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Endogenous Liquidity Constraints in a Dynamic Contest

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Abstract

In this article, I analyze the effects of future liquidity constraints on the investment behavior of two contestants with asymmetric prize valuations in a dynamic contest model. Contestants compete in two consecutive Tullock contests in order to win a contest prize in each period. The loser of the first-period contest can be liquidity constraint in the second period. The model reveils the following four main results: (i) Future liquidity constraints marginally affect today's intensity of competition but rather influence tomorrow's contest. (ii) A higher contest prize in both periods surprisingly decreases aggregate second-period investment in a symmetric contest. (iii) Counterintuitively, a higher asymmetry with respect to the contest prize valuations increases the first-period investment of both contestants. (iv) The effect of a higher asymmetry on second-period investment depends on which contestant won the first-period contest. Further results are derived with respect to the existence and uniqueness of the equilibrium, competitive balance and expected total profits.

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1 Introduction

In contests, agents invest irreversible resources, i.e., money, time or effort, in order to win a prize. A whole bunch of different contest types exist: rent-seeking, lobbying, patent race, litigation, sports tournament, beauty contest and so forth. In this article, I consider the investment behavior of two contestants in two consecutive contests. The special issue is that I analyze the interaction of liquidity constraints and the dynamic characteristic of the contest. Contestants can be endogenously liquidity-constrained on a future stage. The analysis provides insights with respect to the influence of future constraints on today's investment behavior.

An important branch of the contest literature deals with contestants' optimal investment behavior and the contest designer's optimal choice of rules that attenuate or increase incentives to invest in contests. For instance, a government can implement an R&D tax exemption to foster R&D investments and intensify patent races. On the other hand, spending caps in elections have been introduced to alleviate the time politicians devote to fund-raising instead of legislation. Konrad (2009) presents an excellent overview for the different contest types and provides a comprehensive survey on contest theory.

Many contests - as the one presented in this article - have a dynamic characteristic which means that agents plan their investments over time for many consecutive contests. A sports club, for example, can invest its financial resources today or can retain its investments in order to wait for more attractive players on the market in the future. The literature on dynamic contests is steadily increasing. Several articles consider contestants' behavior in multi-stage contests. Baik and Kim (1997) discuss the effect of delegation in contests on aggregate efforts.[1] Contestants decide whether they want to hire a delegate or not. Afterwards, if appropriate, the delegation's types are announced and efforts are expended. Baik and Kim conclude that aggregate efforts are higher (lower) if the low-valuation (high-valuation) player unilaterally delegates compared to the benchmark without delegation. Furthermore, if both contestants delegate, then aggregate efforts are lower compared to the benchmark. Gradstein (1998) discusses the optimal contest structure to maximize aggregate discounted efforts. Even if contestants elicit higher effort in a multistage contest than in a single stage contest it is not clear which contest structure is preferred by the contest designer since efforts are provided slower in the multistage contest. Therefore, the contest designer's time-preference is decisive which contest structure he/she prefers.[2] Amegashie (1999) analyzes the effect of a preliminary contest - which determines the finalists - on effort in a two stage contest. He concludes that the amount of wasteful expenditures in rent-seeking depends on the sensitivity of the contest success function in the preliminary contest as well as in the final contest. More precisely, he derives conditions for the different combinations

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[1]For instance, firms engage lawyers to win a trial.
[2]See also Gradstein and Konrad (1999).

of the sensitivity parameters in the two contests for which social welfare decreases or increases. Amegashie (2000) analyzes the effect of a shortlisting in contests and compares the results with the outcome in a single-stage contests. Baik and Lee (2000) consider a two-stage contest with carryovers. First period efforts are partly useful in the second period. They show that carryovers can imply full rent-dissipation. Moldovanu and Sela (2006) analyze how the introduction of different sub-contests into an overall contest as well as the type of cost function influence aggregate efforts and the highest effort in contests. Grossmann and Dietl (2009) discuss the investment behavior of contestants in a two-stage contest. They find multiple equilibria in the case of closed-loop strategies. One of these equilibria counterintuitively predicts that the underdog invests more than the favorite in both periods.[3]

Some types of contest can have another characteristic. Contestants can be liquidity-constrained due to, for instance, a limited access to the capital market. This means that contestants investment are limited by a cap. If effort is measured as time devoted to a contest, then a similar constraint, i.e., a time budget constraint, emerges (Konrad (2009)). For example, sports teams have to allocate their limited time resources optimally between two consecutive matches to prepare the appropriate tactic for the upcoming match. Constraints are interesting to analyze since they can significantly change the outcome of contests. Several articles consider contests with budget constraints. Che and Gale (1997) compare the effects of liquidity constraints in lotteries and all-pay auctions. They conclude that rent-dissipation is lower in an all-pay auction than in a lottery. Che and Gale (1998) show that caps on political lobbying can surprisingly increase aggregate expenditures in a static all-pay auction. Kaplan and Wettstein (2006) extend the model of Che and Gale (1998). They study a situation in which contestants are able to exceed caps. Even if this exceeding is costly (for example, a luxury tax on exceeding investments in the Major League Baseball (MLB) if a club exceeds the salary cap), they show that the introduction of a cap decreases aggregate expenditures. Gavious et al. (2002) analyze the effect of bid caps for different forms of cost functions in an all-pay auction. Kvasov (2007) considers contestants' allocation of limited resources in a number of simultaneous all-pay auctions. The author finds mixed-strategy equilibria and proves the nonexistence of pure-strategy equilibria.

Even if the individual literature on dynamic contests as well as on contests with liquidity constraints is large, minor attention has been paid on the combination of dynamic contests and liquidity constraints. The analysis of this combination is the main topic in this article. Two contestants compete to win an exogenous prize in each of two consecutive contests. Asymmetry is introduced assuming that contestants can have different contest prize valuations. The contestant with the higher (lower) prize valuation is called favorite

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[3]Other multistage contests are analyzed by Harris and Vickers (1985), Rosen (1986), Klumpp and Polborn (2006), Konrad and Kovenock (2009) and Konrad and Kovenock (2010). Some economists discuss the optimal investment behavior of contestant with an infinite time horizon (Leininger and Yang (1994); Shaffer and Shogren (2008); Grossmann et al. (2010); Grossmann et al. (2011)).

(underdog). Furthermore, contestants are endowed with symmetric initial wealth which they can irreversible invest to increase their probabilities to win the contest in each period. In period 1, both contestants partly invest their (financial) resources and the relative investments determine the probability to win the contest prize in period 1. After period 1, both contestants observe which contestant has won the first contest. The winner of the first contest accumulates his initial wealth due to the realization of the contest prize. By this means, the two consecutive contests are linked together. For similar investments, the winner endogenously has a higher wealth for investments in period 2 compared to the loser. I concentrate on situations in which (only) the loser in period 1 possibly faces a liquidity constraint in period 2. Herewith, the effects of the possibly binding constraint on individual and aggregate investments can be evaluated and analyzed.

The main findings of this paper are as follows.

Symmetric contest: In a symmetric contest, contestants have identical prize valuations. (i) If contestants know that one contestant will be constrained in the second period, a unique symmetric equilibrium exists. Potential future liquidity constraints only marginally reduce the intensity of the present competition. Both contestants decrease investments in both periods compared to an unconstrained situation. However, the decrease is relatively higher in period 2. (ii) A higher contest prize for both periods increases first-period investments, but counterintuitively decreases aggregate second-period investment. (iii) In period 1, a perfect competitive balance results in equilibrium even in the presence of future liquidity constraints. In period 2, however, the contest is imbalanced and this imbalance increases for tighter liquidity constraints. (iv) A tighter liquidity constraint increases expected aggregate profits since the intensity of the competition decreases.

Asymmetric contest: In an asymmetric contest, contestants have different prize valuations. A higher asymmetry, i.e., an increase (decrease) of the favorite's (underdog's) prize valuation, has the following effects. (v) Both contestants surprisingly increase their first-period investment such that the intensity of the competition in the first period increases. (vi) In period 2, contestants behavior depends on who won the first-period contest. If the favorite won the first-period contest, then a higher asymmetry increases (decreases) the second-period investment of the favorite (underdog). However, if the underdog won the first-period contest, then both contestants decrease their second-period investment for a higher asymmetry. (vii) Higher asymmetry marginally affects competitive balance in the first period but decreases competitive balance in the second period. (vii) Expected profit of the favorite (underdog) is increasing (decreasing) for a higher asymmetry. Expected total profits of both contestants increase for a higher asymmetry.

This paper is most closely related to the following three articles. Amegashie (2004) considers a dynamic elimination contest and discusses circumstances under which contestants spend their resources predominantly in early stages of the contest. The main difference to the model presented in this article is that Amegashie

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models the contest in the first stage as an all-pay auction. Stein and Rapoport (2005) discuss the effect of budget constraints in a two-stage game when contestants compete against each other in separate groups and the winners of the first round thereafter compete against each other in a subsequent contest. In contrast to this article, they concentrate on a two-stage contest in which the members of the contest change on the different stages. However, we often observe repeated contests with identical agents. For instance, republicans campaign against the democrats after a four-year term for presidency. Clubs in the major Northern American professional team sports leagues do not change but they play against each other season by season. In this paper I study a dynamic contest with this characteristic. In a static contest model, Grossmann and Dietl conclude that an underdog's profit can increase for a tighter liquidity constraint. This means that an underdog may prefer a situation in which his wealth is lower because the favorite plays less aggressive and, therefore, competition becomes less intense. Dynamic aspects of liquidity constraints are not considered in their paper.

The remainder of this article is structured as follows. Section 2 introduces the model and presents the main assumptions. Furthermore, I discuss the possible equilibria and derive the optimal behavior of contestants in the absence of liquidity constraints as a benchmark. Section 3 starts with a theoretical analysis of potentially binding liquidity constraints. Since the model is not explicitly solvable, I apply a simulation for symmetric contestants. Afterwards, a simulation is provided in the case of asymmetric contestants. Section 4 summarizes the main results and concludes the paper.

2   Model

2.1   Assumptions

Two risk-neutral contestants maximize expected profits. Contestant i ($i = 1, 2$) makes irreversible investments $t_{i,t}$ in period t ($t = 1, 2$) in order to increase the probability to win an exogenous prize R_i in period 1 and an identical prize R_i in period 2. The relative investments in period t determine the winning probability of contestant i in period t according to the following Tullock contest success function[4]:

$$p(t_{i,t}, t_{j,t}) = \frac{t_{i,t}}{t_{i,t} + t_{j,t}} \text{ for } t_{i,t} + t_{j,t} > 0 \tag{1}$$

with $i, j \in \{1, 2\}$, $j \neq i$. If both contestants do not invest, then the winning probability equals zero for each contestant. The linear function $c(t_{i,t}) = t_{i,t}$ characterizes the investment costs.

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[4]Tullock (1980) introduced this contest success function which was axiomated by Skaperdas (1996) and Clark and Riis (1998). In this article, we use a reduced form of this function in order to simplify matters. A more general contest success function has the form $p_i(T_{i,t}, T_{j,t}) = f_i(t_{i,t})/(f_i(t_{i,t}) + f_j(t_{j,t}))$ where f_i and f_j are concave functions.

Expected total profit π_i of contestant i for the whole contest is the sum of expected profit in period 1 ($\pi_{i,1}$) and expected profit in period 2 ($\pi_{i,2}$) such that $\pi_i \equiv \pi_{i,1} + \pi_{i,2}$:[5]

$$\pi_i = \underbrace{\frac{t_{i,1}}{t_{i,1} + t_{j,1}} R_i - t_{i,1}}_{\equiv \pi_{i,1}} + \underbrace{\frac{t_{i,2}}{t_{i,2} + t_{j,2}} R_i - t_{i,2}}_{\equiv \pi_{i,2}} \tag{2}$$

In the beginning of the first contest, contestants are endowed one-time with wealth W which can be invested in period 1 and/or 2 to increase the probability to win the contest. It is important to note that, ex ante, the whole contest can be asymmetric. Contestants have identical initial wealth as well as identical cost functions in each period but can have different valuations of the contest prize.

Competitive balance CB_t is measured by the ratio of the winning probabilities:

$$CB_t = p_{i,t}/p_{j,t}$$

If this ratio gets closer to 1, competitive balance increases.

2.2 Possible Equilibria

Prior to solving the model, preliminary thoughts are presented in terms of possible equilibria. At first glance, one could expect sixteen different types of equilibria. The $(4*4)-$matrix in Table 1 sums up these types depending on which contestant is liquidity-constrained in which period.

liquidity-constrained is/are …		second period			
		no one	only first-period loser	only first-period winner	both
first period	no one	benchmark	core analysis	X	X
	only contestant 1	X	X	X	X
	only contestant 2	X	X	X	X
	both	X	X	X	X

Table 1: Possible Equilibria Depending on Liquidity Constraints

Only two of the sixteen fields represent possible equilibria. The winner of the first-period contest will never be liquidity-constrained in the second period since he receives the contest prize after the first period which is larger than his/her optimal second-period investment independently on the loser's second-period

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[5]Note that there is no time preference for the first-period profit in order to simplify matters. See Grossmann and Dietl (2009) and Grossmann et al. (2011) for an analysis of a time discount factor in a dynamic Tullock contest without liquidity constraints.

behavior.[6] Therefore, columns 3 and 4 in the $(4*4)-$matrix are not possible in equilibrium and therefore marked with an X. Is it possible that one contestant or even both of them are liquidity-constrained in the first-period contest in an optimum? It will never be optimal for a contestant to invest his/her total wealth in the first-period since marginal revenue of investment is infinite in the second period with a positive probability. Therefore, rows 2-4 in the $(4*4)-$matrix are not possible in equilibrium and therefore also marked with an X. As a benchmark, I will analyze contestants' behavior in the case of no liquidity constraints. This means that wealth are sufficiently high in the beginning of the contest. Furthermore, I will show in the core analysis that equilibria exist in which contestants are not constrained in the first-period contest but (only) the loser is constrained in the second-period contest.

2.3 Benchmark

I start with unconstrained contestants with respect to initial wealth in order to receive a benchmark. Contestants are not liquidity-constrained if initial wealth is sufficiently large. In order to receive the optimal investment behavior in a Nash-equilibrium, I apply the conventional concept of backward induction. The following Proposition sums up the main findings:

Proposition 1 *If neither of the two contestants is considered to be liquidity-constrained, then contestant i invests $t_{i,t}^* = R_i^2 R_j / (R_i + R_j)^2$ in period t and has expected total profits of $\pi_i^* = 2R_i^3/(R_i + R_j)^2$.*

Proof. See Appendix 5.1. ∎

I have assumed that contestants' wealth is sufficiently large without mentioning the specific value of the necessary wealth level. After the computation of the equilibrium, it is easy to see that contestant i needs initial wealth $W \geq t_{i,1}^* + t_{i,2}^* = 2R_i^2 R_j/(R_i + R_j)^2$ in order to be unconstrained in equilibrium with probability 1.

3 Endogenous Liquidity Constraint

In this section, I consider a contest in which contestants' wealth are lower compared to the benchmark. After period 1, the loser's wealth limits his spending in period 2. Once again, I apply backward induction to solve the problem.

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[6]This point will be specified in footnote 20 in Appendix 5.1.

3.1 Equilibrium Conditions

3.1.1 Period 2

Contestants observe after period 1 which contestant has won the first contest. Suppose that initial wealths and the realization of the first-period profits imply that only the loser of the first period is liquidity-constrained in period 2.[7] Next, I derive the optimal investment behavior in these circumstances.

Without loss of generality, I assume that contestant i is the winner and contestant j is the loser in period 1. In this case, contestant j is liquidity-constrained which means that his actual wealth (initial wealth minus his first-period investment plus zero revenue from period 1) is lower than the desired second-period investments from the benchmark. Therefore, contestant j invests all his remaining wealth:

$$t_{j,2}(t_{j,1}) = W - t_{j,1} \tag{3}$$

Contestant i maximizes $\pi_{i,2}$ with respect to $t_{i,2}$ and gets the reaction function $t_{i,2}(t_{j,2}) = \sqrt{t_{j,2}R_i} - t_{j,2}$ which depends on contestant j's second-period investment $t_{j,2}$. Contestant i anticipates that contestant j is liquidity-constrained in the second period such that the above-mentioned reaction function reduces to:

$$t_{i,2}(t_{j,1}) = \sqrt{(W - t_{j,1})R_i} - (W - t_{j,1}) \tag{4}$$

3.1.2 Period 1

In period 1, contestants take into account that one contestant will be liquidity-constrained in the second period. But contestants do not know which one it will be. Next, I analyze how contestant 1 will optimally choose his investment in period 1. Note that contestant 2 has a symmetric maximization problem to solve which I neglect for a moment.

Contestant 1 maximizes π_1 with respect to $t_{1,1}$ anticipating the optimal second-period behavior, i.e.,

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[7]In the following subsection, I assume that this condition holds. I derive the necessary parameter conditions for this situation later.

equation (3) and (4).

$$\max_{t_{1,1}} \pi_1 = \underbrace{\frac{t_{1,1}}{t_{1,1} + t_{2,1}} R_1 - t_{1,1}}_{\text{expected profit in period 1}}$$

$$+ \underbrace{\frac{t_{1,1}}{t_{1,1} + t_{2,1}} \left[\frac{\sqrt{(W - t_{2,1})R_1} - (W - t_{2,1})}{\sqrt{(W - t_{2,1})R_1} - (W - t_{2,1}) + W - t_{2,1}} R_1 - \left(\sqrt{(W - t_{2,1})R_1} - (W - t_{2,1}) \right) \right]}_{\text{expected profit in period 2 if contestant 1 wins in period 1}}$$

$$+ \underbrace{\left(1 - \frac{t_{1,1}}{t_{1,1} + t_{2,1}} \right) \left[\frac{W - t_{1,1}}{W - t_{1,1} + \sqrt{(W - t_{1,1})R_2} - (W - t_{1,1})} R_1 - (W - t_{1,1}) \right]}_{\text{expected profit in period 2 if contestant 1 loses in period 1}}$$

The following first-order condition results:

$$
\begin{aligned}
\frac{\partial \pi_1}{\partial t_{1,1}} &= \frac{t_{2,1}}{(t_{1,1} + t_{2,1})^2} R_1 - 1 \\
&\quad + \frac{t_{2,1}}{(t_{1,1} + t_{2,1})^2} \left[\frac{\sqrt{(W - t_{2,1})R_1} - (W - t_{2,1})}{\sqrt{(W - t_{2,1})R_1}} R_1 - \left(\sqrt{(W - t_{2,1})R_1} - (W - t_{2,1}) \right) \right] \\
&\quad - \frac{t_{2,1}}{(t_{1,1} + t_{2,1})^2} \left[\frac{W - t_{1,1}}{\sqrt{(W - t_{1,1})R_2}} R_1 - (W - t_{1,1}) \right] \\
&\quad + \left(1 - \frac{t_{1,1}}{t_{1,1} + t_{2,1}} \right) \left[-0.5(W - t_{1,1})^{-0.5} \frac{R_1}{\sqrt{R_2}} + 1 \right] \\
&= 0
\end{aligned}
\tag{5}
$$

Since there is a symmetric first-order condition for contestant 2, we implicitly get the optimal first-period investments $t_{1,1}^c$ and $t_{2,1}^c$ in equilibrium.[8] These optimal values in the first-period determine the optimal second-period investments $t_{1,2}^c$ and $t_{2,2}^c$ in equations (3) and (4).

3.1.3 Equilibrium

In the general model, it is not possible to prove the existence and uniqueness of the equilibrium. However, if contestants are symmetric, I get the following Proposition:

Proposition 2 *A unique symmetric equilibrium exists in the symmetric contest if the first-period loser is liquidity-constrained in the second period.*

Proof. See Appendix 5.2. ∎

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[8]Note that the superscript c in $t_{1,1}^c$ and $t_{2,1}^c$ stands for the optimal "constrained" first-period investment in order to distinguish it from the benchmark values marked with asterisks. This means that one contestant will be constrained in the second period but not in the first period.

This Proposition states that (i) a symmetric equilibrium always exists and (ii) neither additional symmetric equilibria nor additional asymmetric equilibria exist if contestants have symmetric prize valuations and the first-period loser is liquidity-constrained in the second period. It is important to note that a symmetric equilibrium means that contestants invest symmetrically in the first-period. However, the loser and winner of the first-period contest invest differently in the second contest. Moreover, the Proposition states that we can generally exclude asymmetric equilibria in the symmetric contest in which contestants invest asymmetrically in the first contest. This finding is - ex ante - not an obvious result.[9]

In order to derive and analyze explicit solutions, I have to simulate the model. The necessary conditions which must hold for a maximum with only one constrained contestant in equilibrium are presented in Appendix 5.3. Note that all of these conditions are satisfied for the subsequent simulations.

3.2 Simulation for Symmetric Contestants

In the following subsection, contestants have identical initial wealth and valuation of the contest prize as well as identical cost functions in each period. The reason for this strong assumption is that it allows me to elaborate the asymmetric effects of future liquidity constraints. Liquidity constraints play a major role in the second period since only one of the two contestants win the contest in the first period. The winner receives the prize R and the loser 0 which is observed before contestants choose their investment for the second contest. This profit distribution after period 1 affects the freedom of action in the second period since the winner has more resources to invest in the second contest. I simulate the model for parameters W, R_1 and R_2 as follows: Prize valuation of contestant 1 and 2 are symmetric $R_1 = R_2 \equiv R \in \{3.6, 3.8, 4\}$ and $W \in [1, 1.8]$. According to this constellation, one contestant is liquidity-constrained in equilibrium.[10] It is important to note that, ex ante, the whole contest is fully symmetric.

3.2.1 Investments

In period 1, contestants choose their investments as presented in Figure 1. Both contestant have an identical investment level $t_{1,1}^c = t_{2,1}^c \equiv t_1^c$ due to the ex-ante symmetric starting position.

Higher initial wealth and/or a higher contest prize increase investment in period 1. In the benchmark, unconstrained first-period investments are $t_{1,1}^* = t_{2,1}^* \equiv t_1^* = 0.9$ (for $R = 3.6$), $t_1^* = 0.95$ (for $R = 3.8$) and $t_1^* = 1$ (for $R = 4$). Therefore, the three curves are lower compared to the values in the benchmark.[11]

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[9]See, for instance, Hefti (2011) who discusses the conditions for symmetric contests with asymmetric equilibria.

[10]There is one exemption as a reference to the benchmark and as a proof of consistency: If $W = 1.8$ and $R = 3.6$ - the combination with highest wealth and lowest prize - then there is a corner solution. Both contestants are not constrained in equilibrium but invest their full wealth over the two periods.

[11]Once again, there is one exemption as a reference: If $W = 1.8$ and $R = 3.6$, then both contestants choose $t_1^c = 0.9$ which is consistent to the benchmark $t_1^* = 0.9$ since contestants are not constrained in equilibrium.

Figure 1: First-Period Investment
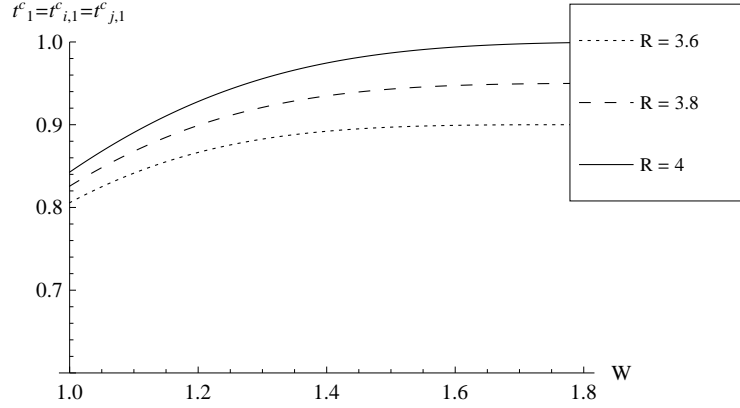
However, the slopes of the three curves in Figure 1 are relatively flat and the investment levels are close to the unconstrained levels for a large range of wealth. Therefore, I conclude that a (small) liquidity constraint has almost no effect on period-1 investment. The intensity of competition attenuates marginally in the first period. In order to stress this point, I have calculated the relative constrained investment t_1^c in period 1 to the unconstrained investment t_1^* in period 1. Thereby, I assume that contestants only have 75% of their required wealth to be unconstrained for the two periods which is a relatively large constraint.[12]

W	1.350	1.425	1.500
R	3.6	3.8	4.0
t_1^c/t_1^*	$\frac{0.890}{0.9} = 99\%$	$\frac{0.937}{0.95} = 99\%$	$\frac{0.987}{1} = 99\%$

Table 2: Low Effects of Future Liquidity Constraints in Period One

Table 2 confirms that contestants respond slightly to the forthcoming constraint even if this constraint is relatively large. Contestants invest 99% relative to their unconstrained investments in period 1.[13]

Figures 2a and 2b present the results in period 2. In the benchmark, unconstrained investments in period 2 are $t_{1,2}^* = t_{2,2}^* \equiv t_2^* = 0.9$ (for $R = 3.6$), $t_2^* = 0.95$ (for $R = 3.8$) and $t_2^* = 1$ (for $R = 4$). In the constrained setting, the loser as well as the winner of the first-period contest invest less in period 2 compared to the benchmark.[14] Contestants' decrease in investment is stronger in period 2 compared to period 1. Especially the loser of the first-period contest reduces severely his/her second-period investment. Overall, contestants

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[12]For $R = 4$, unconstrained contestants invest 1 in each period (see Proposition (5.1)). Therefore, I assume that wealth $W = 1.5$ which corresponds to 75% of the required wealth to be unconstrained for both periods.

[13]Note that this result is robust in a broader sense. I have verified it for various combinations of W and R.

[14]Once again, there is one exemption as a reference: If $W = 1.8$ and $R = 3.6$, then both contestants choose $t_{i,2}^c = t_{j,2}^c = 0.9$ which is consistent to the benchmark since contestants are not constrained in equilibrium.

predominantly react in period 2 such that the competition becomes less intense. Table 3 highlights these effects. Once again, I assume that contestants only have 75% of their required wealth to be unconstrained for the two periods. Table 3 shows that the winner (contestant i) of the first-period contest invests in period 2 92% - whereas the loser (contestant j) of the first-period contest invests in period 2 only 51% - relative to his/her unconstrained investment t_2^* in period 2.

Figure 2: Second-Period Investment

| (a) Winner in Period One | (b) Loser in Period One |
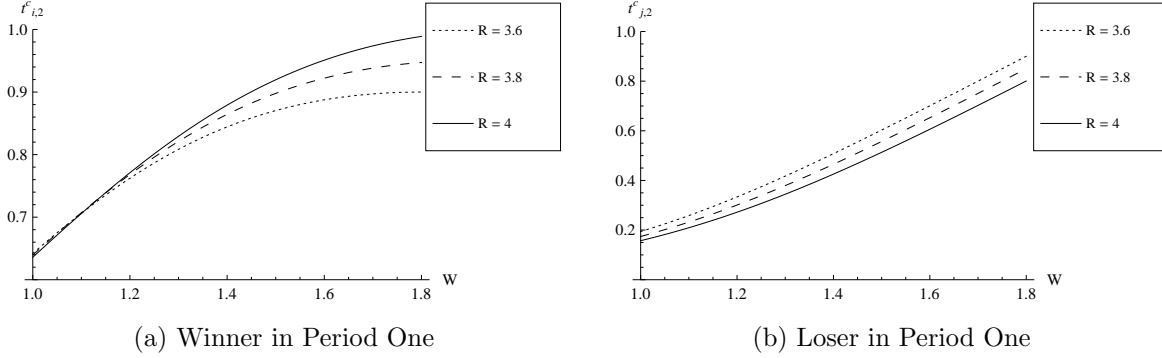
W	1.350	1.425	1.500
R	3.6	3.8	4.0
$t_{i,2}^c/t_2^*$	$\frac{0.828}{0.9} = 92\%$	$\frac{0.874}{0.95} = 92\%$	$\frac{0.920}{1} = 92\%$
$t_{j,2}^c/t_2^*$	$\frac{0.462}{0.9} = 51\%$	$\frac{0.488}{0.95} = 51\%$	$\frac{0.513}{1} = 51\%$

Table 3: High Effects of Liquidity Constraints in Period Two

The characteristics of contests lead to a rather low smoothing behavior over the two periods. This is contradictory to what we could expect from consumption theory. The permanent income hypothesis states that consumers entirely smooth their consumption paths (see Friedman (1957)). So, we could expect with a similar argument, that contestants should reduce their investments in contests if they anticipate future liquidity constraints. In contest, however, contestants anticipate that both contestants will adapt their behavior after period 1 depending on who won the first contest. They play relatively aggressive in the first period since they know the dynamic importance of winning the first contest. Furthermore, they anticipate that the loser in the first contest plays less aggressive in period 2 due to the limited wealth. The strategy of the winner in the first contest is complementary to the loser's strategy (due to the logit formulation of the contest success function) such that he/she also decreases investments in period 2 compared to the benchmark. Because all of this future behavior is anticipated in the beginning of the contest, we have a

relatively low smoothing behavior in contests.

Klumpp and Polborn (2006) also find a relative intense first-period competition in context of the U.S. primaries. Their model simplifies and reduces the complete contest, i.e., the primaries, to a three-stage contest. In each stage, a single contest represents the primary in a specific state. Candidates for the U.S. presidential election have to win at least in two out of the three states in order to win the complete contest. Klumpp and Polborn conclude that candidates have an incentive to invest more in the first contest. The reason is that a candidate has an advantage to win the complete contest as a front-runner.[15]

According to Figure 2a and 2b, it is additionally interesting to see how a contest prize variation affects investments. A higher contest prize, i.e., a higher R in both periods, unambiguously decreases the loser's second-period investment. The reason is as follows: if the contest prize increases, then marginal revenues of first-period investments increase such that both contestants increase their investments in period 1 (see Figure 1). Consequently, the first-period loser's remaining wealth is lower for his/her second-period investment. On the other hand, Figure 2a pretends to show that a higher contest prize always increases the winner's second-period investment. However, a closer look into the data of the simulation shows that a higher prize slightly decreases the second-period investment of the first-period winner for $W < 1.08$.[16] This inversion for a relative tight liquidity constraint, i.e., low initial wealth, appears in all simulations. The reason for this inversion is as follows: Ceteris paribus, a higher contest prize increases the incentives to invest in period 1 and 2 *(direct effect)*. In period 2, the loser has to decrease his/her investment since he/she has increased the first-period investment as mentioned before. The winner anticipates this reaction in the second-period and has an incentive to decrease his/her second-period investments, too *(indirect effect)*. This incentive stems from the complementary strategy due to the logit-formulation of the contest success function. Therefore, there are two opposing effects which are responsible for this ambiguity. It is not astonishing that the indirect effect dominates the direct effect for a tight liquidity constraint since the loser has to decrease his/her second-period investment severely.

In aggregate, second-period investments are always lower for a higher contest prize (see Figure 3a). Thus, the first-period loser's decrease in investment always dominates the possible second-period increase in investment of the first-period winner. In the presence of liquidity constraints, if a contest-designer pursues to increase investments in a contest by increasing the contest prize, he may achieve this aim today, i.e., in the first-period contest, but may fail in the future, i.e., in the second-period contest. This negative dynamic effect on investment has been neglected so far in the contest literature.

---

[15]Even if the result of Klumpp and Polborn has some similarities to the one derived in this section in terms of first period investments, it is important to note that the setting of the models are very different. Klumpp and Polborn do not include liquidity constraints in their model. Furthermore, they assume that there is only one prize in the end of the three sub-contests.

[16]In Figures 2a and 2b, this effect is difficult to see. In Appendix 5.4, I provide an additional example for this phenomenon to highlight this special result graphically in a close-up.

However, even if a higher contest prize reduces aggregate second period investment, aggregate investments over both periods increase (see Figure 3b). Hence, if the contest designer's intention is to increase aggregate investments not in a specific period but rather over the two consecutive contest, then a higher contest prize is an appropriate instrument.

Figure 3: Aggregate Investment

| (a) Only Period Two | (b) Both Periods |
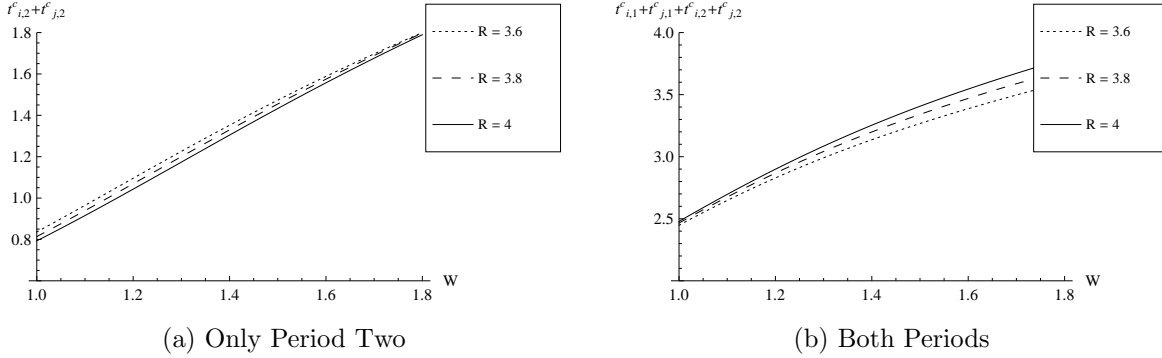
The following Proposition sums up the main findings:

Proposition 3 *In a symmetric contest with one liquidity-constrained contestant in the second period:*

(i) Both contestants decrease investments in both periods compared to an unconstrained situation. However, the decrease is relatively higher in period 2.

(ii) A higher contest prize increases (decreases) the winner's second-period investment in the case of a weak (tight) liquidity constraint but decreases the loser's second-period investment unambiguously. In aggregate, second-period investments decrease for a higher contest prize.

*(iii) Aggregate investments over both periods are higher for a higher contest prize.*

3.2.2 Competitive Balance

Contest designers are often interested in the balance of the contest. In sports economics, for instance, a balanced contest increases interests and revenues (Szymanski (2003)). Therefore, the major Northern American professional team sports leagues introduced mechanism, e.g., draft system, salary caps, revenue sharing, to increase competitive balance. In this model, it is easy to see that the contest is fully balanced in the first period due to the symmetry of the contest. However, the ex-ante symmetric contest leads to an asymmetry in the second period due to the endogenous liquidity constraint. According to Figure 4, we get the following Proposition:

Proposition 4 *In a symmetric contest with one liquidity-constrained contestant in the second period:*

(i) The first-period contest is fully balanced.

(ii) Lower initial wealth or a higher contest prize decreases competitive balance in the second-period contest.

Figure 4: Second-Period Competitive Balance
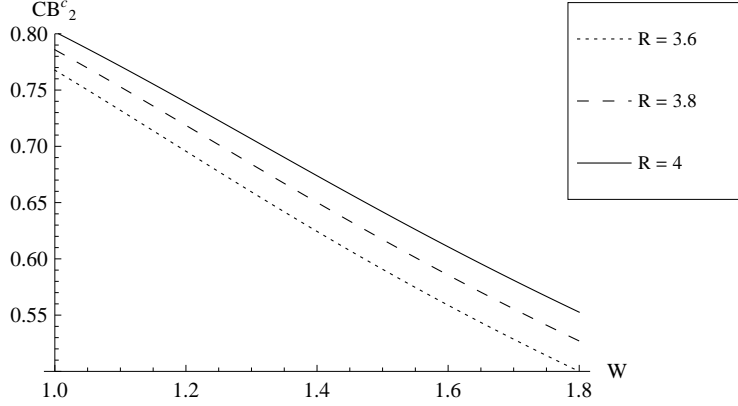
Result *(i)* directly follows from the ex-ante symmetry. Result (ii) is intuitive: a lower initial wealth decreases competitive balance in the second-period since lower wealth decreases the loser's second-period investment more than the winner's second-period investment. A higher contest prize also decreases competitive balance in the second-period. As I have figured out in the last subsection, the loser (of the first-period contest) decreases and the winner (of the first-period) increases or decreases his/her investment in the second period for a higher contest prize. Even if the winner decreases his second-period investment for a higher prize, this reduction never compensates the loser's decrease such that competitive balance in the second-period unambiguously decreases in equilibrium. Therefore, a higher contest prize unambiguously decreases competitive balance in the second-period contest.

These results imply that a contest designer who is primarily interested in a balanced league cannot influence the first-period competitive balance by changing the contest prize. However, a reduction in the contest prize increases competitive balance in the second-period in the presence of liquidity constraints. Otherwise, competitive balance increases in the second period if the contest designer is able to increase initial wealths.

3.2.3 Expected Profits

Contestants are primarily interested in profits. Therefore, it makes sense to consider the effect of liquidity constraints on profits. According to Figure 5, lower wealth increases contestant's expected profits. In case of a liquidity constraint, expected profits are higher than expected profits in the unconstrained case.[17] Furthermore, a higher contest prize increases expected profits. The following Proposition sums up the main findings with respect to profits:

Proposition 5 *In a symmetric contest with one liquidity-constrained contestant in the second period:*

(i) A higher liquidity constraint, i.e., lower wealth, implies higher expected profits.

(ii) A higher contest prize increases expected profits.

Figure 5: Expected Total Profits
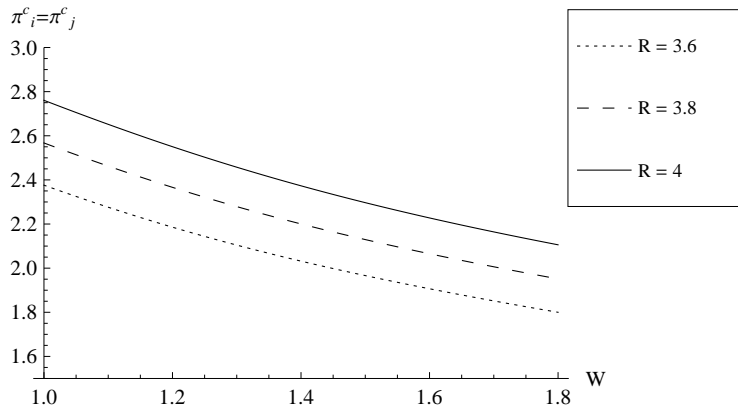
The reason for these results are as follows: *(i)* Potential liquidity constraints reduce incentives to invest. Lower investments imply less competition (especially in the second-period) without decreasing the contest prize. Therefore, expected profits increase for lower wealth. It is important to note that the large positive effect on expected total profits is driven by the second-period contest. Expected profits in the first-period contest increase only marginally due to the slightly lower investment costs. Result *(ii)* is not obvious since aggregate investments over both periods and therefore aggregate costs are higher for a higher contest prize. However, marginal revenues of an investment also increase for a higher contest prize. Altogether, revenues increase more than costs for a higher contest prize.

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[17]As shown in the benchmark (see Proposition (5.1)), each contestant has expected profits of $\pi_i^* = R/2$ for $R_i = R_j \equiv R$. The three curves are higher than $R/2$ for all wealth levels. There is one exception: for $W = 3.6$ and $R = 1.8$, $\pi_i^* = \pi_i^c = 1.8$ since both contestants are unconstrained in this case.

3.3 Simulation for Asymmetric Contestants

In the last section, I assumed that contestants are ex-ante fully symmetric. In this case, I derived a unique symmetric solution which was then simulated to elicit comparative statics. Symmetry was essential to derive and segregate the asymmetric effects of future liquidity constraints on today's behavior. To highlight these effects, I have varied wealth W on the x-axis. In the following three subsections, however, I try to capture the additional effects of ex-ante asymmetry. The prize valuation of contestant 2 is normalized to $R_2 = 1$. Allowing for asymmetry, I assume that the prize valuation of contestant 1 is at least as high as the valuation of contestant 2: $R_1 \in [1, 1.5]$. Thus, contestant 1 is the *favorite* and contestant 2 the *underdog* in this contest as long as $R_1 \neq 1$. To segregate the effect of asymmetry, I will vary the favorite's prize valuation R_1 on the x-axis. Henceforth, a higher asymmetry means a higher value of R_1.[18] In order to satisfy the necessary conditions and have only one constrained contestant in equilibrium (see Appendix 5.3), I choose $W \in \{0.38, 0.40, 0.42\}$. In section 3.1.3, I was not able to theoretically prove the existence and uniqueness in the case of asymmetric contestants. However, all simulations in the following subsections are very robust so that I conjecture that the equilibrium even in the asymmetric model exists and is unique. However, a proof is left for future work.
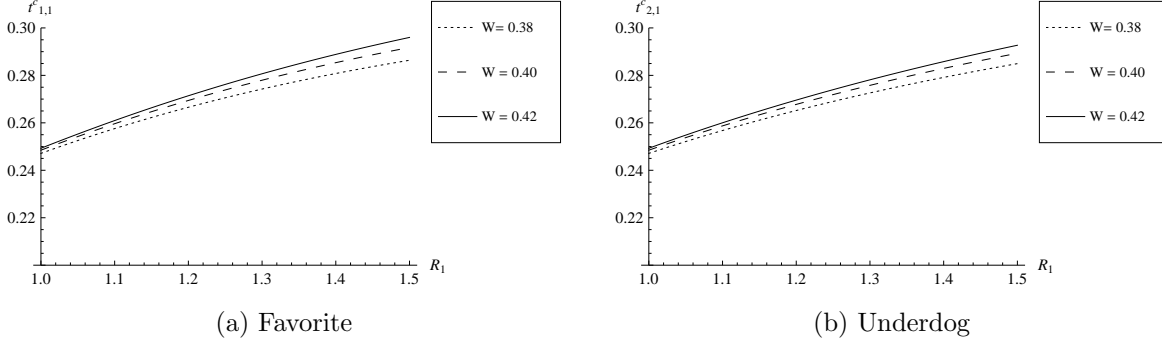
3.3.1 Investments

In a static contest without liquidity constraints, a higher asymmetry increases (decreases) the favorite's (underdog's) investment. In a dynamic setting, however, it is interesting to see that a higher asymmetry, i.e., a higher value R_1, increases first-period investment not only of the favorite but also increases first-period investment of the underdog (see Figures 6a and 6b). This result means that a higher asymmetry intensifies today's competition in the presence of future liquidity constraints. So far, this dynamic effect of liquidity constraints has been neglected in the literature. But why should the underdog increase his/her first-period investment in the case of higher asymmetry? There are two reasons: Ceteris paribus, a higher contest prize for the favorite implies that the underdog's second-period revenues in case of a loss decrease since the favorite increases his/her investments. In short, a first-period loss pays off less for the underdog. Therefore, incentives to invest in the first period increase for the underdog to alleviate the probability to lose the first-period contest. Second, since marginal revenues in the second period decrease for the underdog in the case of a first-period loss, he/she has an incentive to decrease second-period investments. Lower second-period investments in period 2 means higher first-period investment if the underdog is liquidity-constrained in the second period due the first-period loss.

---

[18]Of course, a higher asymmetry could also be interpreted as a lower prize valuation of the underdog keeping the prize valuation of the favorite constant.

According to Figures 6a and 6b, it is hardly to see (especially for small values of R_1) whether the favorite or the underdog invests more. A closer look into the data of the simulation clearly shows that the favorite marginally invests more than the underdog for $R_1 > 1$.

Figure 6: First-Period Investment

(a) Favorite

(b) Underdog

In the second period, I have to differentiate between two cases: Second-period investments depend on which contestant has won the first contest. According to Figures 7a and 7b, contestant 1, i.e., the favorite, increases his/her second-period investment and contestant 2, i.e., the underdog, decreases his/her second-period investment for a higher asymmetry if the favorite has won the first-period contest. The favorite increases his second-period investment due to higher marginal revenues. The underdog decreases his/her second-period investment since a higher asymmetry increases his/her first-period investment which automatically means lower remaining wealth in the second period in the case of a first-period loss. However, higher asymmetry decreases the second-period investment for both contestants if the underdog has won the first-period contest (see Figures 8b and 8b). It is intuitive that the favorite decreases his/her second-period investment in the case of a first-period loss because his/her remaining wealth is lower due to a higher first-period investment.

The effect of higher asymmetry on aggregate second-period investment also depends on who has won the first-period contest. If the favorite has won it, then aggregate second-period investment slightly increases for a higher asymmetry (see Figure 9a). On the other hand, aggregate second-period investment is decreasing in asymmetry if the underdog has won the first-period contest (see Figure 9b).

Aggregate investments over both period increases for a higher asymmetry independently on who has won the first-period contest (see Figure 10a and 10b). However, the increase is stronger if the favorite has won the first-period contest.

In terms of contestants' investment behavior with asymmetry, the following Proposition sums up the main findings:

Figure 7: Second-Period Investment if Favorite won First-Period Contest
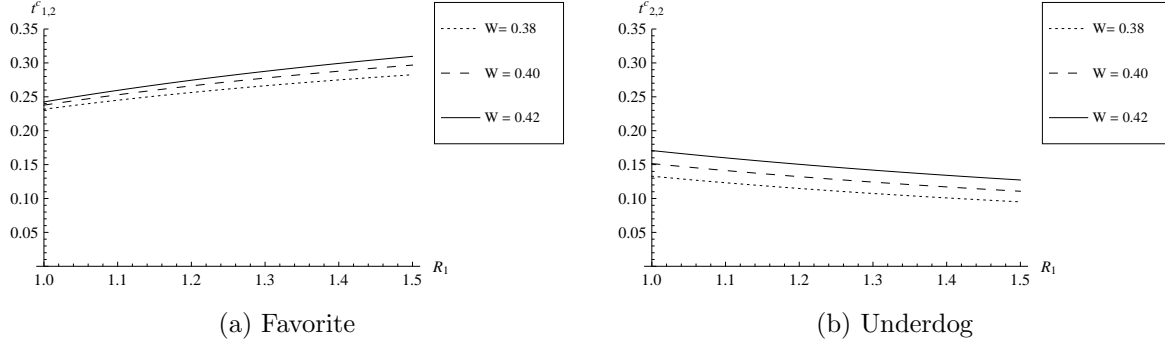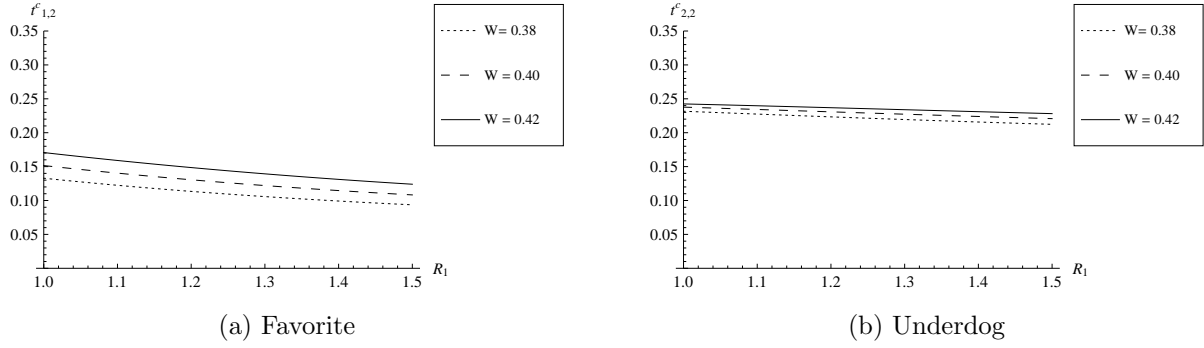

(a) Favorite

(b) Underdog

Figure 8: Second-Period Investment if Underdog won First-Period Contest

(a) Favorite

(b) Underdog

Proposition 6 *In an asymmetric contest with one liquidity-constrained contestant in the second period: higher asymmetry, i.e., a higher value of R_1, implies that:*

(i) the favorite as well the underdog increase their first-period investment such that the competition in the first-period contest intensifies. The favorite has a marginally higher first-period investment than the underdog for $R_1 > 1$.

(iia) the favorite (underdog) increases (decreases) his/her second-period investment if the favorite won the first-period contest.

(iib) second-period investments are lower for both contestants if the underdog won the first-period contest.

(iii) the aggregate second-period investment increases (decreases) if the favorite (underdog) won the first-period contest.

(iv) aggregate investments over both periods increase independently on who won the first-period contest.
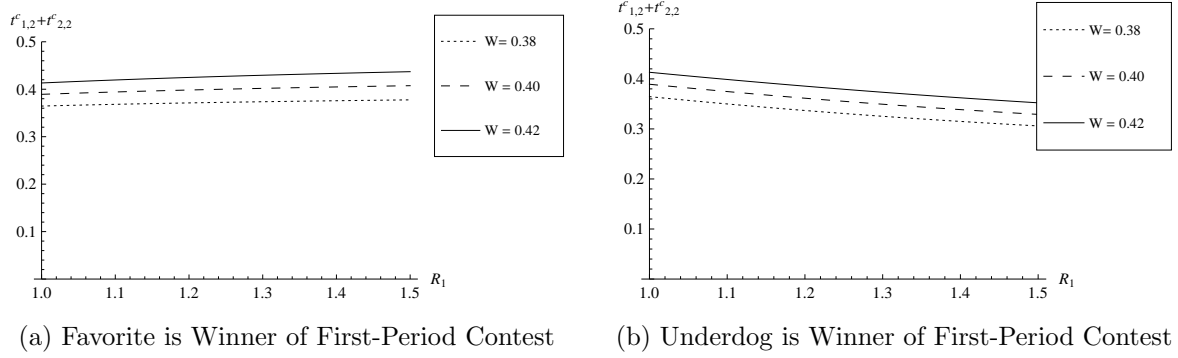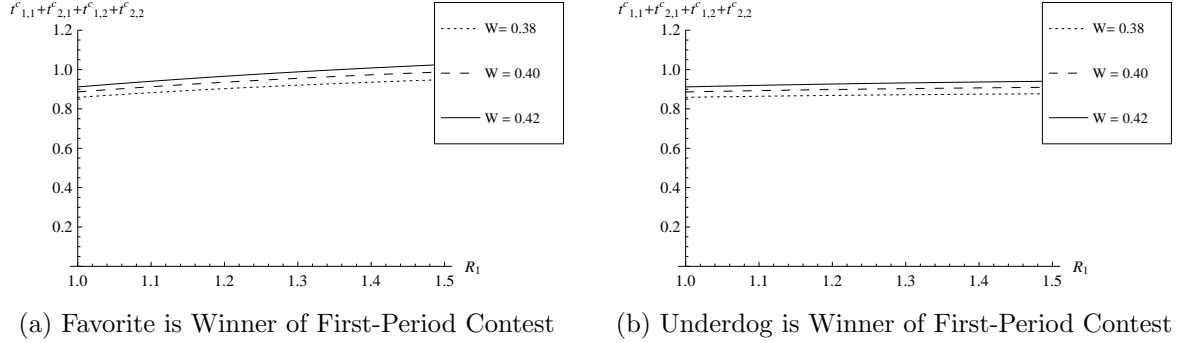
Figure 9: Aggregate Second-Period Investment

(a) Favorite is Winner of First-Period Contest

(b) Underdog is Winner of First-Period Contest

Figure 10: Total Investments

(a) Favorite is Winner of First-Period Contest

(b) Underdog is Winner of First-Period Contest

3.3.2 Competitive Balance

As we have seen in the last subsection, both contestants increase their first-period investment for a higher asymmetry. Therefore, it is not clear whether first-period competitive balance is influenced by asymmetry. According to Figure 11, asymmetry affects first-period competitive balance only marginally. To see the relative small effect of asymmetry, one has to take into account the scale of the y-axis.

Competitive balance in the second-period contest depends on who has won the first-period contest (see Figure 12a and 12b). Competitive balance is decreasing in asymmetry in both cases. The effect is stronger if the favorite has won the first-period contest. Additionally, a lower initial wealth amplifies the negative effect of asymmetry on second-period competitive balance.

The following Proposition sums up the main findings:

Proposition 7 *In an asymmetric contest with one liquidity-constrained contestant in the second period: higher asymmetry, i.e., a higher value of R_1,*

(i) affects first-period competitive balance only marginally.
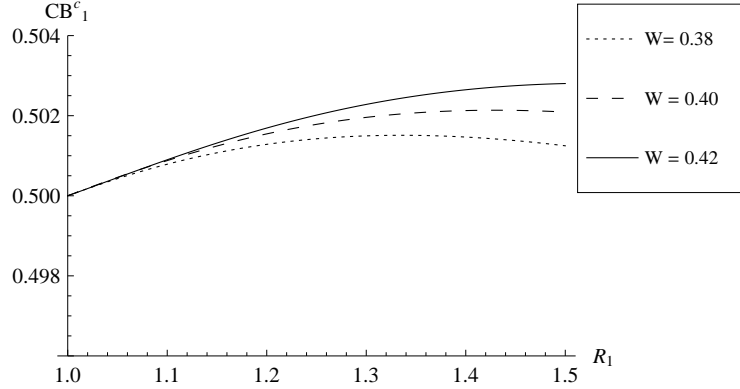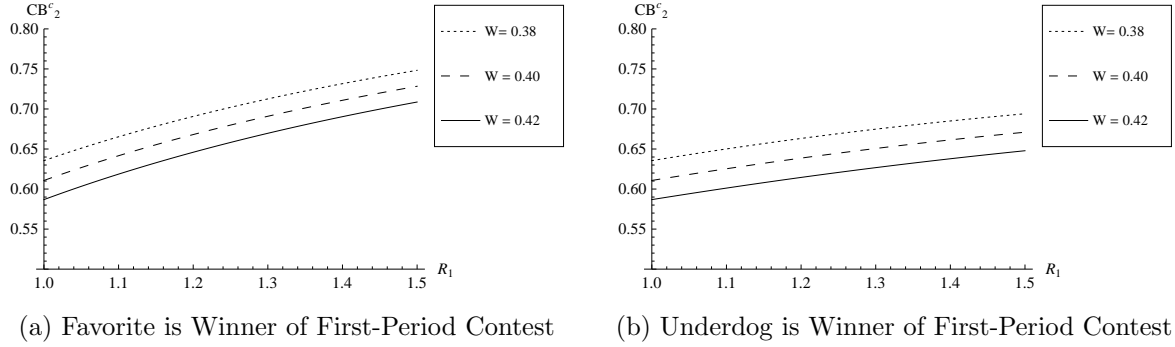
Figure 11: First-Period Competitive Balance

Figure 12: Second-Period Competitive Balance

(a) Favorite is Winner of First-Period Contest

(b) Underdog is Winner of First-Period Contest

(ii) decreases second-period competitive balance independently on who won the first-period contest. This decrease is stronger for a tighter liquidity constraint.

3.3.3 Expected Profits

Figures 13a and 13b show that higher asymmetry increases the favorite's expected total profits and decreases the underdog's expected total profits. The effect of higher asymmetry is relatively stronger for the favorite such that expected total profits for both contestants increase in higher asymmetry (see Figure 13c).

The following Proposition sums up the main findings:

Proposition 8 *In a asymmetric contest with one liquidity-constrained contestant in the second period: higher asymmetry, i.e., a higher value of R_1,*

(i) increases (decreases) the favorite's (underdog's) expected total profits.

(ii) increases expected total profits for both contestants.

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Figure 13: Expected Total Profits

(a) Favorite

(b) Underdog
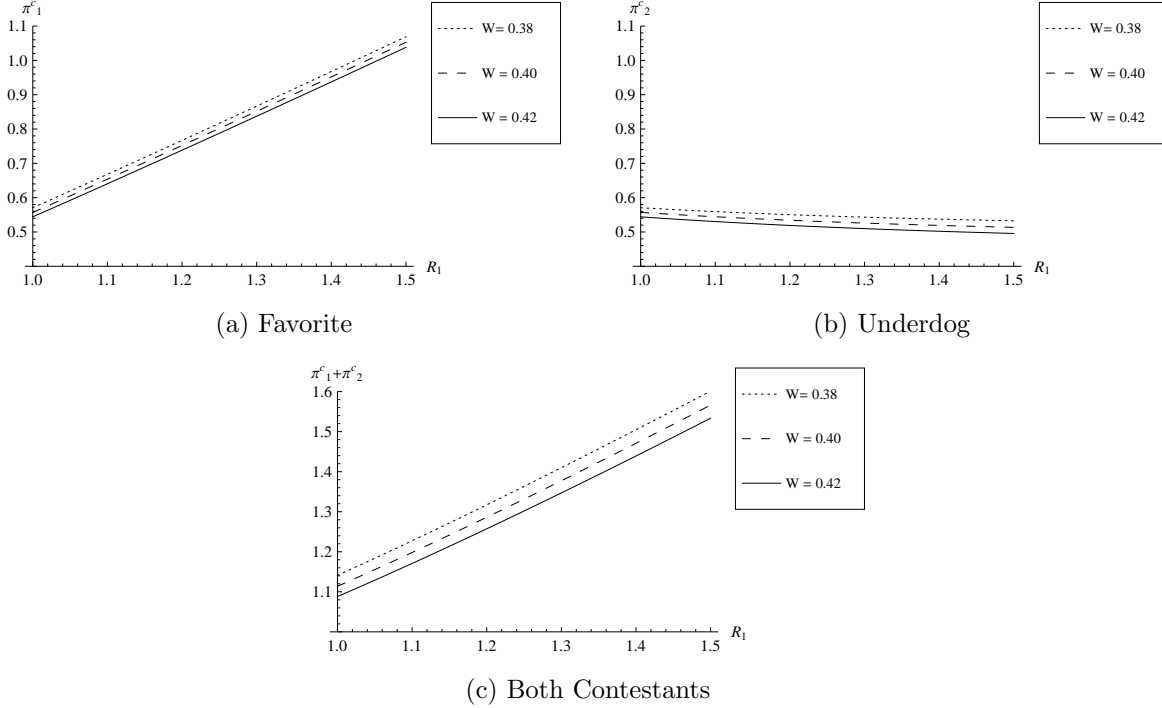
(c) Both Contestants

4 Conclusion

This article presents an analysis of contestants' investment behavior if there is a threat of possible future liquidity constraints. The loser of the first-period contest can be liquidity-constrained in the consecutive second-period contest since he/she does not receive additional revenues as a result of the first-period loss. The analysis reveals dynamic effects of liquidity constraints which are not discussed and understood in static models so far.

A main result of this paper is that a unique symmetric equilibrium exists in a symmetric contest. Furthermore, simulations of the model show that contestants only marginally decrease their investments today in the presence of future liquidity constraints. Therefore, today's contest stays intense even if both contestants know that the loser of today's contest will be liquidity-constrained in the future. Contestants rather decrease their second-period investments. The loser of the first-period contest is forced to reduce his second-period investment due to his/her lack of resources. The winner of the first-period contest decreases the second-period investment since he/she anticipates the loser's reluctance in the second period. These results can be condensed as follows: Contestants reduce their investments as recently as one contestant is really

constrained.

In the major European soccer leagues, several clubs accumulated large debts over the last years as a result of voluminous financial investments in players. The literature revealed some explanations for this phenomenon. Dietl et al. (2008), for instance, argue that an open-league structure with promotion and relegation as it is common in the European leagues - in contrast to the closed-league structure in the major Northern American sports leagues - abets overinvestment. In addition, a high prize differential between the first and second division within open-leagues foster overinvestment. The model in this article can provide an additional channel and explanation, why many clubs in the European soccer leagues still invest large amounts of financial resources in players even if their managers have to take into account possible future shortages. According to the model, there is a high competition in the first period since clubs (still) receive money on the capital market such that these high investments are possible. However, the access to the capital market will get harder and one can expect a transition to the second period of the model in which clubs have to cut their spending heavily. Some of the major Northern American professional team sports leagues introduced salary caps, luxury tax and other mechanism to undermine overinvestment.

A higher contest prize for both periods and both contestants increases first-period investments, but counterintuitively decreases aggregate second-period investment. Therefore, a contest designer should carefully evaluate the dynamic effects of a higher contest prize. In the presence of liquidity constraints, incentives to invest can increase today but simultaneously decrease in the future.

A future liquidity constraint also affects competitive balance and profits. A tighter constraint decreases second-period competitive balance but marginally affects competitive balance in period 1 only if contestants have asymmetric prize valuations. Expected aggregate profits increases if a future liquidity constraint exist since the intensity (primarily) of the second-period contest decreases.

Asymmetry in the prize valuation also changes the outcome of the contest. A higher contest prize for the favorite increases investments of both contestants in the first-period contest. On the one hand, it is intuitive that the favorite increases his/her first-period investment. On the other hand, the surprising result is that also the underdog increases his first-period investment. The reason for this counterintuitive result is that a first-period loss is less attractive for the underdog. The underdog's marginal revenue decreases in the second period in case of a first-period loss. The underdog therefore increases his first-period investment in equilibrium to reduce the probability to lose the first-period contest. This result may contribute an additional argument to the previous discussion regarding the high investments in the European soccer leagues. Since there are less automatic stabilizer for redistribution (like a draft system or revenue sharing) in the European leagues compared to the major Northern American professional team sports leagues and, additionally, the leagues in Europe are usually open (with promotion and relegation) in contrast to the closed-league structure

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in the major Northern American leagues, there should be a higher asymmetry between the clubs in the European league which can amplify the high (first-period) investments in the European leagues.[19]

In the second-period, the effect of a higher asymmetry depends on who won the first-period contest. If the favorite, i.e., the contestant with the higher prize valuation, won the first-period contest, then a higher asymmetry increases (decreases) the second period investment of the favorite (underdog).

Asymmetric prize valuations marginally affect first-period competitive balance but decrease competitive balance in the second period. A higher prize valuation of the favorite increases his/her expected profits more than it decreases the underdog's expected profits.

This article shows that future liquidity constraints can significantly change the outcome on different stages in a dynamic contest. Even if symmetric contestants ex-ante start with identical initial conditions, the outcome of the contest can be asymmetric ex-post due to endogenous liquidity constraints. There is a broad range for future theoretical and empirical research in this area. For instance, a generalization of the cost function or the Tullock contest success function or a consideration of a contest with more than two contestants could reveal further insights with respect to the interaction of dynamics and liquidity constraints.

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[19]Buzzacchi et al. (2003) find empirical evidence for a higher competitive balance in the Northern American leagues.

5   Appendix

5.1   Proof of Proposition 1

Period 2:

In period 2, contestant i maximizes $\pi_{i,2}$ with respect to $t_{i,2}$ and takes $t_{j,2}$, $t_{i,1}$ as well as $t_{j,1}$ as given:

$$\frac{\partial \pi_{i,2}}{\partial t_{i,2}} = \frac{t_{j,2}}{(t_{i,2} + t_{j,2})^2} R_i - 1 = 0$$

Therefore, contestant i's reaction function $t_{i,2}(t_{j,2})$ is[20]

$$t_{i,2}(t_{j,2}) = \sqrt{t_{j,2} R_i} - t_{j,2}.$$

Since both contestants have symmetric first-order conditions it is easy to derive that

$$t_{i,2}^* = \frac{R_i^2 R_j}{(R_i + R_j)^2} \text{ and } \pi_{i,2}^* = \frac{R_i^3}{(R_i + R_j)^2}$$

hold in equilibrium.[21]

Period 1:

In period 1, contestants take into account the second-period behavior and maximize their expected total profits π_i with respect to $t_{i,1}$ as follows:

$$\max_{t_{i,1}} \pi_i = \frac{t_{i,1}}{t_{i,1} + t_{j,1}} R_i - t_{i,1} + \frac{t_{i,2}^*}{t_{i,2}^* + t_{j,2}^*} R_i - t_{i,2}^* = \frac{t_{i,1}}{t_{i,1} + t_{j,1}} R - t_{i,1} + \frac{R_i^3}{(R_i + R_j)^2}$$

$$\frac{\partial \pi_i}{\partial t_{i,1}} = \frac{t_{j,1}}{(t_{i,1} + t_{j,1})^2} R_i - 1 = 0$$

Once again, both contestants have symmetric first-order conditions such that

$$t_{i,1}^* = \frac{R_i^2 R_j}{(R_i + R_j)^2} \text{ and } \pi_{i,1}^* = \frac{R_i^3}{(R_i + R_j)^2}$$

hold in equilibrium. Thus, contestant i's expected total profits are

$$\pi_i^* = \pi_{i,1}^* + \pi_{i,2}^* = \frac{2R_i^3}{(R_i + R_j)^2}.$$

---

[20]Note that the maximum value $t_{i,2} = R_i/4$ for $t_{j,2} = R_i/4$ according to contestant i's reaction function. Therefore, contestant i always wants to invest less than his/her valuation of the contest prize independent of contestant j's choice of $t_{j,2}$.

[21]Note that the second-order conditions are satisfied.

5.2 Existence of Unique Symmetric Equilibrium in a Symmetric Contest

For a symmetric contest, I show in part (i) that there exists exactly one symmetric equilibrium. In part (ii), I prove that the second-order condition is always satisfied for this symmetric equilibrium. In part (iii), I derive that no additional asymmetric equilibria exist for symmetric contestants.

Part (i):

In this part, I show that a symmetric solution exists and that there are no additional symmetric solutions. The two first-order conditions (5) for contestant 1 and 2 and the assumption of identical prize valuations, i.e., $R_1 = R_2 \equiv R$, imply that $t_{1,1}^c = t_{2,1}^c \equiv t_1^c$ and therefore:

$$2(R+W) - 3\sqrt{(W - t_1^c)R} = 4t_1^c + \frac{Rt_1^c}{\sqrt{(W - t_1^c)R}}$$

In the following, I only consider $0 \le t_1^c \le W$ since contestants can invest in maximum their initial wealth in the first period. The left (right) hand side of the last equality is henceforth abbreviated by LHS (RHS). Both sides are continuous functions in t_1^c.

Analysis of LHS:

Left bound:

$$LHS\big|_{t_1^c=0} = 2(R+W) - 3\sqrt{WR} > 0$$

Therefore, I conclude that the LHS starts with a positive value at $t_1^c = 0$.

Right bound:

$$LHS\big|_{t_1^c=W} = 2(R+W) > 0$$

Therefore, I conclude that the LHS ends with a positive value at $t_1^c = W$. It is easy to see that this positive value is larger than the left bound.

The slope:

$$\frac{\partial LHS}{\partial t_1^c} = \frac{3}{2}\frac{R}{\sqrt{(W - t_1^c)R}} > 0$$

The curvature:

$$\frac{\partial^2 LHS}{\partial t_1^{c2}} = \frac{3}{4}\frac{\sqrt{R}}{(W - t_1^c)^{1.5}} > 0$$

Thus, the LHS is a strictly increasing continuous function in t_1^c for $0 \le t_1^c \le W$.

Analysis of RHS:

Left bound:

$$RHS\big|_{t_1^c=0} = 0$$

Therefore, I conclude that the RHS starts in the origin.

Right bound:

$$\lim_{t_1^c \to W} RHS = \infty$$

Therefore, I conclude that the RHS converges to infinity as t_1^c converges to W.

The slope:

$$\frac{\partial RHS}{\partial t_1^c} = 4 + \frac{R\sqrt{(W-t_1^c)R} + 0.5R^2 t_1^c ((W-t_1^c)R)^{-0.5}}{(W-t_1^c)R} > 0$$

The curvature:

$$
\begin{aligned}
\frac{\partial^2 RHS}{\partial t_1^{c2}} &= \frac{\left[-0.5R^2\left((W-t_1^c)R\right)^{-0.5} + 0.5R^2((W-t_1^c)R)^{-0.5} + 0.25R^3 t_1^c((W-t_1^c)R)^{-1.5}\right](W-t_1^c)R}{((W-t_1^c)R)^2} \\
&\quad + \frac{R\left[R\sqrt{(W-t_1^c)R} + 0.5R^2 t_1^c((W-t_1^c)R)^{-0.5}\right]}{((W-t_1^c)R)^2} \\
&= \frac{R^2\sqrt{(W-t_1^c)R} + 0.75R^3 t_1^c((W-t_1^c)R)^{-0.5}}{((W-t_1^c)R)^2} > 0
\end{aligned}
$$

Thus, the RHS is a strictly increasing continuous function in t_1^c for $0 \le t_1^c < W$.

Figure 14 qualitatively presents the LHS and RHS as functions of t_1^c. The two curves have only one intersection. Therefore, I conclude that there exist a unique symmetric equilibrium for t_1^c in the contest with symmetric contestants. Moreover, the unique values for the second-period investments immediately follow from equations (3) and (4).

Figure 14: Existence and Uniqueness in a Symmetric Contest
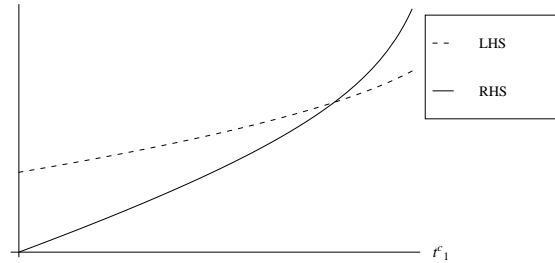
Part (ii):

In this part, I show that the second-order condition is always satisfied for the symmetric equilibrium. If contestants are symmetric, i.e., $R_1 = R_2 \equiv R$, the first-order condition (5) of contestant 1 reduces to

$$
\begin{aligned}
\frac{\partial \pi_1}{\partial t_{1,1}} &= \frac{t_{2,1}}{(t_{1,1} + t_{2,1})^2} R - 1 \\
&+ \frac{t_{2,1}}{(t_{1,1} + t_{2,1})^2} \left[\frac{\sqrt{(W - t_{2,1})R} - (W - t_{2,1})}{\sqrt{(W - t_{2,1})R}} R - \left(\sqrt{(W - t_{2,1})R} - (W - t_{2,1}) \right) \right] \\
&- \frac{t_{2,1}}{(t_{1,1} + t_{2,1})^2} \left[\frac{W - t_{1,1}}{\sqrt{(W - t_{1,1})R}} R - (W - t_{1,1}) \right] \\
&+ \left(1 - \frac{t_{1,1}}{t_{1,1} + t_{2,1}} \right) \left[-0.5(W - t_{1,1})^{-0.5}\sqrt{R} + 1 \right] \\
&= 0
\end{aligned}
\tag{6}
$$

The second-order condition of contestant 1 is derived as follows:

$$
\begin{aligned}
\frac{\partial \pi_1^2}{\partial t_{1,1}^2} &= -\frac{2t_{2,1}}{(t_{1,1} + t_{2,1})^3} R \\
&- \frac{2t_{2,1}}{(t_{1,1} + t_{2,1})^3} \left[\frac{\sqrt{(W - t_{2,1})R} - (W - t_{2,1})}{\sqrt{(W - t_{2,1})R}} R - \left(\sqrt{(W - t_{2,1})R} - (W - t_{2,1}) \right) \right] \\
&+ \frac{2t_{2,1}}{(t_{1,1} + t_{2,1})^3} \left[\sqrt{(W - t_{1,1})R} - (W - t_{1,1}) \right] - \frac{t_{2,1}}{(t_{1,1} + t_{2,1})^2} \left[-\frac{1}{2} \left((W - t_{1,1})R \right)^{-0.5} R + 1 \right] \\
&- \frac{t_{2,1}}{(t_{1,1} + t_{2,1})^2} \left[1 - 0.5(W - t_{1,1})^{-0.5}\sqrt{R} \right] - \frac{t_{2,1}}{t_{1,1} + t_{2,1}} \left[0.25(W - t_{1,1})^{-1.5}\sqrt{R} \right] \\
&< 0
\end{aligned}
$$

In equilibrium, contestants invest $t_{1,1}^c = t_{2,1}^c \equiv t_1^c$ in period 1. Therefore, the second-order condition reduces to

$$
\begin{aligned}
\frac{\partial \pi_1^2}{\partial t_{1,1}^2} \bigg|_{t_{1,1}^c = t_{2,1}^c \equiv t_1^c} &= -\frac{2t_1^c}{(t_1^c + t_1^c)^3} R \\
&- \frac{2t_1^c}{(t_1^c + t_1^c)^3} \left[\frac{\sqrt{(W - t_1^c)R} - (W - t_1^c)}{\sqrt{(W - t_1^c)R}} R - \left(\sqrt{(W - t_1^c)R} - (W - t_1^c) \right) \right] \\
&+ \frac{2t_1^c}{(t_1^c + t_1^c)^3} \left[\sqrt{(W - t_1^c)R} - (W - t_1^c) \right] - \frac{t_1^c}{(t_1^c + t_1^c)^2} \left[1 - \frac{1}{2} \left((W - t_1^c)R \right)^{-0.5} R \right] \\
&- \frac{t_1^c}{(t_1^c + t_1^c)^2} \left[1 - 0.5(W - t_1^c)^{-0.5}\sqrt{R} \right] - \frac{t_1^c}{t_1^c + t_1^c} \left[0.25(W - t_1^c)^{-1.5}\sqrt{R} \right] \\
&< 0
\end{aligned}
$$

$$\Leftrightarrow \quad -\frac{2R}{4\left(t_1^c\right)^2} - \frac{W-t_1}{2\left(t_1^c\right)^2} + \frac{3\sqrt{(W-t_1^c)R}}{4t_1^{c2}} - \frac{1}{2t_1^c} + \frac{\sqrt{R}}{4t_1^c\sqrt{(W-t_1^c)}} - \frac{\sqrt{R}}{8(W-t_1^c)^{1.5}} < 0$$

$$\Leftrightarrow \quad 3W\sqrt{R} - 2t_1^c\sqrt{R} < 2(R+W)\sqrt{(W-t_1^c)} + \frac{(t_1^c)^2\sqrt{R}}{2(W-t_1^c)}$$

Replacing the first term on the right hand side of the last inequality by the first-order condition

$$2(R+W)\sqrt{(W-t_1^c)} = 4t_1^c\sqrt{(W-t_1^c)} + \sqrt{R}t_1^c + 3(W-t_1^c)\sqrt{R},$$

we get:

$$3W\sqrt{R} - 2t_1^c\sqrt{R} \quad < \quad 4t_1^c\sqrt{(W-t_1^c)} + \sqrt{R}t_1^c + 3(W-t_1^c)\sqrt{R} + \frac{(t_1^c)^2\sqrt{R}}{2(W-t_1^c)}$$

$$\Leftrightarrow \quad 0 < 4t_1^c\sqrt{(W-t_1^c)} + \frac{(t_1^c)^2\sqrt{R}}{2(W-t_1^c)}$$

The last condition is always satisfied. Therefore, the implicit solution for t_1^c given by the first-order condition is a maximum.

Part (iii):

In this part, I show that no (additional) asymmetric equilibrium exists in the symmetric contest. Contestant 1's first-order condition (6) reduces to the following simplified first-order condition in period 1:

$$\frac{t_{2,1}}{(t_{1,1}+t_{2,1})^2}\left(2(R+W) - t_{1,1} - t_{2,1} - 2\sqrt{(W-t_{2,1})R} - \sqrt{(W-t_{1,1})R}\right)$$
$$+\frac{t_{2,1}}{t_{1,1}+t_{2,1}}\left[1 - 0.5(W-t_{1,1})^{-0.5}\sqrt{R}\right]$$
$$= \quad 1$$

Symmetrically, contestant 2 has the following simplified first-order condition in period 1:

$$\frac{t_{1,1}}{(t_{1,1}+t_{2,1})^2}\left(2(R+W) - t_{1,1} - t_{2,1} - 2\sqrt{(W-t_{1,1})R} - \sqrt{(W-t_{2,1})R}\right)$$
$$+\frac{t_{1,1}}{t_{1,1}+t_{2,1}}\left[1 - 0.5(W-t_{2,1})^{-0.5}\sqrt{R}\right]$$
$$= \quad 1$$

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Combining the two simplified first-order conditions, the following condition must hold in equilibrium:

$$\frac{t_{2,1}}{(t_{1,1}+t_{2,1})^2}\left(2(R+W)-t_{1,1}-t_{2,1}-2\sqrt{(W-t_{2,1})R}-\sqrt{(W-t_{1,1})R}\right)$$
$$+\frac{t_{2,1}}{t_{1,1}+t_{2,1}}\left[1-0.5(W-t_{1,1})^{-0.5}\sqrt{R}\right]$$
$$=\frac{t_{1,1}}{(t_{1,1}+t_{2,1})^2}\left(2(R+W)-t_{1,1}-t_{2,1}-2\sqrt{(W-t_{1,1})R}-\sqrt{(W-t_{2,1})R}\right)$$
$$+\frac{t_{1,1}}{t_{1,1}+t_{2,1}}\left[1-0.5(W-t_{2,1})^{-0.5}\sqrt{R}\right]$$

$$\Leftrightarrow\quad t_{2,1}\left(2(R+W)-t_{1,1}-t_{2,1}-2\sqrt{(W-t_{2,1})R}-\sqrt{(W-t_{1,1})R}\right)$$
$$+t_{2,1}(t_{1,1}+t_{2,1})\left[1-0.5(W-t_{1,1})^{-0.5}\sqrt{R}\right]$$
$$=\quad t_{1,1}\left(2(R+W)-t_{1,1}-t_{2,1}-2\sqrt{(W-t_{1,1})R}-\sqrt{(W-t_{2,1})R}\right)$$
$$+t_{1,1}(t_{1,1}+t_{2,1})\left[1-0.5(W-t_{2,1})^{-0.5}\sqrt{R}\right]$$

Next, I factor out $t_{2,1}$ on the LHS and $t_{1,1}$ on the RHS in the last equation. Afterwards, I divide both sides by $t_{1,1}$ as well as by the term within the bracket on the LHS. After some manipulations, I receive the following condition:

$$\frac{t_{2,1}}{t_{1,1}}=\frac{2(R+W)-\sqrt{(W-t_{1,1})R}-\sqrt{(W-t_{2,1})R}-\sqrt{(W-t_{1,1})R}-0.5(t_{1,1}+t_{2,1})(W-t_{2,1})^{-0.5}\sqrt{R}}{2(R+W)-\sqrt{(W-t_{1,1})R}-\sqrt{(W-t_{2,1})R}-\sqrt{(W-t_{2,1})R}-0.5(t_{1,1}+t_{2,1})(W-t_{1,1})^{-0.5}\sqrt{R}}$$

The last equation must hold in equilibrium. The question is whether an asymmetric equilibrium is possible with $t_{1,1}>t_{2,1}$ or $t_{1,1}<t_{2,1}$. Due to the symmetry, it is sufficient to show that $t_{1,1}>t_{2,1}$ is not possible in equilibrium. In order to do that, I apply a proof by contradiction:

Proof by contradiction:

Suppose that $t_{1,1}>t_{2,1}$. Then, the last equation implies that the LHS is smaller than 1. In equilibrium, it follows that the RHS must be smaller than 1, too. Therefore,

$$\underbrace{\frac{t_{2,1}}{t_{1,1}}}_{<1}=\underbrace{\frac{2(R+W)-\sqrt{(W-t_{1,1})R}-\sqrt{(W-t_{2,1})R}-\sqrt{(W-t_{1,1})R}-0.5(t_{1,1}+t_{2,1})(W-t_{2,1})^{-0.5}\sqrt{R}}{2(R+W)-\sqrt{(W-t_{1,1})R}-\sqrt{(W-t_{2,1})R}-\sqrt{(W-t_{2,1})R}-0.5(t_{1,1}+t_{2,1})(W-t_{1,1})^{-0.5}\sqrt{R}}}_{<1}$$

It is easy to see that the denominator is larger than the numerator on the LHS of the last equation if the

following condition holds:

$$2(R+W) - \sqrt{(W-t_{1,1})R} - \sqrt{(W-t_{2,1})R} - \sqrt{(W-t_{2,1})R} - 0.5(t_{1,1}+t_{2,1})(W-t_{1,1})^{-0.5}\sqrt{R}$$
$$> \quad 2(R+W) - \sqrt{(W-t_{1,1})R} - \sqrt{(W-t_{2,1})R} - \sqrt{(W-t_{1,1})R} - 0.5(t_{1,1}+t_{2,1})(W-t_{2,1})^{-0.5}\sqrt{R}$$
$$\Leftrightarrow \quad 2\sqrt{(W-t_{1,1})} + \frac{t_{1,1}+t_{2,1}}{\sqrt{W-t_{2,1}}} > 2\sqrt{(W-t_{2,1})} + \frac{t_{1,1}+t_{2,1}}{\sqrt{W-t_{1,1}}}$$

The last inequality holds if $t_{1,1} < t_{2,1}$. However, this condition contradicts the previously made assumption that $t_{1,1} > t_{2,1}$. Therefore, no additional asymmetric equilibrium exists.

5.3   Necessary Conditions

The following necessary conditions must hold for a maximum with only one constrained contestant in equilibrium:

(i) In equilibrium, the loser j of the first-period contest must be constrained in the second period:

$$\frac{\partial \pi_{j,2}}{\partial t_{j,2}}\Big|_{t_{i,1}=t^c_{i,1},t_{j,1}=t^c_{j,1},t_{i,2}=t^c_{i,2},t_{j,2}=t^c_{j,2}} = \frac{t^c_{i,2}}{(t^c_{i,2}+t^c_{j,2})^2}R_j - 1 > 0$$

(ii) In equilibrium, the winner i of the first contest is not constrained in the second period:

$$W + R_i - t^c_{i,1} - t^c_{i,2} \geqslant 0$$

(iii) In period 1, contestants $k = 1, 2$ are not constrained with respect to their first-period investment in equilibrium:

$$W - t^c_{k,1} \geqslant 0$$

(iv) The second-order conditions for the total contest must be satisfied for both contestants $k = 1, 2$ in equilibrium:

$$\frac{\partial^2 \pi_k}{\partial t^2_{k,1}}\Big|_{t_{i,1}=t^c_{i,1},t_{j,1}=t^c_{j,1},t_{i,2}=t^c_{i,2},t_{j,2}=t^c_{j,2}} < 0$$

5.4   Second-Period Investment of the First-Period Winner

This subsection provides a close-up of the second-period investment of the first-period winner. Figure 15 highlights that a tight (weak) liquidity constraint, i.e. a low (high) value of initial wealth W, implies that the winner of the first-period contest decreases (increases) his/her second period investment for a higher contest prize R.

Figure 15: Close-Up of Second-Period Investment of First-Period Winner
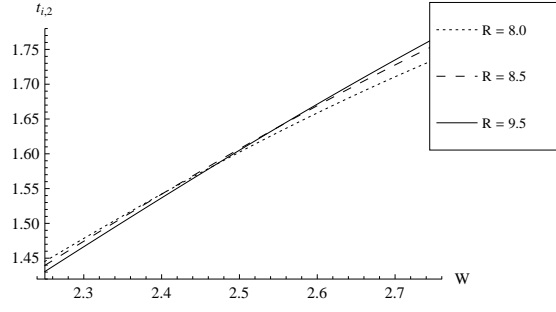
In this example, the parameters are chosen as follows to satisfy the necessary conditions derived in Appendix 5.3: $R_1 = R_2 \equiv R \in \{8.0, 8.5, 9.0\}$ and $W \in [2.25, 2.75]$.

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