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Professional sports leagues have witnessed the appearance of so-called "sugar daddies" - people who invest enormous amounts of money into clubs and become their owners. This paper presents a contest model of a professional sports league that incorporates this phenomenon. We analyze how the appearance of a sugar daddy alters competitive balance and social welfare compared to a league with purely profit-maximizing club owners. We further show that the welfare effect of revenue sharing in a sugar daddy league is ambiguous and depends on the degree of redistribution and on whether the sugar daddy invests in a small or large club.
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1 Introduction

”Only two financing models now remain. The first works for about six clubs, chiefly Manchester United and Barcelona: Have such a big global brand that you can generate money to pay great players. The second and rising model is the sugar daddy. Find an Arab sheikh to buy your club as a toy.”

Simon Kuper, Financial Times

Financial support for football clubs from wealthy investors is a part of the football business. In the Premier League, the top tier of professional English football, rich individuals invest enormous sums of money to support the club and sometimes become de facto club owners. A model case of this is Jack Walker's investment in the second tier Blackburn Rovers F.C. in the early 1990s, which led the club to the Premier League championship in 1994-1995.

As the cost of such investments has increased, the group of possible investors has become more exclusive such that several very wealthy non-British sheikhs and tycoons are now among the so-called ”sugar daddy” club owners. For example, Roman Abramovich invested about £701 million in Chelsea London from 2003 to June 2008. Sheik Mansour invested about £395 million in Manchester City from August 2008 to the beginning of 2010. Mike Ashley invested about £250 million in Newcastle United from May 2007 to May 2009, and Mohamed Al-Fayed invested about £175 million in Fulham FC from 1997 to mid-2008.[1]

According to Deloitte and Touche [2009], this finance model is becoming a very important trend in the English Premier League. Further, sugar daddies' investment in football clubs is not a uniquely British phenomenon. In the Italian Serie A, Silvio Berlusconi and Massimo Moratti regularly transfer great sums of money to the clubs AC Milan and FC Internazionale Milano, respectively, at the end of the season. The ”sugar daddy concept” also appears to have become the dominant finance model for clubs in Eastern Europe since the decline of communism [Franck, 2010b].

The difference between this type of investment and more typical investment by profit-maximizing investors outside the sports industry is that these sugar daddies invest enormous amounts of money and seem not to take the resulting financial losses into account. The utility derived from sporting success appears to compensate for the financial losses.[2] Because these sugar daddies not only financially support their clubs but also become actual club owners with ”full control”, Kuper [2009] may be right in the statement that football clubs become the toys of wealthy persons.

In short, sugar daddies invest in a football club and become club owners with the right to control all business and sporting aspects of the club. The invested money is used to hire talent on the player market to improve a team's playing strength. The illustrative

[1]The indicated sums in the above examples are taken from Conn [2009a,b, 2010] and Kelso [2009].

[2]See Franck [2010a], who outlines the possible reasons for a sugar daddy to invest in football clubs in greater detail. He also provides more information about the governance structures and ownership of clubs within European football leagues.

examples provided above suggest that clubs in which a sugar daddy has invested do not have a fixed budget constraint in a narrow sense. As a result, these clubs have more resources to invest in talent compared to other clubs without a sugar daddy. With his appearance in a sports league, a sugar daddy alters the competition in the league. This paper develops a contest model of a professional sports league and analyzes how the appearance of a sugar daddy alters competitive balance and social welfare compared to a league with purely profit-maximizing club owners.

Before proceeding with the model, we provide a short literature overview. Fort and Quirk [1995] and Vrooman [1995] have formalized the insights developed in the early sports economics literature by Rottenberg [1956] and Neale [1964] in a general economic model of a sports league. This "Fort, Quirk, and Vrooman model" is considered as the common origin of the early fixed supply model of a sports league. Since then economists have repeatedly tried to contribute to the development of a better understanding of a sports league by seeking to capture the interaction between clubs in formal league models. The recent sports economics literature has suggested modeling a team sports league using contest theory [Szymanski, 2003].

The existing literature mainly considers football clubs to be either pure profit-maximizers [Fort and Quirk, 1995, Szymanski and Késenne, 2004, Dietl, Lang, and Rathke, 2010] or pure win-maximizers [Zimbalist, 2003, Fort and Quirk, 2004, Késenne, 2006]. Sloane [1971] was the first to suggest that the owner of a sports club actually maximizes utility, which may include inter alia playing success and profits. Sloane defines the different objective functions of a club owner but he does not formalize his insights in an analytical league model. Rascher [1997] takes up Sloane's idea and assumes that clubs maximize a linear combination of profits and wins. The crucial difference with respect to our model is that Rascher [1997] applies "Walrasian conjectures" and assumes a fixed supply of talent in the league [see also Késenne, 2007].

Vrooman [1997, 2000] presents a model of ownership and financial structures among the Major League Baseball (MLB) franchises. In his model, a so-called sportsman owner jointly maximizes the franchise value and the satisfaction derived from winning. Vrooman finds that a sportsman owner sacrifices franchise value for winning, but he does not explicitely model the interactions between the franchises. We extend Vrooman's model by developing a contest model and by deriving welfare implications of the emergence of a sugar daddy. Finally, Dietl, Lang, and Werner [2009] analyze mixed sports leagues in which one club owner is a pure profit maximizer and the other is a pure win maximizer. However, their model does not capture the concept of a sugar daddy because the club owners do not trade off profits and win percentages. The characteristic of a sugar daddy as an agent within a league competition has not yet been studied in the context of a contest model, and this is the focus of our paper. Note that the purpose of this article is not to speculate about the motivation for a sugar daddy to enter the league but rather, given the existence of a sugar daddy, to analyze his effect on the league competition.

Based on a contest model, we show that the competitive balance within a league with a sugar daddy may be higher or lower than in a league with pure profit-maximizing clubs depending on the market sizes of the clubs and the win preference of the sugar daddy.

Furthermore, social welfare in a sugar daddy league may be higher in comparison to a league with pure profit-maximizing clubs if the sugar daddy invests in a large-market club. In contrast, social welfare will always be lower if the sugar daddy invests in a small-market club. Finally, by focusing on the sugar daddy league, we disclose the effect of revenue sharing on competitive balance and social welfare. We show that competitive balance always decreases through revenue sharing. The welfare effect of revenue sharing is ambiguous and depends on the degree of redistribution and on whether the sugar daddy invests in a small- or a large-market club.

The remainder of the paper is structured as follows. In Section 2, we introduce the model and derive club profits via fan demand. We further specify the utility function of the sugar daddy and derive social welfare. In Section 3, we present the equilibrium in a league with pure profit-maximizing clubs and a sugar daddy league, and compare both leagues. In Section 4, we analyze the effect of revenue sharing on competitive balance and social welfare in the sugar daddy league. Finally, Section 5 summarizes the main findings and concludes the paper.

2 Model Setup

We model a two-club league in which both clubs participate in a non-cooperative game and independently invest a certain amount in playing talent. Each club $i \in \{1, 2\}$ generates its own revenues, denoted by R_i, according to a fan demand function that depends on the match quality. Talent investments, denoted by $x_i \in \mathbb{R}_0^+$ for club i, determine the match quality, and therefore, through fan demand, they determine the revenues of both clubs.

2.1 Fan Demand and Club Revenues

Fan demand for a match with quality q_i and club revenues are derived as in Dietl et al. [2009]. Suppose that there is a continuum of fans that differ in their willingness to pay for a match between club i and club j with quality q_i.[3] The parameter $\theta_k \sim U[0, 1]$ measures the preference for match quality of fan k. We define the net utility of fan type θ_k as $\max\{\theta_k q_i - p_i, 0\}$. At price $p_i \in \mathbb{R}^+$,[4] the fan who is indifferent to the consumption of the product or not is given by $\theta^* = p_i/q_i$. It follows that $1 - \theta^* = (q_i - p_i)/q_i$ denotes the measure of fans who purchase at p_i. Thus, the fan demand function of club $i \in \{1, 2\}$ is given by

$$d_i(p_i, q_i) = m_i \left(1 - \frac{p_i}{q_i}\right),$$

where $m_i \in \mathbb{R}^+$ represents the market size parameter of club i. We assume that clubs are heterogeneous with respect to their market size or drawing potential. Without loss of

[3] Note that quality q_i represents the quality of the competition in the stadium of club i. The quality q_i is specified below by equation (4).

[4] For instance, the price p_i can be interpreted as the subscription fee for TV coverage of the match or the gate price.

generality, we assume that club 1 is the large-market club and club 2 is the small-market club, i.e., $m_1 > m_2$. As a result, the large-market club 1 generates higher demand for a given set of parameters (p_i, q_i) than the small-market club 2. For notational sake, we write $m_1 = \sigma m$ and $m_2 = m$ with $\sigma > 1$.

By normalizing all other costs to zero (e.g., stadium and broadcasting costs), club i's revenue is given by[5]

$$R_i = \frac{m_i}{4} q_i. \tag{1}$$

Following DIETL ET AL. [2009], we assume that match quality q_i depends on two factors: the probability of club i's success and the uncertainty of the outcome. We further assume that both factors enter the quality function as a linear combination with equal weights, that is: quality = probability of success + uncertainty of outcome. This specification of the quality function gives rise to a quadratic revenue function that is widely used in the sports economic literature.

We measure the probability of club i's success by the win percentage w_i of this club. The win percentage is characterized by the contest-success function (CSF). Applying the logit approach, the win percentage of club $i \in \{1, 2\}$ is given by

$$w_i(x_i, x_j) = \frac{x_i}{x_i + x_j}, \tag{2}$$

with $i, j \in \{1, 2\}$, $i \neq j$. The logit CSF is probably the most widely used functional form of a CSF in sporting contests. This CSF was introduced by TULLOCK [1980] and was subsequently axiomatized by SKAPERDAS [1996] and CLARK AND RIIS [1998]. An alternative functional form would be the probit CSF [LAZEAR AND ROSEN, 1981, DIXIT, 1987] and the difference-form CSF [HIRSHLEIFER, 1989].

Given that the sum of the win percentages must equal unity, we obtain the adding-up constraint: $w_j = 1 - w_i$. In our model, we adopt the "Contest-Nash conjectures" $\partial x_i/\partial x_j = 0$ and compute the derivative of (2) as $\partial w_i/\partial x_i = x_j/(x_i + x_j)^2$. The so-called "Walrasian conjectures" $\partial x_i/\partial x_j = -1$ were applied in the traditional literature [EL-HODIRI AND QUIRK, 1971, FORT AND QUIRK, 1995, RASCHER, 1997] for leagues with a fixed supply of talent. The recent literature, however, proposes the use of the Contest-Nash conjectures $\partial x_i/\partial x_j = 0$ to characterize non-cooperative behavior between clubs [SZYMANSKI, 2003, 2004, SZYMANSKI AND KÉSENNE, 2004]. For a discussion regarding the Walrasian and Contest-Nash conjectures, see SZYMANSKI [2004], ECKARD [2006], and FORT AND QUIRK [2007].

The uncertainty of the outcome is measured by the competitive balance CB in the league and is specified by the product of the winning percentages:

$$CB(x_i, x_j) = w_i(x_i, x_j) w_j(x_i, x_j) = \frac{x_i x_j}{(x_i + x_j)^2}, \tag{3}$$

with $i, j \in \{1, 2\}$ and $i \neq j$. Note that competitive balance CB attains its maximum of 1/4 for a completely balanced league in which both clubs invest the same amount in

[5] Note that club i chooses the price $p_i^* = q_i/2$ to maximize revenues $R_i = p_i \cdot d_i(p_i, q_i)$.

talent such that $w_1 = w_2 = 1/2$. A less balanced league is then characterized by a lower value of CB.

With win percentage specified by equation (2) and competitive balance specified by equation (3), the quality function is derived as

$$q_i(x_i, x_j) = w_i(x_i, x_j) + w_i(x_i, x_j)w_j(x_i, x_j), \tag{4}$$

with $i, j \in \{1, 2\}$, $i \neq j$. Plugging (4) into (1) and noting that $w_j = 1 - w_i$, we derive the revenue function of club $i \in \{1, 2\}$ as[6]

$$R_i(x_i, x_j) = \frac{m_i}{4} q_i(x_i, x_j) = \frac{m_i}{4} \left[2w_i(x_i, x_j) - w_i(x_i, x_j)^2\right].$$

Assuming a competitive labor market and following the sports economics literature, the market clearing cost of a unit of talent, denoted by c, is the same for every club. The cost function of club $i \in \{1, 2\}$ is thus given by $C(x_i) = cx_i$, where c is the marginal unit cost of talent.

2.2 Club Profits and the Sugar Daddy's Utility Function

The profit function of club $i \in \{1, 2\}$ is given by revenue minus costs:

$$\pi_i(x_i, x_j) = R_i(x_i, x_j) - C(x_i) = \frac{m_i x_i (x_i + 2x_j)}{4(x_i + x_j)^2} - cx_i,$$

with $i, j \in \{1, 2\}$ and $i \neq j$.

We will analyze two different leagues: first, a league with pure profit-maximizing clubs (PM-league) and second, a league in which a sugar daddy invests in one of the two clubs (SD-league). If the sugar daddy invests in the large-market club 1, we call it a type 1 SD-league, while a type 2 SD-league represents a league in which the sugar daddy invests in the small-market club 2.

We assume that the sugar daddy's utility function exhibits diminishing marginal utility of winning and is given by

$$\begin{aligned} u_i(x_i, x_j) &= \pi_i(x_i, x_j) + \gamma w_i(x_i, x_j) \\ &= \frac{m_i}{4} \left[2w_i(x_i, x_j) - w_i(x_i, x_j)^2\right] - cx_i + \gamma w_i(x_i, x_j), \end{aligned}$$

where $\gamma \in \mathbb{R}^+$ represents the sugar daddy's preference for winning.[7]

2.3 Social Welfare

We assume that social welfare is given by the sum of aggregate consumer (fan) surplus, aggregate player salaries and aggregate club profits [see Dietl and Lang, 2008]. Aggregate consumer surplus is computed by adding the consumer surplus from fans of club

[6] This revenue function is consistent with the revenue functions used, e.g., in Hoehn and Szymanski [1999], Késenne [2007], and Vrooman [2007, 2008].

[7] Note that the sugar daddy's utility is linear with respect to the win preference part.

1 to that of club 2. The consumer surplus CS_i from fans of club $i \in \{1,2\}$ in turn corresponds to the integral of the demand function $d_i(p_i, q_i)$ from the equilibrium price $p_i^* = q_i/2$ to the maximal price $\overline{p}_i = q_i$ that fans are willing to pay for quality q_i:

$$CS_i = \int_{p_i^*}^{\overline{p}_i} d_i(p_i, q_i) dp_i = \int_{\frac{q_i}{2}}^{q_i} m_i \frac{q_i - p_i}{q_i} dp_i = \frac{m_i}{8} q_i.$$

Assuming that players' utility corresponds to their salary, total players' utility is given by the aggregate salary payments $PS = cx_1 + cx_2$ in the league.

The addition of aggregate consumer surplus, aggregate salary payments, and aggregate club profits produces social welfare as follows:[8]

$$W(x_1, x_2) = \frac{3m}{8} \left[\sigma q_1(x_1, x_2) + q_2(x_1, x_2) \right].$$

Salary payments do not directly influence social welfare because salaries merely represent a transfer from clubs to players. Moreover, note that we neglect the win preference part of the sugar daddy's utility in the welfare function. Economically, one would have to add this term. However, the reason to neglect this term is that we will compare a league without a sugar daddy to a league with a sugar daddy. To compare the two leagues, it makes sense to ignore the win preference part of the sugar daddy's utility. If this term were included in the welfare function, it could be easily argued that a sugar daddy league increases social welfare compared to a profit-maximizing league by assuming a high parameter γ. We will show that under specific parameter conditions, even without the win preference part of the sugar daddy's utility, the sugar daddy league yields higher social welfare compared to the league with profit-maximizing clubs.

In the next lemma, we derive the welfare-maximizing win percentages.

Lemma 1 *Social welfare is maximized for*

$$\left(w_1^W, w_2^W\right) = \left(\frac{\sigma}{\sigma+1}, \frac{1}{\sigma+1}\right), \tag{5}$$

such that the welfare-optimal level of competitive balance is given by

$$CB^W = \frac{\sigma}{(\sigma+1)^2}. \tag{6}$$

Proof See Appendix A.1.

This lemma shows that social welfare is maximized in an unbalanced league in which the large-market club has a higher win percentage than the small-market club. Note that a greater difference in market sizes requires a greater imbalance to maximize social welfare. This result is intuitive because (i) the large-market club has higher marginal revenues than the small-market club and (ii) fan demand and thus the consumer welfare are higher for the large-market club due to its larger market size.

[8] Recall that $m_1 = \sigma m$ and $m_2 = m$.

3 Equilibrium Analysis

3.1 Pure Profit-Maximizing Clubs (PM-league)

In this section, we analyze the benchmark case in which both clubs maximize their profits. Club $i \in \{1, 2\}$ solves the following maximization problem:

$$\max_{x_i \geq 0} \left\{ \pi_i = \frac{m_i x_i (x_i + 2x_j)}{4(x_i + x_j)^2} - c x_i \right\}.$$

The corresponding first-order conditions yield[9]

$$\frac{\partial \pi_i}{\partial x_i} = \frac{m_i x_j^2}{2(x_i + x_j)^3} - c = 0,$$

with $i, j \in \{1, 2\}$ and $i \neq j$.

Lemma 2 *In the PM-league, the equilibrium win percentages are given by*

$$(w_1^{PM}, w_2^{PM}) = \left(\frac{\sqrt{\sigma}}{1 + \sqrt{\sigma}}, \frac{1}{1 + \sqrt{\sigma}} \right). \tag{7}$$

Proof See Appendix A.2.

In equilibrium, $w_1^{PM} > w_2^{PM}$, i.e., the large-market club 1 invests more in playing talent due to its higher marginal revenue of talent investment than the small-market club 2. Competitive balance in the PM league is given by

$$CB^{PM} = \frac{\sqrt{\sigma}}{(1 + \sqrt{\sigma})^2}.$$

By comparing (7) with the welfare-maximizing win percentages (5), we see that the degree of competitive balance is higher in the PM-league compared to the welfare optimum. Thus, from a welfare point of view a more unbalanced PM-league is desirable in which the small-market club 2 wins less often and the large-market club 1 wins more often.[10]

Through its talent investments, each club imposes a negative externality on the other club's expected revenue. Because of the asymmetric market sizes, the small-market club imposes a larger externality on the large-market club than vice versa [c.f. Dietl and Lang, 2008]. This result is because the increase in revenue from a given increase in win percentage is higher for the large-market club than for the small-market club. Neither club, however, internalizes this negative externality. As a result, in the PM-league, the

[9] It can be easily verified that the second-order conditions for a maximum are satisfied.

[10] An interesting analysis regarding the optimal degree of competitive balance is provided by Groot [2009]. He shows that the optimal degree of competitive balance is lower in open leagues than in closed leagues. For more details, see also Vrooman [2007, 2008], who compares the effects of open and closed league models.

marginal revenue of talent is equalized between the two clubs, but the marginal revenue of a win is not. More precisely, the marginal revenue of a win is higher for the large-market club than for the small-market club. Thus, a decrease in the win percentage of the small-market club and an increase in the win percentage of the large-market club results in higher social welfare. Note that in the case of symmetric clubs, i.e., $\sigma = 1$, social welfare is maximized in the PM-league.

3.2 The Sugar Daddy League (SD-league)

In this section, we derive the equilibrium in the sugar daddy league, in which club $i \in \{1, 2\}$ maximizes utility and club $j \in \{1, 2\}$, $j \neq i$ maximizes profits. Recall that in the type 1 SD-league, the sugar daddy invests in the large-market club (i.e., $i = 1$ and $j = 2$), while in the type 2 SD-league the sugar daddy invests in the small-market club (i.e., $i = 2$ and $j = 1$). The maximization problem of club $i \in \{1, 2\}$ in which the sugar daddy has invested is thus given by

$$\max_{x_i \geq 0} \left\{ u_i(x_i, x_j) = \frac{m_i x_i \left(x_i + 2x_j\right)}{4(x_i + x_j)^2} - c x_i + \gamma \frac{x_i}{x_i + x_j} \right\},$$

and for the profit-maximizing club $j \in \{1, 2\}$, $j \neq i$ it yields

$$\max_{x_j \geq 0} \left\{ \pi_j(x_i, x_j) = \frac{m_j x_j \left(x_j + 2x_i\right)}{4(x_i + x_j)^2} - c x_j \right\}.$$

The corresponding first-order conditions are given by[11]

$$\frac{\partial u_i}{\partial x_i} = \frac{x_j(m_i x_j + 2\gamma(x_i + x_j))}{2(x_i + x_j)^3} - c = 0 \text{ and } \frac{\partial \pi_j}{\partial x_j} = \frac{m_j x_i^2}{2(x_i + x_j)^3} - c = 0.$$

Lemma 3 *In the type 1 SD-league, the equilibrium win percentages are*

$$(w_1^{SD1}, w_2^{SD1}) = \left(\frac{\gamma + \kappa_1}{\gamma + m + \kappa_1}, \frac{m}{\gamma + m + \kappa_1} \right) \text{ with } \kappa_1 \equiv \left(\gamma^2 + 2m\gamma + \sigma m^2\right)^{1/2},$$

while in the type 2 SD-league they are given by

$$(w_1^{SD2}, w_2^{SD2}) = \left(\frac{\sigma m}{\gamma + \sigma m + \kappa_2}, \frac{\gamma + \kappa_2}{\gamma + \sigma m + \kappa_2} \right) \text{ with } \kappa_2 \equiv \left(\gamma^2 + 2\sigma m\gamma + \sigma m^2\right)^{1/2}.$$

Proof See Appendix A.3.

In the type 1 SD-league, the win percentage of club 1 (club 2) always increases (decreases) through the higher win preference of the sugar daddy, i.e., $\partial w_1^{SD1}/\partial\gamma > 0$ and $\partial w_2^{SD1}/\partial\gamma < 0$. As a result, the revenues of club 1 (club 2) increase (decrease). Conversely, for the type 2 SD-league, i.e., $\partial w_1^{SD2}/\partial\gamma < 0$ and $\partial w_2^{SD2}/\partial\gamma > 0$.

Lemma 4 *In the type 1 SD-league, the large-market club 1 will always be the dominant team in equilibrium, whereas in the type 2 SD-league, the small-market club 2 will be the dominant team in equilibrium if the sugar daddy's win preference γ is sufficiently large with $\gamma > \gamma' \equiv (m/4)(\sigma - 1)$.*

[11] It can be easily verified that the second-order conditions for a maximum are satisfied.

Proof Straightforward.

It is clear that the large-market club will always be the dominant team with a higher win percentage in equilibrium if the sugar daddy invests in the large-market club (type 1 SD-league). If the sugar daddy invests in the small-market club (type 2 SD-league), however, this club can become the dominant team in equilibrium through a sufficiently strong win-orientated behavior of the sugar daddy.

Competitive balance in the two types of SD-league yields

$$CB^{SD1} = \frac{m(\gamma + \kappa_1)}{(\gamma + m + \kappa_1)^2} \text{ and } CB^{SD2} = \frac{\sigma m(\gamma + \kappa_2)}{(\gamma + \sigma m + \kappa_2)^2}.$$

The type 2 SD-league becomes more balanced through a higher win preference of the sugar daddy until the league is completely balanced for $\gamma = \gamma'$. Note that in the type 1 SD-league, competitive balance always decreases through a higher win preference of the sugar daddy.

3.3 Comparison of the PM-League and SD-League

In this section, we compare the outcomes of the PM-league with the outcomes of the two different types of SD-leagues. We are interested in the competitive balance and welfare of the two leagues.

We establish the following proposition regarding competitive balance:

Proposition 1 (Competitive Balance) *The type 1 SD-league is less balanced than the PM-league, whereas the type 2 SD-league is more balanced than the PM-league as long as the sugar daddy's win preference γ is sufficiently small with $\gamma < \gamma'' \equiv m/\left[2(\sigma+1)(\sigma^{1/2}-1)\right]$.*

Proof See Appendix A.4.

It is clear that the PM-league is more balanced than the type 1 SD-league because the sugar daddy invests in the large-market club 1. As a result, the sugar daddy will further unbalance the league through his more win-orientated behavior. Conversely, in the case that the sugar daddy invests in the small-market club 2 (type 2 SD-league), his more win-orientated behavior further balances the league in the beginning (i.e., for small values of γ). If, however, his win preference becomes too large, i.e., $\gamma > \gamma''$, the sugar daddy induces his club to spend so much on talent that the league becomes even less balanced than the PM-league. These results are illustrated in Figure 1 which qualitatively depicts the win percentages as a function of the sugar daddy's win preference parameter.

[Figure 1 about here]

We compare social welfare in the next proposition:

Proposition 2 (Social Welfare) *(i) Social welfare increases in the type 1 SD-league with the appearance of a sugar daddy and is higher than in the PM-league as long as the win preference of the sugar daddy is not too large. The welfare-maximizing level of the win preference of the sugar daddy in the type 1 SD-league is given by* $\gamma^W \equiv [\sigma m(\sigma-1)] / [2(\sigma+1)]$.

(ii) Social welfare decreases in the type 2 SD-league with the appearance of the sugar daddy and is always lower than in the PM-league.

Proof See Appendix A.5.

Part (i) shows that the appearance of a sugar daddy can increase social welfare. This is the case in the type 1 SD-league when the sugar daddy invests in the large-market club. We know from Section 3.1 that the PM-league is too balanced from a welfare perspective. A league in which the large-market club wins more often and the small-market club wins less often would be socially desirable. This is exactly what happens in the type 1 SD-league. The sugar daddy induces the large (small) club to increase (decrease) its win percentage such that social welfare increases. In particular, there exists a hump-shaped relationship between the win preference of the sugar daddy, γ, and the welfare of that league. That is, social welfare increases with a higher win preference of the sugar daddy until the welfare maximum is reached for $\gamma = \gamma^W$. Note that if $\gamma = \gamma^W$, then $w_1^{SD1} = \sigma/(\sigma+1)$, $w_2^{SD2} = 1/(\sigma+1)$ and thus corresponds to the welfare-maximizing win percentages given by equation (5). If the sugar daddy becomes too win-orientated with $\gamma > \gamma^W$ social welfare starts to decrease and can even be lower than in the PM-league.

[Figure 2 about here]

In Figure 2, we compare social welfare in the PM-league with social welfare in the type 1 SD-league. The parameters are chosen as $m = 5$ and $\sigma = 2$. Maximized social welfare is indicated as a benchmark. According to the simulation and consistent with the theoretical derivations, there exists one parameter $\gamma^W = 5/3$ for which the type 1 SD-league attains the social welfare maximum. Social welfare is lower in the profit-maximizing league than in the type 1 SD-league as long as the sugar daddy's win-orientation is not too high.

Part (ii) shows that it is socially undesirable for the sugar daddy to invest in the small-market club because he induces the "wrong" club to increase its win percentage. It follows that social welfare will be lower in the type 2 SD-league than in the PM-league independent of the win preference γ. Note, however, that there exists a win preference γ such that the league's competitive balance corresponds to the welfare-maximizing level of competitive balance CB^W given by equation (6). Formally, if $\gamma = \gamma''' \equiv [m(\sigma^3-1)] / [2(\sigma+1)]$, then $w_1^{SD2} = w_2^W$ and $w_2^{SD2} = w_1^W$ (see Figure 1). However, in this case, the "wrong" club - namely the small-market club - wins too often such that social welfare is not maximized.[12]

[12]An extension would be a league with two sugar daddies. In such a pure SD-league, the win ratio

4 The Effect of Revenue Sharing in the SD-League

In this section, we analyze the effect of revenue sharing on competitive balance and social welfare in the SD-league. Revenue sharing plays an important role in the redistribution of revenue and has long been accepted as an exemption from antitrust law.[13] The basic idea behind this cross-subsidization policy is to guarantee a reasonable competitive balance in the league by redistributing revenues from large-market clubs to small-market clubs because large-market clubs have a higher revenue-generating potential than do small-market clubs [Késenne, 2000, Szymanski and Késenne, 2004]. Current revenue-sharing schemes vary widely among professional sports leagues around the world. The most prominent may be the scheme operated by the National Football League (NFL), in which the visiting club receives 40% of the locally-earned television and gate receipt revenues. In some European football leagues, the system for sharing the revenue generated by a centralized broadcast contract resembles a pool revenue-sharing system (e.g., in the English Premier League or the German Bundesliga).

In our model, we introduce a pool revenue-sharing system. Under a pool-sharing arrangement, club i receives an α-share of its revenue R_i and an $(1-\alpha)/2$-share of the league revenue pool $R_i + R_j$.[14] The after-sharing revenues of club i, denoted by R_i^* can be written as:

$$R_i^* = \alpha R_i + \frac{(1-\alpha)}{2}(R_i + R_j),$$

with $\alpha \in (0, 1]$ and $i, j \in \{1, 2\}$, $i \neq j$. Note that a higher parameter α represents a league with a lower degree of redistribution. Thus, the limiting case of $\alpha = 1$ describes a league without revenue sharing.

The maximization problem of club $i \in \{1, 2\}$ in which the sugar daddy has invested is thus given by

$$\max_{x_i \geq 0} \left\{ u_i^*(x_i, x_j) = R_i^*(x_i, x_j) - cx_i + \gamma w_i(x_i, x_j) \right\},$$

and for the profit-maximizing club $j \in \{1, 2\}$, $j \neq i$ it is given by

$$\max_{x_j \geq 0} \left\{ \pi_j^*(x_i, x_j) = R_j^*(x_i, x_j) - cx_j \right\}.$$

We establish the following proposition:

---

w_1/w_2 in equilibrium is given by $(\sigma m+2\gamma)^{1/2}/(m+2\gamma)^{1/2}$. It follows that competitive balance increases with higher win preferences of the sugar daddies. Moreover, one can show that the pure SD-league is more competitively balanced and thus yields a lower level of social welfare than the PM-league. This result is consistent with the findings in Vrooman [1997, 2000], who shows that the sportsman league is more balanced than the PM-league.

[13] Professional team sports leagues often find themselves under antitrust surveillance [Flynn and Gilbert, 2001]. Most revenue-sharing arrangements, however, have not been challenged in the courts because revenue sharing is supposed to enhance competitive balance and thus is in the interest of the consumer [Szymanski, 2003].

[14] Note that our results are robust with respect to gate revenue-sharing, which is another popular form of revenue-sharing in sports leagues. From the home match, club i obtains share α of its own revenues R_i, and from the away match, it obtains share $(1-\alpha)$ of club j's revenues R_j. In this case, the after-sharing revenues of club i are given by $R_i^* = \alpha R_i + (1-\alpha)R_j$ with $\alpha \in (0.5, 1]$.

Proposition 3 (Revenue Sharing) *(i) Greater revenue sharing decreases competitive balance in both types of SD-leagues.*

(ii) The welfare effect of revenue sharing is ambiguous and depends on whether the sugar daddy invests in the large-market or the small-market club. In particular, revenue sharing increases social welfare in the type 1 SD-league until the welfare optimum is reached for $\alpha = \alpha^W \equiv [2\gamma(\sigma+1)] / [m\sigma(\sigma-1)]$. *In the type 2 SD-league, revenue sharing increases social welfare if and only if the sugar daddy's win preference* γ *is sufficiently small with* $\gamma < \gamma'$.

Proof See Appendix A.6.

Part (i) of the proposition shows that greater revenue sharing produces a less balanced league independent of whether the sugar daddy invests in the small-market or large-market club. That is, greater revenue sharing (a lower parameter α) results in a higher win percentage for the dominant team and a lower win percentage for the subordinate team. Remember that in the type 1 SD-league, the large-market club is always the dominant team in equilibrium, whereas in the type 2 SD-league, the large-market club is the dominant team in equilibrium if and only if $\gamma < \gamma'$.[15] The result of Part (i) is due to the so-called "dulling effect" of revenue sharing as introduced by Szymanski and Késenne [2004]. The dulling effect describes the well-known finding in the sports economics literature that revenue sharing reduces incentives to invest in playing talent. This is because the marginal benefit of an investment has to be shared with the other club through the revenue-sharing arrangement. Due to the logit formulation of the CSF the negative effect of revenue sharing on marginal revenue is stronger for the underdog than for the dominant team.[16] As a result, the underdog will decrease its investment level relatively more than the dominant team such that the competitive balance in the league decreases through revenue sharing.

The theoretical literature in sports economics regarding the effect of revenue sharing on the competitive balance in sports leagues can be grouped along two dimensions of assumptions: profit- versus win-maximizing behavior and fixed versus flexible supply of talent (i.e., closed versus open leagues). According to this categorization, the invariance proposition with respect to revenue sharing is derived under the assumptions of profit-maximizing club owners and a fixed supply of talent [El-Hodiri and Quirk, 1971, Fort and Quirk, 1995]. There is wide agreement that the invariance proposition does not hold in leagues with either win-maximizing club owners or a flexible supply of talent [Késenne, 2000, Szymanski, 2003, Vrooman, 2007, 2008, Grossmann, Dietl,

[15] Note that in standard, static contest models without a sugar daddy, the large club is always the dominant team at equilibrium. In a dynamic setting, however, this may no longer hold. For example, Grossmann and Dietl [2009] show in a two-period contest model of a PM-league that an equilibrium exists in which the small club invests more than the large club in both periods.

[16] The dulling effect is stronger for the underdog than for the dominant team because the (positive) marginal impact on the dominant team's revenues of a reduction in talent investments by the underdog is greater than the (positive) marginal impact on the underdog's revenues of a reduction in talent investments by the dominant team.

and Lang, 2010]. There is disagreement, however, as to whether the invariance proposition holds in a league with profit-maximizing club owners and a fixed talent supply. El-Hodiri and Quirk [1971] and Fort and Quirk [1995] show that the invariance proposition does hold with respect to revenue sharing, whereas Szymanski and Késenne [2004] conclude that revenue sharing results in a more uneven distribution of talent and thus contradict the invariance proposition. Because all of these models use the same assumptions, namely, profit-maximizing club owners and a fixed supply of talent, the contradiction results from methodological differences. El-Hodiri and Quirk and Fort and Quirk utilize Walrasian conjectures, whereas Szymanski and Késenne employ Contest-Nash conjectures. Our model shows that the invariance proposition with regard to revenue sharing does not hold in a league with a profit-maximizing club and a sugar daddy club under the Contest-Nash conjectures.

Part (ii) of the proposition shows that the welfare effect of revenue sharing crucially depends on the type of SD-league and the magnitude of the sugar daddy's win preference. In the type 1 SD-league, a higher degree of revenue sharing yields an increase in social welfare. This is because greater revenue sharing induces the large-market club 1 to increase its win percentage in equilibrium, whereas the small-market club 2 decreases its win percentage. As we know from Section 3.3, social welfare increases in the type 1 SD league if the large (small) club increases (decreases) its win percentage. The welfare-maximizing level of revenue sharing is then reached at a level of revenue sharing given by $\alpha = \alpha^W$.

In the type 2 SD-league, social welfare increases through a higher degree of revenue sharing only if the large-market club is also the dominant team in equilibrium. This is the case, if the win preference of the sugar daddy is sufficiently small ($\gamma < \gamma'$). In this case, the win percentages of both clubs approach the welfare-maximizing win percentages (w_1^W, w_2^W) through a lower small-market club parameter α. In contrast, if $\gamma > \gamma'$, then the small-market club is the dominant team in equilibrium and revenue sharing results in a higher win percentage for this club and a lower win percentage for the large-market club. This is detrimental to social welfare, however.

5 Conclusion

There have always been rich individuals who financially support football clubs. Recently, some extraordinarily rich people have invested great sums of money into clubs and become club owners with full control. These so-called sugar daddies have changed the face of football. With the appearance of a sugar daddy within a league, several questions may arise. In particular, how does the appearance of a sugar daddy change the competition (competitive balance) in the league, and what is its effect on social welfare? How do cross-subsidization schemes such as revenue sharing work in a sugar daddy league, and what is their welfare effect?

In this paper, we try to answer these questions by extending the existing "sportsman" literature through a welfare analysis based on a contest model of a professional team sports league. In particular, we compare a league in which both clubs are profit maximizers (PM-league) with a league in which one club is a profit maximizer and the other club is owned by a sugar daddy (SD-league). We distinguish between two cases: the sugar daddy invests in a large-market club (type 1 SD-league) and a small-market club (type 2 SD-league).

Our analysis shows that the type 1 SD-league is always less balanced than the PM-league. When the sugar daddy invests in the large-market club, this club will become dominant within the league. In the type 2 SD-league, however, competitive balance can be higher or lower than in the PM-league depending on the win preference of the sugar daddy. Moreover, we find that social welfare in the type 1 SD-league can be higher or lower than in the PM-league. With the appearance of a sugar daddy who invests in a large-market club (type 1 SD-league), social welfare is initially higher than in the PM-league. If, however, the win preference of the sugar daddy increases above a certain threshold, welfare will decrease and may even fall below the welfare level of the PM-league. Alternatively, a sugar daddy who invests in a small-market club (type 2 SD-league) will always induce lower welfare than in the PM-league. From a welfare perspective, a sugar daddy should thus always invest in the club with the larger market size in the league.

We further show that pool revenue sharing in a SD-league will always decrease competitive balance independent of whether the sugar daddy has invested in a small-market or a large-market club. This result shows that the invariance proposition with regard to revenue sharing does not hold in a league with one profit-maximizing club and one club that is owned by a sugar daddy. The welfare effect of pool revenue sharing in a sugar daddy league, however, is ambiguous and depends on the clubs' market sizes and on the win preference of the sugar daddy. In particular, the introduction of pool revenue sharing always increases social welfare in the type 1 SD-league. In the type 2 SD-league, however, pool revenue sharing only increases social welfare if the sugar daddy's win preference is below a certain threshold.

Our study can be seen as a further step in the analysis of the interesting real-world phenomenon of sugar daddies and their effects. We encourage further research in this area. For example, one promising avenue for further research might be the analysis of salary restrictions in a sugar daddy league and their effects on competitive balance, club profits, and social welfare.

Figures

Figure 1

The Effect of the Win Preference on Win Percentages

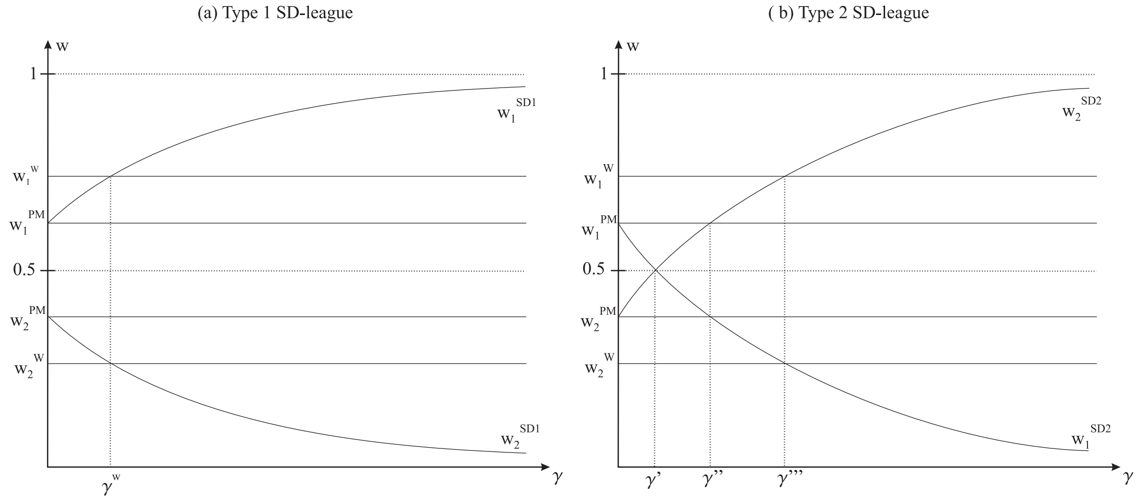
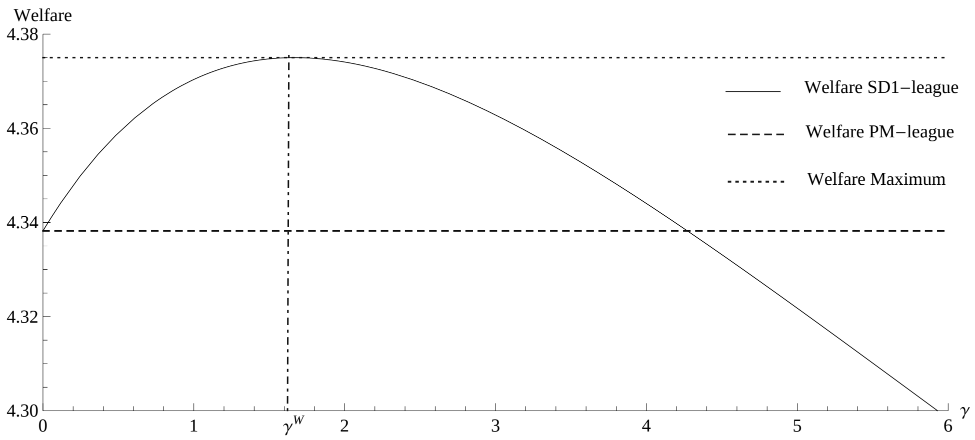


Figure 2

The Welfare Effect of the Sugar Daddy



Appendix

A.1 Proof of Lemma 1

Social welfare $W(w_1, w_2) = [(3m)/8]\,[\sigma q_1(w_1, w_2) + q_2(w_1, w_2)]$ with $q_i = w_i + w_i(1 - w_i)$ can be expressed in terms of $w_1 = (1 - w_2)$ as

$$W(w_1) = \frac{3m}{8}\left[\sigma(2w_1 - w_1^2) + (1 - w_1^2)\right].$$

Maximizing social welfare with respect to w_1 yields

$$w_1^W = \frac{\sigma}{\sigma + 1} = \arg\max_{w_1 \geq 0} W(w_1).$$

It follows that

$$w_2^W = (1 - w_1^W) = \frac{1}{\sigma + 1}.$$

The welfare-maximizing level of competitive balance CB^W is then given by $w_1^W w_2^W$.

A.2 Proof of Lemma 2

From the first-order conditions

$$\frac{\partial \pi_1}{\partial x_1} = \frac{\sigma m x_2^2}{2(x_1 + x_2)^3} - c = 0 \text{ and } \frac{\partial \pi_2}{\partial x_2} = \frac{m x_1^2}{2(x_1 + x_2)^3} - c = 0,$$

we derive that in equilibrium (x_1^{PM}, x_2^{PM}) it must hold that $\sigma = (x_1^{PM})^2/(x_2^{PM})^2$. It follows that the equilibrium win percentages in the PM-league amount to

$$(w_1^{PM}, w_2^{PM}) = \left(\frac{\sqrt{\sigma}}{1 + \sqrt{\sigma}}, \frac{1}{1 + \sqrt{\sigma}}\right).$$

A.3 Proof of Lemma 3

Recall that $m_1 = \sigma m$ and $m_2 = m$. From the first-order conditions

$$\frac{\partial u_i}{\partial x_i} = \frac{x_j(m_i x_j + 2\gamma(x_i + x_j))}{2(x_i + x_j)^3} - c = 0 \text{ and } \frac{\partial \pi_j}{\partial x_j} = \frac{m_j x_i^2}{2(x_i + x_j)^3} - c = 0,$$

we derive that in equilibrium (x_i^{SD}, x_j^{SD}) it must hold that

$$\frac{\partial u_i}{\partial x_i} - \frac{\partial \pi_j}{\partial x_j} = \frac{-m_j\left(x_i^{SD}\right)^2 + 2\gamma x_i^{SD} x_j^{SD} + \left(x_j^{SD}\right)^2 [m_i + 2\gamma]}{2(x_i^{SD} + x_j^{SD})^3} = 0.$$

It follows that

$$x_i^{SD} = \frac{\gamma + (\gamma^2 + 2m_j\gamma + m_i m_j)^{1/2}}{m_j} x_j^{SD},$$

such that the equilibrium win percentages in the SD-league amount to

$$(w_i^{SD}, w_j^{SD}) = \left(\frac{\gamma + \kappa}{\gamma + m_j + \kappa}, \frac{m_j}{\gamma + m_j + \kappa} \right)$$

with $\kappa \equiv (\gamma^2 + 2m_j\gamma + m_i m_j)^{1/2}$. Thus, in the type 1 SD-league ($i = 1$ and $j = 2$)

$$(w_1^{SD1}, w_2^{SD1}) = \left(\frac{\gamma + \kappa_1}{\gamma + m + \kappa_1}, \frac{m}{\gamma + m + \kappa_1} \right) \text{ with } \kappa_1 \equiv \left(\gamma^2 + 2m\gamma + \sigma m^2\right)^{1/2},$$

and in the type 2 SD-league ($i = 2$ and $j = 1$)

$$(w_1^{SD2}, w_2^{SD2}) = \left(\frac{\sigma m}{\gamma + \sigma m + \kappa_2}, \frac{\gamma + \kappa_2}{\gamma + \sigma m + \kappa_2} \right) \text{ with } \kappa_2 \equiv \left(\gamma^2 + 2\sigma m\gamma + \sigma m^2\right)^{1/2}.$$

A.4 Proof of Proposition 1

Type 1 SD-league (the sugar daddy invests in the large-market club): In this case, the sugar daddy amplifies the clubs' investment inequality, such that the type 1 SD-league becomes increasingly unbalanced compared to the PM-league. Formally, $\partial w_1^{SD1}/\partial\gamma > 0$ and $\partial w_2^{SD1}/\partial\gamma < 0$.

Type 2 SD-league (the sugar daddy invests in the small-market club): We derive that if $\gamma = \gamma'' \equiv m/\left[2(\sigma + 1)(\sigma^{1/2} - 1)\right]$ then $w_1^{SD2} = w_2^{PM}$ and $w_2^{SD2} = w_1^{PM}$. In this case, the small-market club 2 is the dominant team and has the same win percentage as the large-market club 1 from the PM-league. If $\gamma > \gamma''$ then club 1 has an even higher win percentage such that the type 2 SD-league is even more unbalanced than the PM-league. If $\gamma < \gamma''$ then the type 2 SD-league is more balanced than the PM-league.

A.5 Proof of Proposition 2

ad (i) Recall that the PM-league is too balanced compared to the social welfare maximum and a higher value of γ in the type 1 SD-league leads to higher imbalance, social welfare increases in the type 1 SD-league through the appearance of a sugar daddy. However, this holds true only if the win preference of the sugar daddy is not too large. In particular, there exists a welfare-maximizing win preference γ^W, computed as follows:

$$w_1^W = w_1^{SD1} \Leftrightarrow \frac{\sigma}{\sigma + 1} = \frac{\gamma + \kappa_1}{\gamma + m + \kappa_1} \Leftrightarrow \gamma = \gamma^W \equiv \frac{\sigma m(\sigma - 1)}{2(\sigma + 1)}.$$

By increasing the win preference above γ^W, social welfare starts to decrease and can even be lower than in the PM-league.

ad (ii) In the type 2 SD-league, the sugar daddy decreases the imbalance for a small value of γ such that social welfare decreases. For large values of γ, the small-market club may even have a higher win percentage than the large-market club and possibly the imbalance increases. In this case, however, the "wrong" club (i.e., the small-market club) has a higher win percentage, such that social welfare decreases even if the imbalance increases.

A.6 Proof of Proposition 3

The first-order conditions are derived as[17]

$$\begin{aligned}\frac{\partial u_i^*}{\partial x_i} &= \alpha\frac{\partial R_i}{\partial w_i}\frac{\partial w_i}{\partial x_i}+\frac{1-\alpha}{2}\left(\frac{\partial R_i}{\partial w_i}\frac{\partial w_i}{\partial x_i}+\frac{\partial R_j}{\partial w_j}\frac{\partial w_j}{\partial x_i}\right)-c+\gamma\frac{\partial w_i}{\partial x_i}=0,\\ \frac{\partial \pi_j^*}{\partial x_j} &= \alpha\frac{\partial R_j}{\partial w_j}\frac{\partial w_j}{\partial x_j}+\frac{1-\alpha}{2}\left(\frac{\partial R_i}{\partial w_i}\frac{\partial w_i}{\partial x_j}+\frac{\partial R_j}{\partial w_j}\frac{\partial w_j}{\partial x_j}\right)-c=0,\end{aligned}$$

with $i,j \in \{1,2\}$ and $i \neq j$. By combining the first-order conditions and using the adding-up constraint $(\partial w_i)/(\partial x_i) = -(\partial w_j)/(\partial x_i)$, we obtain

$$\left[\alpha\frac{\partial R_i}{\partial w_i}-\frac{1-\alpha}{2}\left(\frac{\partial R_j}{\partial w_j}-\frac{\partial R_i}{\partial w_i}\right)+\gamma\right]\frac{\partial w_i}{\partial x_i}=\left[\alpha\frac{\partial R_j}{\partial w_j}-\frac{1-\alpha}{2}\left(\frac{\partial R_i}{\partial w_i}-\frac{\partial R_j}{\partial w_j}\right)\right]\frac{\partial w_j}{\partial x_j}.$$

Thus, in equilibrium $(\widehat{x}_i^{SD}, \widehat{x}_j^{SD})$ it must hold that

$$\widehat{x}_i^{SD} = \frac{(1-\alpha)(m_i-m_j)+4\gamma+\widehat{\kappa}}{2(1+\alpha)m_j}\widehat{x}_j^{SD},$$

with $\widehat{\kappa} \equiv \left[4m_j(1+\alpha)\left[(1+\alpha)m_i+4\gamma\right]+(m_i(1-\alpha)-m_j(1-\alpha)+4\gamma)^2\right]^{1/2}$. It follows that the equilibrium win percentages in the SD-league with revenue sharing are given by

$$\begin{aligned}\widehat{w}_i^{SD} &= \frac{(1-\alpha)(m_i-m_j)+4\gamma+\widehat{\kappa}}{(1-\alpha)(m_i-m_j)+4\gamma+\widehat{\kappa}+2(1+\alpha)m_j},\\ \widehat{w}_j^{SD} &= \frac{2(1+\alpha)m_j}{(1-\alpha)(m_i-m_j)+4\gamma+\widehat{\kappa}+2(1+\alpha)m_j}.\end{aligned}$$

Moreover, we calculate

$$\widehat{w}_i^{SD} = w_i^W = \frac{m_i}{m_i+m_j} \Leftrightarrow \alpha = \alpha^W \equiv \frac{2\gamma(m_i+m_j)}{m_i(m_i-m_j)}.$$

Thus, the welfare-maximizing win percentages coincide with the win percentage in the SD-league if $\alpha = \alpha^W$.

We further compute the partial derivative of $\widehat{w}_i^{SD}$ with respect to α at $\alpha = 1$ and derive

$$\left.\frac{\partial \widehat{w}_i^{SD}}{\partial \alpha}\right|_{\alpha=1} > 0 \Leftrightarrow \gamma \in \left(-\frac{m_i}{2}, \frac{1}{4}(m_j-m_i)\right).$$

Recall that in the type 1 SD-league the sugar daddy invests in the large-market club (i.e., $i = 1$ and $j = 2$), while in the type 2 SD-league the sugar daddy invests in the small-market club (i.e., $i = 2$ and $j = 1$).

(a) In the type 1 SD-league with $i = 1$ and $j = 2$, we conclude that $(\partial \widehat{w}_1^{SD1})/(\partial\alpha)\mid_{\alpha=1} < 0$ for all $\gamma > 0$.

[17] It is easy to verify that the second-order conditions for a maximum are satisfied.

(b) In the type 2 SD-league with $i = 2$ and $j = 1$, we conclude that $(\partial \widehat{w}_2^{SD2})/(\partial \alpha) \mid_{\alpha=1} > 0$ for $\gamma \in (0, \gamma')$ and $(\partial \widehat{w}_2^{SD2})/(\partial \alpha) \mid_{\alpha=1} < 0$ for $\gamma > \gamma'$ with $\gamma' = (m/4)(\sigma - 1)$. Remember that γ' is the threshold parameter above which the small-market club 2 is the dominant team in equilibrium in the type 2 SD-league (Lemma 4).

From (a), we deduce that the introduction of revenue sharing increases the win percentage of the large-market club 1 and consequently decreases the win percentage of the small-market club 2. As a result, competitive balance decreases.

From (b), we deduce that the introduction of revenue sharing decreases the win percentage of the small-market club 2 and consequently increases the win percentage of the large-market club 1 if and only if $\gamma \in (0, \gamma')$. As a result, competitive balance decreases in this case. If, however, $\gamma > \gamma'$, the small-market club 2 is the dominant team in equilibrium and revenue sharing increases this club's win percentage. The result is again a decrease in competitive balance. Numerical simulations have shown that our claims hold for all parameters $\alpha \in (0, 1]$.[18]

To prove that revenue sharing has a negative effect on marginal revenue, we compute the partial derivative of club i's marginal revenue $MR_i = \partial R_i^* / \partial x_i$ with respect to the revenue-sharing parameter α as:

$$\frac{\partial MR_i}{\partial \alpha} = \frac{x_j}{(x_1 + x_2)^2}(m_i(1 - w_i) + m_j w_i) > 0,$$

with $i, j \in \{1, 2\}$ and $i \neq j$. It follows that a higher degree of revenue sharing (i.e., a lower parameter α) has a negative effect on marginal revenue.

The proof of part (ii) is given verbally in Section 4 below Proposition 3. Note that in the type 1 SD-league, the welfare-maximizing degree of revenue sharing α^W is given by $\alpha^W = [2\gamma(\sigma + 1)] / [m\sigma(\sigma - 1)]$.

[18] Detailed simulation results are available from the corresponding author upon request.

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