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**Evolutionarily Stable Strategies in Sports Contests**

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# Evolutionarily Stable Strategies in Sports Contests

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**Abstract**

In the recent years, many clubs in the biggest European soccer leagues have run into debts. The sports economic literature provides several explanation for this development, e.g., the league structure (open versus closed league), club constitutions, ruinous rat races between clubs. While the majority of the articles presume the well-known Nash equilibrium concept, I apply evolutionary game theory in a sports contest model. If clubs follow evolutionarily stable strategies (ESS), then ESS generate higher investments and lower profits than predicted by Nash strategies independent of win maximizing or profit maximizing clubs. Overdissipation of the rent is possible for Nash strategies as well as for ESS.

**JEL Classification:** C72, C73, D74, L13, L83

**Keywords:** Contest, evolutionary stable strategies, utility maximization, team sports league

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# 1 Introduction

## 1.1 Overinvestment in Sports Leagues

It is a matter of fact that many soccer clubs in the major European soccer leagues have accumulated vast debts in the last years. The net debt in the English Premier league was £3.1 billion in the end of 2007/08. In this season, the French League (Ligue 1) was confronted by operating losses due to major wage increases (Deloitte and Touche (2009)). Furthermore, the Italian League (Serie A) generated losses even if the this league was the fastest growing one - revenues increased by 34% in one year- of the five biggest European leagues. In recent years, some clubs even went bankrupt (e.g., AC Fiorentina (Serie A) in 2002, AC Parma (Serie A) in 2004, Servette Genf (Swiss Football League) in 2005).

The economic literature provides several explanation for this development. Ruinous rat race between clubs, the league structure (open league versus closed league), and inadequate club constitutions have been identified as possible sources for overinvestment. In this article, I provide another theoretical channel which can be responsible for overinvestment. Ideas from evolutionary game theory are applied to a general model in sports economics and the results are compared to the outcome applying traditional game theory. I show that the concept of evolutionarily stable strategies can explain the race to the bottom as observed for some clubs in professional soccer leagues.

So far, the sports economic literature explained the "paradox of rising revenues and declining profits" as follows. Whitney (1993) argues that clubs' striving for stars in professional sports leads to a destructive competition and suboptimal welfare since some participants drop out of the market even if this leaving is inefficient. Dietl et al. (2008a) argue that overinvestment originates from the ruinous interaction between clubs in competition, i.e. the negative externalities of an investment on the opponent. They theoretically derive that the league structure - open league or closed league - also affects the extent of revenue-dissipation. A system with promotion and relegation (i.e., an open league) fosters overinvestment. On the other hand, Dietl and Franck (2000) emphasize that inadequate club constitutions generate incentives for overinvestment. Traditional clubs are often constituted as an organization without residual claimant. Club managers try to rather maximize the probability to win the championship instead of maximizing profits due to the lower control.[1] However, this argument can only partly explain overinvestment since we also observe high debts for clubs as capitalistic corporations.

[1] Garcia-del Barrio and Szymanski (2009) find empirical evidence that clubs trade-off profits and wins with a tendency for win maximization in the Spanish and English football leagues. Atkinson et al. (1988) also find for data from the National Football League, that clubs maximize a mix of profits and wins.

## 1.2 Evolutionarily Stable Strategies

The majority of research in the field of sports economics applies traditional game theory with the Nash equilibrium concept to analyze the interaction of clubs. It is assumed that club owners are fully rational and maximize their payoffs with an adequate choice of their investment level. Following Nash strategies (NS), clubs inherently maximize their *absolute* payoffs. Evolutionary game theory uses a different approach as originally introduced by Smith (1982), Maynard Smith (1974) and Maynard Smith and Price (1973) in the context of animal conflicts. These articles analyze the strategies and survival of different types in a population. Adopting evolutionary game theory to an economic environment, Schaffer (1989) argues that profit-maximizing firms are not the best survivor since they are not immune against spiteful behavior of their competitors.

In the context of evolutionary game theory, a competition between players should be rather interpreted as a struggle than an exchange of goods in a market (Leininger (2003)). Therefore, this approach fits well to contest but especially to sports contests.

Strategies are evolutionarily stable if no player invades the population. Based on Schaffer (1988), Leininger (2003) defines an evolutionarily stable strategy (ESS) for a finite population of $n$ individuals as follows:

**Definition 1** *Let a strategy $x$ be adapted by all players $i$, $i = 1, ..., n$.*

*i) A strategy $\bar{x} \neq x$ can invade $x$, if the pay-off for a single player using $\bar{x}$ (against $(n-1)$ players using $x$) is strictly higher than the pay-off of a player using $x$ (against $(n-2)$ other players using $x$ and the deviationist using $\bar{x}$).*

*ii) A strategy $x^{ESS}$ is evolutionary stable, if it cannot be invaded by any other strategy.*

The main difference to the Nash strategy is that players may increase their investments even if their absolute payoffs decrease as long as the other players' payoffs decrease even more. ESS implies that players consider *relative* payoffs and not their *absolute* payoffs in contrast to NS.

Hehenkamp et al. (2001) and Leininger (2003) introduced and advocated the concept of evolutionary games in contest theory. Leininger (2003) argues:

*"[...] contest or more general conflict-theory portrays and emphasizes competition through struggle and contention in order to allocate economic goods and is thus opposed to the market-oriented exchange paradigma. Bringing contests and evolutionary analysis - both concepts that entered economics via biology - together in a (micro-)economic setting may therefore offer some promising perspectives."*

Thus, if evolutionary game theory fits to general contests due to the characteristics of conflicts, then this theory should especially be suitable for sports contest. The concept of ESS understands participants' behavior as a result of a dynamic process in which participants do not have to be fully rational. Agents will survive in the long run if their payoffs are *relatively* higher compared to their opponents. Thus, participants may have an incentive to increase their efforts in order to harm their competitors. Leininger (2003) proves that the outcome with ESS always differs from the outcome with NS. However, for an infinite population, the two concepts coincide.[2] Furthermore, Leininger shows that an ESS contest can be transformed and interpreted as a contest in which participants try to beat the average competitor.

Hehenkamp et al. (2001) find a simple rule for ESS regarding the possibility of overdissipation of the rent depending on the sensitivity of the winning probability. They show that overdissipation holds for a high sensitivity of the winning probability, full rent dissipation holds for a medium sensitivity of the winning probability, and underdissipation holds for a low sensitivity of the winning probability. Moreover, they are able to exclude full dissipation or overdissipation for NS in their model.

In this article, I apply the concept of ESS in sports contests. In order to do that, I will consider two important peculiarities in sports which influence the outcome of the contest. First, clubs' objective, i.e., the payoff, does not necessarily have to be profits. Clubs possibly maximize their winning probability, their profits or a mix out of the two possibilities. Second, different to the assumptions used in general contest theory, clubs revenue are influenced by competitive balance. Large inequalities in team strength negatively affect (gate and broadcasting) revenues due to the lack of suspense of the game (see Szymanski (2003)).

### 1.3 Main Results and Outline of This Article

The main findings of this article are as follows: (i) Clubs' investments with NS are always smaller than investments with ESS independent of win maximizing or profit maximizing clubs. Hence, ESS can even better explain the high investments in professional sports leagues than NS. (ii) However, in contrast to Hehenkamp et al. (2001), overdissipation of the rent is possible for NS as well as for ESS. The possibility for overdissipation increases if the weight of the win maximization is sufficiently high and, simultaneously, competitive balance has a minor influence on clubs' revenues. Similar to Hehenkamp et al. (2001), a higher sensitivity of the winning probability increases the possibility of overdissipation. (iii) Profits are higher for NS than ESS. This means that ESS provide an additional explanation for low or even negative profits in sports leagues.

The remainder of this article structured as follows: Section 2 presents the model. In Section 3, I derive

[2]Due to this result, it is often argued that ESS is a *refinement* ot the Nash equilibrium concept. However, this statement is an inadequate description of ESS (see Hehenkamp et al. (2001)).

clubs ' investment behavior with NS and ESS. The outcomes of the two concepts are compared in Section 4. Section 5 sums up the main findings and concludes.

## 2 Model

In this contest, $n \geqslant 2$ symmetric clubs compete to win a contest. Club $i$, $i \in \{1, ..., n\}$, invest $x_i$ in order to increase the probability $p_i$ to win the contest. Club $i$'s probability to win depends on relative investments and is determined according to the Tullock contest success function (CSF):

$$p_i(x_1, ..., x_n) = \frac{x_i^r}{\sum_{j=1}^{n} x_j^r}$$

The parameter $r$ determines the so-called discriminatory power of the CSF, i.e., the sensitivity of the probability function with respect to investments. A high $r$ means that a small difference of efforts between contestants has a large effect on the differences in the winning probabilities. Otherwise, a low $r$ implies that a small difference in efforts has a little impact on the winning probabilities.

Club $i$'s revenue is a concave function in the success function and increasing in the market potential $m_i$:

$$R_i(m_i, x_1, ..., x_n) = m_i p_i - \frac{b}{2} p_i^2$$

The parameter $b$ determines the effect of competitive balance on club revenues. If $b = 0$, competitive balance has no effect on revenue. Club $i$'s expected profit $\pi_i$ is therefore given by

$$\pi_i(x_1, ..., x_n) = R_i(m_i, x_1, ..., x_n) - c x_i.$$

In the sports economic literature, it is often assumed that clubs' market potential differs (e.g., Szymanski (2003), Szymanski and Késenne (2004), Grossmann et al. (2008), Grossmann and Dietl (2009), Lang et al. (2011)). It is a commonly accepted form to introduce asymmetry and discuss the effect of different mechanism (like salary caps, revenue sharing, luxury taxes) to influence competitive balance in sports leagues. In this article, however, the primary aim is to analyze individual and aggregate investments under different equilibrium concepts. The main research question is under which equilibrium concept do clubs invest more. Therefore, I assume identical market sizes and set $m_i = m \ \forall \ i = 1, ..., n$ in order to isolate the effect of the equilibrium concept.[3] Consequently, the contest is fully symmetric.

As widely discussed in the sports economic literature, club owners may not only be interested in profit,

[3]This assumption will be especially helpful for the determination of the ESS.

but also in the win percentage (Késenne (1996), Késenne (2000), Kesenne (2006), Dietl et al. (2011)). Similar to Dietl et al. (2011), I assume that a club owner $i$ maximizes his expected utility $u_i(\pi_i, p_i)$ which contains his profit as well as his win probability. The weight attached to profit is $\alpha$ whereas the weight attached to the win probability is $1-\alpha$. Therefore, the club owner $i$'s objective function is

$$\begin{aligned} u_i(x_{1,...,}x_n) &= \alpha\pi_i + (1-\alpha)p_i \\ &= \alpha\left[R_i(m, x_{1,...,}x_n) - cx_i\right] + (1-\alpha)p_i \end{aligned}$$

Note that - at the margin - the model incorporates win maximization ($\alpha = 0$) as well as profit maximization ($\alpha = 1$).[4]

To measure rent-dissipation, I follow Chung (1996). The ratio between the sum of investments in equilibrium and the contest prize measures the extent of rent dissipation. In this article, the value $m$ can be interpreted as the contest prize. Therefore, $m$ does not only represent the clubs' market potential but also the championship prize. The extent of rent-dissipation $D$ is defined as[5]

$$D = \frac{\sum_{i=1}^{n} x_i}{m}.$$

Within this context, it will be interesting to analyze whether clubs invest more (equal) ((less)) in aggregate than the championship prize $m$ such that $D > 1$ ($D = 1$) (($D < 1$)) and the rent is overdissipated (fully dissipated) ((underdissipated)) .

# 3 Solution

## 3.1 Nash Strategies

Club $i$ chooses its investment according to the Nash strategy and therefore maximizes its expected utility $u_i$ with respect to $x_i$. The following Proposition sums up the main findings:

**Proposition 1** *Under Nash strategies,*

*(i) club i invests* $x_i^* = \frac{r(n-1)}{\alpha c n^2}\left(\alpha\left(m - b/n\right) + (1-\alpha)\right)$.

*(ii) club i's investment* $x_i^*$ *is increasing in* $r$ *and* $m$, *decreasing in* $c, b$ *and* $\alpha$, *and ambiguous in* $n$.

[4] Note that there are different possibilities to formalize win maximization (see Késenne (1996) and Késenne (2000)). Kesenne (2006), for instance, assumes that clubs invest their whole budgets in order to maximize their investments, and therefore their win percentages. In this case, clubs' budgets directly determine total investment in the league. In this article, I follow Dietl et al. (2011) who allow for an objective function which includes a mix of profit and win maximization.

[5] Note that Dietl et al. (2008b) use another measure: They calculate the percental deviation of the league optimum investments to the investments in a Nash equilibrium and call this the "ratio of revenue dissipation".

*(iii) overdissipation of the rent, i.e., $D > 1$, is possible for small values of $c, b$ and $\alpha$, and for large values of $n$ and $r$.*

***Proof.*** *See Appendix 6.1* ■

(i) It is easy to see that the condition $\alpha\left(m - \frac{b}{n}\right) + (1-\alpha) > 0$ is required to guarantee positive investments. Henceforth, we assume that this condition holds. (ii) A higher sensitivity of the probability function increases marginal revenue of an investment and, therefore, club $i$'s investment increases in equilibrium. The same intuition holds for the market potential $m$. On the other hand, higher marginal costs decrease investments. If competitive balance has a large impact on revenues, i.e., for a high $b$, clubs decrease their investments. It is also intuitive that clubs' investment are higher if they care more about win maximizing, i.e., for a lower value of $\alpha$.

Result (iii) is interesting since Hehenkamp et al. (2001) can rule out overdissipation for NS in their model with profit maximization and without competitive balance. However, I find that the introduction of a large weight of win maximization in combination with a small effect of competitive balance on revenues, overdissipation of the rent is possible. The intuition for this result is that a higher weight of win maximization as well as a lower influence of competitive balance on revenues foster incentive to invest. However, even if the weight for win maximization is large, a high effect of competitive balance on revenue could overcompensate incentives to invest such that underdissipation results. Therefore, the combination of win maximization and competitive balance are an important source for the extent of rent dissipation. In addition, a high number of clubs, a high sensitivity of the probability function or low marginal costs increase investments and therefore foster overdissipation.

## 3.2 Evolutionarily Stable Strategies

In order to derive the ESS, I consider player 1. Under ESS, this player tries to maximize his/her relative utility compared to the opponent $j \neq 1$.[6] Suppose that all other players choose the ESS $\hat{x}$, then player 1's optimal choice $x$ must equal this $\hat{x}$ in the following maximization problem in order to verify $\hat{x}$ to be ESS:[7]

$$\max_x U_1(x, \hat{x}) \equiv u_1(x, \hat{x}, ..., \hat{x}) - u_j(x, \hat{x}, ..., \hat{x})$$

[6] Since clubs are symmetric, $u_j(x, \hat{x}, ..., \hat{x}) = u_k(x, \hat{x}, ..., \hat{x})\ \forall\ j, k \neq 1$. Therefore, I only have to consider one condition for invadibility with representative $j$ as an opponent of player 1.

[7] Note that this maximization problem is only a tool to find the ESS. As mentioned in the introduction, one could interpret ESS as a dynamic evolutionary process in which group members learn and imitate strategies. The member who develops a superior strategy will survive and will reproduce faster than the rest. The ESS is a stationary point in which no member can improve relative performance (see Leininger (2003)).

with $j \neq 1$. Thus, if $\hat{x}$ is an ESS , then club 1 does not find an $x \neq \hat{x}$, such that his/her utility $u_1$ is larger than the utility of one of the other clubs $j$. To find an $\hat{x}$ in which no club wants to invade, I maximize $U_1(x, \hat{x})$ with respect to $x$. Next, I derive $U_1(x, \hat{x})$ as follows:

$$
\begin{aligned}
& U_1(x,\hat{x}) \\
&= u_1(x,\hat{x},...,\hat{x}) - u_j(x,\hat{x},...,\hat{x}) \\
&= \underbrace{\alpha\left[m\frac{x^r}{x^r+(n-1)\hat{x}^r} - \frac{b}{2}\left(\frac{x^r}{x^r+(n-1)\hat{x}^r}\right)^2 - cx\right] + (1-\alpha)\left[\frac{x^r}{x^r+(n-1)\hat{x}^r}\right]}_{u_1(x,\hat{x},...,\hat{x})} \\
&\quad - \left(\underbrace{\alpha\left[m\frac{\hat{x}^r}{x^r+(n-1)\hat{x}^r} - \frac{b}{2}\left(\frac{\hat{x}^r}{x^r+(n-1)\hat{x}^r}\right)^2 - c\hat{x}\right] + (1-\alpha)\left[\frac{\hat{x}^r}{x^r+(n-1)\hat{x}^r}\right]}_{u_j(x,\hat{x},...,\hat{x})}\right)
\end{aligned}
$$

The first-order condition is given by

$$
\begin{aligned}
\frac{\partial U_1}{\partial x} &= \alpha\left[m\frac{rx^{r-1}(x^r+(n-1)\hat{x}^r-x^r)}{(x^r+(n-1)\hat{x}^r)^2} - b\left(\frac{x^r}{x^r+(n-1)\hat{x}^r}\right)\frac{rx^{r-1}(x^r+(n-1)\hat{x}^r-x^r)}{(x^r+(n-1)\hat{x}^r)^2} - c\right] \\
&\quad +(1-\alpha)\left[\frac{rx^{r-1}(x^r+(n-1)\hat{x}^r-x^r)}{(x^r+(n-1)\hat{x}^r)^2}\right] \\
&\quad -\alpha\left[-m\frac{rx^{r-1}\hat{x}^r}{(x^r+(n-1)\hat{x}^r)^2} + b\left(\frac{\hat{x}^r}{x^r+(n-1)\hat{x}^r}\right)\frac{rx^{r-1}\hat{x}^r}{(x^r+(n-1)\hat{x}^r)^2}\right] \\
&\quad +(1-\alpha)\left[\frac{rx^{r-1}\hat{x}^r}{(x^r+(n-1)\hat{x}^r)^2}\right] \\
&= 0
\end{aligned}
$$

In a symmetric solution, $x = \hat{x} \equiv x^{ESS}$ we get the following Proposition:

**Proposition 2** *Under evolutionarily stable strategies (ESS),*

*(i) club $i$ invests $x_i^{ESS} = \frac{r}{cn^2}\left(n(m-1+\frac{1}{\alpha}) - b\right)$.*

*(ii) club $i$'s investment $x_i^{ESS}$ is increasing in $r$ and $m$, decreasing in $c, b$ and $\alpha$, and ambiguous in $n$.*

*(iii) overdissipation of the rent ($D > 1$) is possible for small values of $c, b$ and $\alpha$, and for large values of $n$ and $r$.*

**Proof.** See Appendix 6.2 ■

(i) It is easy to see that the condition $n(m-1+\frac{1}{\alpha}) - b > 0$ is required to obtain positive investment $x_i^{ESS}$.[8] (ii) A higher $r$ and $m$, or a lower $c, b$ and $\alpha$, imply higher club investments. Result (iii) is interesting

[8] Note that this condition is the same (after some mathematical manipulations) as the condition for a positive investment with NS.

since Hehenkamp et al. (2001) show in their model with profit maximization and without any effects of competitive balance on revenues that overdissipation for ESS depends only on $r \gtreqless 1$ . According to result (iii), there could be two additional sources for overdissipation in the context of sports leagues. Similar to NS, a combination of a large weight on win maximization (i.e., a low value of $\alpha$) and a low effect of competitive balance on revenues, i.e., a low value of $b$, foster overdissipation for ESS. In addition, a high number of clubs, a high sensitivity of the probability function or low marginal costs increase investments and therefore induce overdissipation.

# 4 Comparison

In the last section, I derived clubs' behavior within the different concepts of NS and ESS. In the following two subsections, I compare the outcomes of NS with ESS. First, I compare investments and afterwards profits.

## 4.1 Investment

It is interesting to analyze, whether the NS or the ESS produces higher investments. The following Proposition sums up the main findings:

**Proposition 3** *(i) Club i's investment is lower with NS compared to ESS for* $\alpha > 0$*.*

*(ii) The ratio of ESS investment to NS investment* $x_i^{ESS}/x_i^*$ *equals* $n/(n-1)$*.*

**Proof.** See Appendix 6.3. ■

(i) This result implies that the concept of ESS provides a new explanation for the large investment in sports leagues. As long as club have a positive weight on profits, ESS investments are higher than NS investments. If $\alpha = 0$, then investments converge to infinity for NS as well as for ESS. In this extreme case, clubs only care about their win probability and invest as much as possible. Implicitly, it is assumed that clubs can borrow as much as they want in this situation.[9] (ii) It is interesting that the relative investments between ESS and NS $x_i^{ESS}/x_i^*$ only depend on the number of participating clubs. Neither the weight of win maximization nor the sensitivity of the probability function influence this ratio. The result implies that more clubs in the league decrease relative investments between ESS and NS with a decreasing rate. As the number of clubs converges to infinity, the ratio $x_i^{ESS}/x_i^*$ converges to 1.

[9] Of course, unlimited debts are not possible in reality since clubs cannot credibly commit to pay back the loan. Nonetheless, Manchester United in the English Premier League was able to accumulate £500 million debt (The Guardian Online, 24 May 2011). Apparently, there (still) exist creditors who believe that even this large debt can be paid back in the future.

### 4.2 Profit

Even if clubs consider their utility, I focus on clubs' profit in this subsection. The reason is that I aim to discuss the source for low or even negative profits. The following Proposition sums up the main findings:

**Proposition 4** *(i) For $\alpha > 0$, club $i$'s profit is higher for NS compared to ESS.*

**Proof.** See Appendix 6.4 ■

According to this result, the concept of ESS provides a new explanation for the low profits in sports leagues. Clubs invest more with ESS than NS such that this destructive competition leads to relatively low profits in the league. For $\alpha = 0$ as an extreme case, clubs' profit converges to $-\infty$ independent of NS or ESS. The intuition for this result is that clubs have an incentive to invest as much as possible in this situation since they only care about win maximization.

## 5 Conclusion

Sports economic models are often analyzed with traditional game theory. The Nash equilibrium concept proves to be a useful tool to analyze the players' behavior. Nonetheless, it bases on strong assumptions regarding the players' rationality. Evolutionary game theory is an approach which rather focus on the outcome in a dynamic process. As the notion "evolutionary" suggest, types in a population survive who developed the best strategies in terms of *relative* payoffs. It is quite intuitive that contestants in sports also think in relative terms. Usually, the absolute result is less important than the relative result. For instance, a marathon runner is primarily interested to be the first at the finish line and not to beat a specific time limit.

In this article, I analyze a sports contest in which $n$ clubs compete in a championship. Club owners' objective function includes a mix of win maximization as well as profit maximization. I show that club investments are larger and profits lower if clubs pursue evolutionarily stable strategies (ESS) compared to Nash strategies (NS). Hence, ESS can even better explain than NS why many clubs in the European soccer leagues are plunged in debt in recent years. Furthermore, I conclude that overdissipation of the rent is possible for NS as well as for ESS. The possibility for overdissipation increases if the weight of the win maximization is sufficiently high and, simultaneously, competitive balance has a minor influence on clubs' revenues. Moreover, a higher sensitivity of the winning probability increases the possibility of overdissipation.

As evolutionary game theory has not been applied in sports contest so far, this study can be seen as a first step in this field of research. I would like to invoke further research on this topic in the future. Especially, the effect of asymmetry with ESS could be an interesting issue in order to analyze, for instance, effects of different redistributive mechanism (like revenue sharing) on competitive balance.

# 6 Appendix

## 6.1 Proof of Proposition 1

(i) The maximization problem with NS is

$$\max_{x_i} u_i(x_1, ..., x_n).$$

The first-order condition is given by

$$\frac{\partial u_i}{\partial x_i} = \alpha \left[ m \frac{r x_i^{r-1} \left( \sum\limits_{j=1}^{n} x_j^r - x_i^r \right)}{\left( \sum\limits_{j=1}^{n} x_j^r \right)^2} - b \frac{x_i^r}{\sum\limits_{j=1}^{n} x_j^r} \frac{r x_i^{r-1} \left( \sum\limits_{j=1}^{n} x_j^r - x_i^r \right)}{\left( \sum\limits_{j=1}^{n} x_j^r \right)^2} - c \right] + (1-\alpha) \left[ \frac{r x_i^{r-1} \left( \sum\limits_{j=1}^{n} x_j^r - x_i^r \right)}{\left( \sum\limits_{j=1}^{n} x_j^r \right)^2} \right] = 0.$$

Each club has a symmetric optimization problem. Therefore, club $i$ chooses its optimal investment $x_i^*$ in the symmetric equilibrium as follows:[10]

$$x_i^* = \frac{r(n-1)}{\alpha c n^2} \left( \alpha \left( m - \frac{b}{n} \right) + (1-\alpha) \right)$$

(ii) Straightforward and therefore omitted.

(iii) Aggregate investments in the league are

$$n x_i^* = \frac{r(n-1)}{\alpha c n} \left( \alpha \left( m - \frac{b}{n} \right) + (1-\alpha) \right).$$

The rent $m$ is overdissipated if aggregate investments are larger than this rent:

$$\begin{aligned} D \quad &> \quad 1 \\ \Longleftrightarrow \quad & \frac{\sum_{i=1}^{n} x_i^*}{m} > 1 \\ \Longleftrightarrow \quad & n x_i^* > m \\ \Longleftrightarrow \quad & \frac{r(n-1)}{\alpha c n} \left( \alpha \left( m - \frac{b}{n} \right) + (1-\alpha) \right) > m \\ \Longleftrightarrow \quad & \alpha n m (nr - r - nc) - \alpha b r (n-1) + r n (n - 1 + \alpha) > 0 \end{aligned}$$

[10]The second-order condition is satisfied for $[(r-1)n - 2r][\alpha m n + n - n\alpha - \alpha b] - \alpha b r(n-1) < 0$.

For small values of $a$ and $b$, this condition holds. At the margin, suppose that $\alpha = b = 0$, then this condition reduces to

$$\iff rn(n-1) > 0$$

which always holds. Since $nx_i^*$ is increasing in $n$ and $r$, overdissipation of the rent is possible for large values of $n$ and $r$. Since $nx_i^*$ is decreasing in $c$, overdissipation of the rent is possible for small values of $c$. The parameter $m$ has an ambiguous effect on rent dissipation since $(nr - r - nc) \gtreqless 0$.

## 6.2 Proof of Proposition 2

Result (i) is derived in front of the Proposition. Result (ii) is straightforward and therefore omitted. (iii) Aggregate investments in the league are

$$nx_i^{ESS} = \frac{r}{cn}\left(n(m - 1 + \frac{1}{\alpha}) - b\right).$$

It is easy to see that aggregate investments $nx_i^*$ are increasing in $r, m$ and $n$, but decreasing in $c, b$ and $\alpha$. The rent $m$ is overdissipated if aggregate investments are higher than this rent:

$$\begin{aligned} D &> 1 \\ \iff & \frac{\sum_{i=1}^{n} x_i^*}{m} > 1 \\ \iff & nx_i^{ESS} > m \\ \iff & r(nm + n\frac{1-\alpha}{\alpha} - b) > cmn \end{aligned}$$

Thus, overdissipation of the rent is possible for small values of $\alpha, c$ and $b$, for large values of $n$ and $r$.

## 6.3 Proof of Proposition 3

(i) It is easy to see that $x_i^*$ as well as $x^{ESS}$ converge to infinity if $\alpha = 0$. For $\alpha > 0$, club $i$'s investment is lower for NS compared to the ESS iff:

$$\begin{aligned} x_i^* &< x_i^{ESS} \\ \iff & \frac{r(n-1)}{\alpha c n^2}\left(\alpha\left(m - \frac{b}{n}\right) + (1-\alpha)\right) < \frac{r}{cn^2}\left(n(m - 1 + \frac{1}{\alpha}) - b\right) \\ \iff & 0 < n + a(mn - n - b) \end{aligned}$$

It is now easy to see that the last inequality always holds since $n + a(mn - n - b)$ is positive due to the required condition for positive investments $x_i^*$.

(ii) The ratio of ESS and NS investments is derived as follows:

$$\begin{aligned}\frac{x_i^{ESS}}{x_i^*} &= \frac{\frac{r}{cn^2}\left(n(m-1+\frac{1}{\alpha})-b\right)}{\frac{r(n-1)}{\alpha cn^2}\left(\alpha\left(m-\frac{b}{n}\right)+(1-\alpha)\right)} \\ &= \frac{n}{n-1}\end{aligned}$$

Therefore, the first and second derivative of this ratio with respect to $n$ is determined as follows:

$$\frac{\partial(x_i^{ESS}/x_i^*)}{\partial n} == -\frac{1}{(n-1)^2} < 0$$

$$\frac{\partial^2(x_i^{ESS}/x_i^*)}{\partial n^2} = \frac{2}{(n-1)^3} > 0$$

Thus, a higher number of clubs decreases the ratio $x_i^{ESS}/x_i^*$ with a decreasing rate. As $n$ converges to infinity, this ratio $x_i^{ESS}/x_i^* = \frac{n}{n-1}$ converges to 1.

## 6.4 Proof of Proposition 4

With NS, club $i$'s profit is given by

$$\pi_i^* = \frac{m}{2} - \frac{b}{8} - \frac{r(n-1)}{\alpha n^2}\left(\alpha\left(m - b/n\right) + (1-\alpha)\right).$$

With ESS, club $i$'s profit is given by

$$\pi_i^{ESS} = \frac{m}{2} - \frac{b}{8} - \frac{r}{n^2}\left(n(m-1+\frac{1}{\alpha}) - b\right).$$

For $\alpha > 0$,

$$\begin{aligned}\pi_i^* &> \pi_i^{ESS} \\ &\Longleftrightarrow n > a(b+n-mn).\end{aligned}$$

Note that this condition always holds since we required the condition $\alpha\left(m - b/n\right) + (1-\alpha) > 0$ to have positive investment for $x_i^*$. Thus, profits are higher for NS compared to ESS. For $\alpha = 0$, clubs investment converge to infinity for NS as wells as for ESS. In this case, profits converge to minus infinity independent of the concept.

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