (i) of proposition 3, that is, the free media platform realizes a higher profit in regime A than in regime B ($\pi_f^{A*} > \pi_f^{B*}$), we have to show that

$$\frac{2t\theta^2\beta^2\left(3t+\lambda\right)^2}{\left(2t+\lambda\right)\left(8t+\lambda\right)^2} > \frac{t\theta^2\beta^2\left(12t+5\lambda\right)^2}{32\left(2t+\lambda\right)\left(4t+\lambda\right)^2}.$$

We rearrange the inequality in the following way and obtain

$$2t\theta^2\beta^2\left(3t+\lambda\right)^2 32\left(2t+\lambda\right)\left(4t+\lambda\right)^2-\left(2t+\lambda\right)\left(8t+\lambda\right)^2 t\theta^2\beta^2\left(12t+5\lambda\right)^2>0.$$

After simplifying we get

$$t\theta^2\beta^2\left(2t+\lambda\right)\left(768t^3\lambda+1008t^2\lambda^2+376t\lambda^3+39\lambda^4\right)>0.$$

Equivalently, for the proof of the other claim in part (i), i.e., the profit of the pay media platform is higher in regime B than in regime A ($\pi_p^{A*} < \pi_p^{B*}$), one has to show

$$\frac{\theta t^2\left(4t+3\lambda\right)^2}{\left(2t+\lambda\right)\left(8t+\lambda\right)^2} < \frac{\theta t\left(4t+3\lambda\right)^2}{16\left(2t+\lambda\right)\left(4t+\lambda\right)}.$$

We rearrange the inequality in the following way and obtain

$$\theta t^2\left(4t+3\lambda\right)^2 16\left(2t+\lambda\right)\left(4t+\lambda\right)-\left(2t+\lambda\right)\left(8t+\lambda\right)^2\theta t\left(4t+3\lambda\right)^2<0.$$

After simplifying we get

$$-\theta\lambda^2 t\,(4t+3\lambda)^2\,(2t+\lambda)<0.$$

and it follows that part (i) of proposition 3 holds.

Part (ii). To prove part (ii) of proposition 3 ($\Pi^{A*}>\Pi^{B*}\Leftrightarrow\gamma<\gamma^{\pi}$), first let $\Pi^{A*}-\Pi^{B*}\equiv H$ with $\Pi^{A*}=\pi_p^{A*}+\pi_f^{A*}$ and $\Pi^{B*}=\pi_p^{B*}+\pi_f^{B*}$. One can calculate that $H=\frac{\kappa_1}{\kappa_2}I$, where

$$\begin{aligned}\kappa_1 &= \theta^3\beta^2\gamma t\,(4t+3\lambda)\ \text{ and } \kappa_2=32\,(2t+\lambda)\,(4t+\lambda)^2\,(8t+\lambda)^2,\\ I &= 32t^2\,(6\beta-\gamma)+4t\,(27\beta-8\gamma)\,\lambda+(13\beta-6\gamma)\,\lambda^2.\end{aligned}$$

We now derive the following properties:

(1). $\frac{\kappa_1}{\kappa_2}>0\forall\gamma\in(0,\infty)$.

(2). $\lim_{\gamma\to\infty}H=\lim_{\gamma\to\infty}I=-\infty$.

(3). $I(\gamma=0)=192t^2\beta>0$.

(4). $I(\gamma)$ has two critical points with $\gamma_1=\frac{-32\theta t\beta+13\theta^2\beta^3-\sqrt{448\theta^2t^2\beta^2+1112t\theta^3\beta^4+169\theta^4\beta^6}}{18\theta^2\beta^2}$ and $\gamma_2=\frac{-32\theta t\beta+13\theta^2\beta^3+\sqrt{448\theta^2t^2\beta^2+1112\theta^3t\beta^4+169\theta^4\beta^6}}{18\theta^2\beta^2}$. Moreover, γ_1 is a global minimum and γ_2 is a global maximum because $\left.\frac{\partial^2 I}{\partial\gamma^2}\right|_{\gamma=\gamma_1}=2\sqrt{\theta^2\beta^2\left(448t^2+1112\theta t\beta^2+169\theta^2\beta^4\right)}>0$ and $\left.\frac{\partial^2 I}{\partial\gamma^2}\right|_{\gamma=\gamma_2}=-2\sqrt{\theta^2\beta^2\left(448t^2+1112\theta t\beta^2+169\theta^2\beta^4\right)}<0$.

(5). $\left.\frac{\partial I}{\partial\gamma}\right|_{\gamma=0}=4t\left(27\theta\beta^2-8t\right)$,

we thus distinguish three different cases:

(5.1). If $\left.\frac{\partial I}{\partial\gamma}\right|_{\gamma=0}<0$, then $\gamma_1<0$ and $\gamma_2<0$ with $\gamma_1<\gamma_2$.

(5.2). If $\left.\frac{\partial I}{\partial\gamma}\right|_{\gamma=0}>0$, then $\gamma_1<0$ and $\gamma_2>0$ with $\gamma_1<\gamma_2$.

(5.3). If $\left.\frac{\partial I}{\partial\gamma}\right|_{\gamma=0}=0$, then $\gamma_1<0$ and $\gamma_2=0$ with $\gamma_1<\gamma_2$.

From (1)-(5) it follows that $\Pi^{A*}<\Pi^{B*}$ if and only if γ exceeds a critical level γ_{crit}. For all values of γ smaller than γ_{crit} it is the case that $\Pi^{A*}>\Pi^{B*}$. This completes the proof of proposition 3.

B.4 Proof of Proposition 4

Part (i). This part of proposition 4 states that media consumers enjoy higher surpluses in regime B than in regime A ($CS^{A*}<CS^{B*}$). To prove it, we need to derive

$$\left(v-\frac{5t}{2}+\mu_1\right)\theta<\left(v-\frac{103t}{64}+\mu_2\right)\theta.$$

We rearrange the inequality in the following way and obtain

$$-\frac{5t}{2}+\mu_1+\frac{103t}{64}-\mu_2<0,$$

with

$$\mu_1 = \frac{19t^3 + 8\lambda t^2}{27\left(2t+\lambda\right)^2} + \frac{10\left(326t^3 + 37\lambda t^2\right)}{27\left(8t+\lambda\right)^2} \text{ and } \mu_2 = \frac{11t^3 + 5\lambda t^2}{16\left(2t+\lambda\right)^2} + \frac{4t^2}{4t+\lambda}.$$

After simplifying, we get

$$-\frac{t\lambda\left(4t+3\lambda\right)\left(256t^3 + 528\lambda t^2 + 224t\lambda^2 + 19\lambda^3\right)}{64\left(2t+\lambda\right)^2\left(4t+\lambda\right)\left(8t+\lambda\right)^2} < 0.$$

It follows that part (i) of proposition 4 is true.

Part (ii). This part of proposition 4 indicates a higher advertisers surplus in regime A than in regime B, i.e., $AS^{A*} > AS^{B*}$. Hence, we have to show that

$$\frac{2\theta^2 t^2 \beta^2 \left(3t+\lambda\right)^2}{\left(2t+\lambda\right)^2\left(8t+\lambda\right)^2} > \frac{\theta^2 t^2 \beta^2 \left(12t+5\lambda\right)^2}{32\left(2t+\lambda\right)^2\left(4t+\lambda\right)^2}.$$

We rearrange the inequality in the following way and obtain

$$2\theta^2 t^2 \beta^2 \left(3t+\lambda\right)^2 32\left(2t+\lambda\right)^2\left(4t+\lambda\right)^2 - \left(2t+\lambda\right)^2\left(8t+\lambda\right)^2 \theta^2 t^2 \beta^2 \left(12t+5\lambda\right)^2 > 0.$$

After simplifying, we get

$$\theta^3 \beta^3 t^2 \gamma \left(2t+\lambda\right)^2 \left(4t+3\lambda\right)\left(192t^2 + 108t\lambda + 13\lambda^2\right) > 0.$$

It follows that part (ii) of proposition 4 is true. This completes the proof of proposition 4.

B.5 Proof of Proposition 5

We derive social welfare in regimes A and B as

$$W^{A*} = \frac{\theta}{54}\left(54v - 135t + \Psi_1 + \Psi_2\right) \text{ and } W^{B*} = \frac{\theta}{64}\left(64v - 67t + \Psi_3 + \Psi_4\right)$$

with

$$\begin{aligned}
\Psi_1 &= \frac{3t^2\left(2t+\theta\beta^2\right)}{\left(2t+\lambda\right)^2} + \frac{2t\left(11t+4\theta\beta^2\right)}{\left(2t+\lambda\right)},\ \Psi_2 = \frac{20t\left(61t+5\theta\beta^2\right)}{\left(8t+\lambda\right)} - \frac{375t^2\left(8t+\theta\beta^2\right)}{\left(8t+\lambda\right)^2},\\
\Psi_3 &= \frac{2t^2\left(2t+\theta\beta^2\right)}{\left(2t+\lambda\right)^2} + \frac{2t\left(14t+5\theta\beta^2\right)}{\left(2t+\lambda\right)},\ \Psi_4 = \frac{8t\left(16t+5\theta\beta^2\right)}{\left(4t+\lambda\right)} - \frac{32t^2\theta\beta^2}{\left(4t+\lambda\right)^2}.
\end{aligned}$$

If $\gamma = 0$, then $W^{A*} = W^{B*} = \frac{\theta}{128}\left(128v + 27\theta\beta^2 - 40t\right)$. Moreover, we derive

$$\frac{\partial W^{A*}}{\partial\gamma} = \frac{\theta^2 t\beta}{27}\left(\Psi_5 - \Psi_6\right) < 0 \text{ and } \frac{\partial W^{B*}}{\partial\gamma} = \frac{\theta^2 t\beta}{64}\left(\Psi_7 - \Psi_8\right) < 0$$

with

$$\Psi_5 = \frac{375t\left(8t+\theta\beta^2\right)}{\left(8t+\lambda\right)^3} - \frac{10\left(61t+5\theta\beta^2\right)}{\left(8t+\lambda\right)^2},\ \Psi_6 = \frac{3t\left(2t+\theta\beta^2\right)}{\left(2t+\lambda\right)^3} + \frac{11t+4\theta\beta^2}{\left(2t+\lambda\right)^2},$$

$$\Psi_7 = \frac{64t\theta\beta^2}{\left(4t+\lambda\right)^3} - \frac{8\left(16t+5\theta\beta^2\right)}{\left(4t+\lambda\right)^2},\ \Psi_8 = \frac{4t\left(2t+\theta\beta^2\right)}{\left(2t+\lambda\right)^3} + \frac{28t+10\theta\beta^2}{\left(2t+\lambda\right)^2}.$$

In addition,

$$\frac{\partial^2 W^{A*}}{\partial\gamma^2} = \frac{\theta^3 t\beta^2}{27}\left(\Psi_9+\Psi_{10}\right) > 0 \text{ and } \frac{\partial^2 W^{B*}}{\partial\gamma^2} = \frac{\theta^3 t\beta^2}{32}\left(\Psi_{11}+\Psi_{12}\right) > 0,$$

with

$$\Psi_9 = \frac{20\left(61t+5\theta\beta^2\right)}{\left(8t+\lambda\right)^3} - \frac{1125t\left(8t+\theta\beta^2\right)}{\left(8t+\lambda\right)^4},\ \Psi_{10} = \frac{9t\left(2t+\theta\beta^2\right)}{\left(2t+\lambda\right)^4} + \frac{22t+8\theta\beta^2}{\left(2t+\lambda\right)^3},$$

$$\Psi_{11} = \frac{6t\left(2t+\theta\beta^2\right)}{\left(2t+\lambda\right)^4} + \frac{28t+10\theta\beta^2}{\left(2t+\lambda\right)^3},\ \Psi_{12} = \frac{8\left(16t+5\theta\beta^2\right)}{\left(4t+\lambda\right)^3} - \frac{96t\theta\beta^2}{\left(4t+\lambda\right)^4}.$$

Hence, welfare is a strictly concave function in γ. Moreover,

$$\left.\frac{\partial W^{A*}}{\partial\gamma}\right|_{\gamma=0} = -\frac{\theta^2\beta\left(136t+27\theta\beta^2\right)}{512t} < 0 \text{ and } \left.\frac{\partial W^{B*}}{\partial\gamma}\right|_{\gamma=0} = -\frac{\theta^2\left(32t\beta+9\theta\beta^3\right)}{128t} < 0.$$

For $\theta > \theta' = \frac{8t}{9\beta^2}$, we derive $\left.\frac{\partial W^{A*}}{\partial\gamma}\right|_{\gamma=0} > \left.\frac{\partial W^{B*}}{\partial\gamma}\right|_{\gamma=0}$. Hence, at $\gamma = 0$, welfare in regime A decreases less strongly in γ than welfare in regime B. It follows that $W^{A*} > W^{B*}$ for low values of γ. Increasing the parameter γ, numerical simulations show that there exists a critical value γ^* such that $\frac{\partial W^{B*}}{\partial\gamma} > \frac{\partial W^{A*}}{\partial\gamma}$ $\forall\gamma > \gamma^*$. It follows that there must exist another critical value γ^W such that $W^{B*} > W^{A*}$ $\forall\gamma > \gamma^W$. For $\theta \leq \theta' = \frac{8t}{9\beta^2}$, we derive $\left.\frac{\partial W^{A*}}{\partial\gamma}\right|_{\gamma=0} < \left.\frac{\partial W^{B*}}{\partial\gamma}\right|_{\gamma=0}$. Hence, at $\gamma = 0$, welfare in regime B decreases less strongly in γ than welfare in regime A. Furthermore, numerical simulations show that this is true for all $\gamma > 0$. It follows that $W^{B*} > W^{A*}$ $\forall\gamma > 0$. This completes the proof of proposition 5.

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